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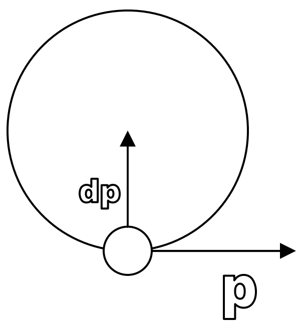
Physics 1 – Demonstration

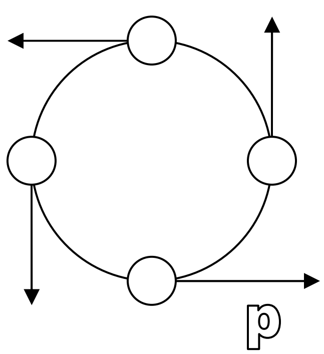
Problem: An object is moving with constant speed in a circle. Infer the $\frac{\Delta p_{\perp}}{\Delta t}$**.**

Challenge: Use no calculus, no acceleration, no rotational motion concepts.

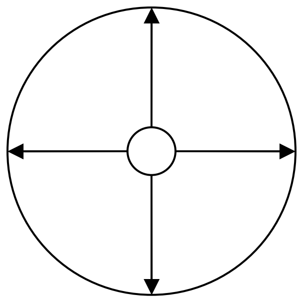
First, let me draw a picture of this situation:



I know that the change in momentum $\Delta\vec{p}$ must be towards the center, since if it was not towards the center my object would be speeding up or slowing down (contrary to the assumptions). But how much towards the center is $\left|\frac{\Delta\vec{p}}{\Delta t}\right| = \frac{\Delta p_{\perp}}{\Delta t}$? Let me draw two pictures. First, consider how much the momentum changes in total (the "arc length") over one revolution. Draw a picture:



So I see that the momentum vectors trace out a full circle in one revolution: that is, looking at the momentum vectors,



Since $p = mv$ is the radius of this circle, its circumference is $\Delta p_{\perp} = 2\pi mv$ in one revolution.

Now consider the time it takes for one revolution. The particle is traveling at a constant speed v, and it is traveling a distance equal to the circumference of the circle $2\pi r$. Dividing distance by speed, I get time: $\Delta t = \dfrac{2\pi r}{v}$. Now putting these together, I have:

$$\frac{\Delta p_{\perp}}{\Delta t} = \frac{2\pi mv}{\frac{2\pi r}{v}} = \frac{mv^2}{r} = \frac{v}{r}p.$$