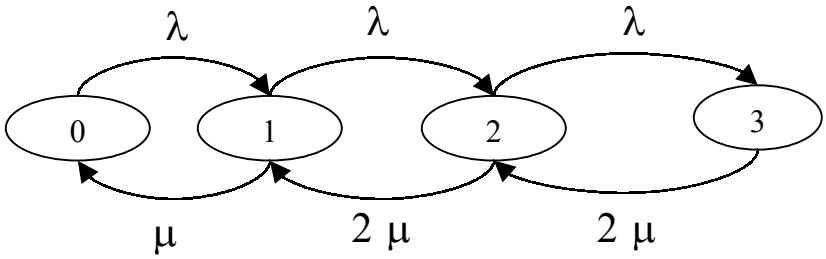
QUIZ 9_SOLUTION KEY

Customers arrive at a two-server system at a Poisson rate λ. An arrival finding the system empty is equally likely to enter service with either server. An arrival finding one customer in the system will enter service with the idle server. An arrival finding two others in the system will wait in line for the first free server. An arrival finding three in the system will not enter. All service times are exponential with rate μ, and once a customer is served (by either server), he departs the system.

(a) Define the states and write down the transition-rate diagram. Is this model similar to the ones that we have studied so far?

Let's define the state of the system as the number of customers in the system at any time. Then, the state space is given by M={0,1,2,3}.



In fact, this is a Markovian M/M/2/3 queuing system.

Assuming that you have already calculated the long-run probabilities, answer the questions below.

(b) Suppose a customer arrives and finds two others in the system What is the expected time he spends in the system?

$$(2\mu)^{-1} + (\mu)^{-1}$$

(c) What proportion of customers enter the system?

$$(1-\pi_3)$$

(d) What is the average time an entering customer spends in the system?

$$w = [\ 0*\pi_0 + 1*\pi_1 + 2*\pi_2 + 3*\pi_3\ ]/[\ \lambda*(1-\pi_3)\ ]$$