Quiz 7

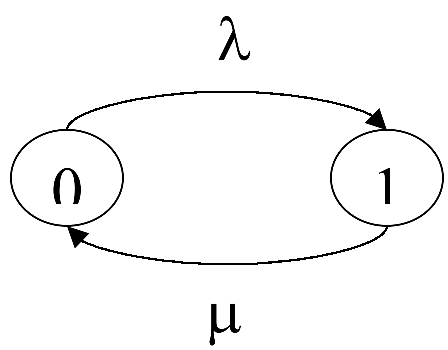
Consider a single machine-station with its repairman. When the machine fails, the repairman repairs and it goes back to the production again. During this time, there is no backup machine to replace the failed machine and therefore, the station would be done.

The time to failure is well-modeled by an exponential distribution with rate 1 per hour and the repair time is well-modeled by an exponential distribution with rate 2 per hour.

<u>Calculate the long-run fraction of time the repairman would be busy.</u>

We will try to formulate this problem through a queuing model:

Consider the failure of a machine as an arrival and the repair of a machine as a service. If we define the system state S_n as the number of customers in the system just after the n^{th} system event, then we end up with the following transition rate diagram:



where $\lambda = 1$ per hour and $\mu = 2$ per hour. Next, we will read the information on this diagram into our balance equations:

$$\text{Rate into state } 0 = \text{Rate out of state } 0$$
$$\pi_1 \mu = \pi_0 \lambda$$

This equation is what we would get if we had tried to set the rate into and out of state 1 equal to each other. We also need $\pi_0 + \pi_1 = 1$. Then, we have two linear equations to solve in two unknowns:

$$\pi_1 \mu = \pi_0 \lambda$$
$$\pi_0 + \pi_1 = 1$$

Then, $\pi_0 = 2/3$ and $\pi_1 = 1/3$. We are asked what the long-run fraction of time the server is busy is. The server is busy when there is a down machine in our queuing system; therefore, the answer is given by π_1, which 1/3.