

QUIZ 6: SOLUTION KEY

Problem Consider a customer who buys beer at regular intervals, say once a week, and every time he has to decide among three brands: A, B, and C. From his buying records so far it has been determined that his brand choice in week $n+1$ depends only on his brand choice in week n , regardless of previous purchases.

From a sample of purchases, representing a set of brand choices the customer has made, the following count data were collected:

next brand choice		A	B	C
	A	20	40	90
	B	30	40	20
current brand choice	C	15	90	30

Construct a Markov chain model for this problem. Define your state space, time index, and estimate the transition matrix.

Answer

Let S_n be the brand he purchases in week n . Then it is obvious that $\{ S_n, n \geq 0 \}$ is discrete-time Markov chain with state space $M = \{ A, B, C \}$. Its one-step transition probability matrix will look as follows:

$$P = \begin{pmatrix} \frac{20}{20+40+90} & \frac{40}{20+40+90} & \frac{90}{20+40+90} \\ \frac{30}{30+40+20} & \frac{40}{30+40+20} & \frac{20}{30+40+20} \\ \frac{15}{15+90+30} & \frac{90}{15+90+30} & \frac{30}{15+90+30} \end{pmatrix}$$

Thus if he purchases beer of brand A in week n , he will purchase beer of brand C in week $n+1$ with probability 0.6 ($=90/150$), etc. Such ***brand-switching models*** are used quite often in practice by industries to predict market shares, etc. A major effort is involved in obtaining the transition probabilities – this involves observing a large number of customers in a carefully controlled environment.

If the arm were currently