

Quiz 2

Let X be a random variable with probability density

$$f(x) = \begin{cases} c(1-x^2), & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) What is the value of c ?
- (b) What is the cumulative distribution function of X ?

Solution

- (a) Here is the key:

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{1}$$

You can develop the intuition as follows: The cumulative distribution is basically a *probability* whose maximum value is one. Since the total area under the probability density function gives the cumulative distribution function, you would expect the equality (1) to hold.

Then,

$$\int_{-1}^1 c(1-x^2) dx = c \frac{4}{3} = 1 \rightarrow c = \frac{3}{4}.$$

- (b)

$$F(x) = \begin{cases} 0, & x \leq -1 \\ \int_{-1}^x \frac{3}{4}(1-y^2)dy = \frac{3}{4}x \left(1 - \frac{x^2}{3}\right) + \frac{1}{2}, & -1 < x < 1 \\ 1, & x \geq 1 \end{cases}$$