Example Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.

If we let the state at time n depend only on whether or not it is raining at time n, then the above model is not a Markov chain. Why? Because the Markov property requires that what the weather is tomorrow depends only on the weather today, but in this example, the weather tomorrow depends on the today's and yesterday's weather. However, we can transform the above model into a Markov chain by saying that the state at any time is determined by the weather conditions during both that day and the previous day. In other words, we can say that the process is in

state 0 if it rained both today and yesterday,
state 1 if it rained today and but not yesterday,
state 2 if it rained yesterday but not today,
state 3 if it did not rain either yesterday or today.

The preceding would then represent a four-state Markov chain having the following transition probability matrix:

$$P = \begin{bmatrix} 0.7 & 0 & 0.3 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.4 & 0 & 0.6 \\ 0 & 0.2 & 0 & 0.8 \end{bmatrix}$$