PROJECT HINTS

- Is a simulation part of the project?

Your project must include a mathematical queueing model of the type being covered in Chapter 8; it may also include a simulation analysis, if that seems appropriate.

- What is a "sensitivity analysis?"

When you estimate parameters for your queuing model, such as the arrival rate λ, there is error in the estimate since it is based only on a sample. We have been measuring this error by the *standard error (se)*.

Thus, it is important to evaluate the effect of this potential estimation error on the results of your queuing analysis. One approach is to recompute and reevaluate the results at *+se* and *-se* from the parameter estimate to see how your conclusions might change. Note that in some situations only a one sided sensitivity analysis is needed; for example, λ+se and μ-se is often the worst case (most congested).

- How to assess the homogeneity of different data sets?

Sometimes an analyst collects *k* sets of observations on a random phenomenon independently and would like to know whether these data sets are homogeneous and thus can be merged. For example, it might be of interest to know whether service times of customers in a bank collected on different days are homogeneous. If they are, then the service times from the different days can be merged and the combined sample used to find the service-time distribution. Otherwise, more than one service-time distribution is needed. Next, there is the presentation of the Kruskal-Wallis hypothesis test for homogeneity. It is a non-parametric test since no assumptions are made about the distributions of the data.

Suppose that we have k independent samples of possibly different sizes, and that the samples themselves are independent. Denote the i^{th} sample of size n_i by

$$X_{i1}, X_{i2}, \ldots, X_{in_i}$$

for $i = 1,2, \ldots, k$, and let n denote the total number of observations,

$$n = \sum_{i=1}^{k} n_i$$

The, we would like to test the null hypothesis

H_0 : All of the population distribution functions are identical against the alternative hypothesis.

H_1 : At least one of the populations tends to yield larger observations than at least one of the other populations.

To construct the Kruskal-Wallis (K-W) statistic, assign rank 1 to the smallest of the n observations, rank 2 to the second smallest, and so on to the largest of the n observations, which receives rank n. Let $R(X_{ij})$ represent the rank assigned to X_{ij} and let R_i be the sum of the ranks assigned to the i^{th} sample, that is,

$$R_i = \sum_{j=1}^{n_i} R(X_{ij}) \; j = 1, 2, \ldots, k$$

Then the K-W test statistic T is defined as

$$T = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1)$$

We reject the null hypothesis H_0 at level α if

$$T \quad > \quad \chi^2_{k-1,1-\alpha}$$

where the term on the right-hand side is the upper 1-α critical value for a chi-square distribution with k-1 degrees of freedom. The above expression for T assumes that no two observations are equal.

Reference: Simulation Modeling & Analysis by Averill M. Law and W. David Kelton (2000).

- What if we need the chi-square statistic, but it is not listed on the chi-square table as k is very large?

We can use the approximation

$$\chi^2_{k-1,1-\alpha} \approx (k-1)\left\{1 - \frac{2}{9(k-1)} + z_{1-\alpha}\sqrt{2/\left[9(k-1)\right]}\right\}^3$$

where $z_{1-\alpha}$ is the upper 1-α critical point of the N(0,1) distribution.