BLP Model (Random Coefficient Logit Model) for Demand Estimation

(SMART 2014)

Agenda

- Overview
- Model
- Identification
- Estimation
- Nevo (2000)
- Matlab Demo

Review: Consumer-Level Demand Models

- Data: panel of I consumers purchasing one of J products over T_i periods
- Model: consumer utility function

$$U_{itj} = \beta_i' x_{tj} - \alpha_i p_{tj} + \varepsilon_{itj}$$

- Estimation principle: assuming ε_{itj} are extreme value, integrate over an individual's errors to get individual purchase probabilities

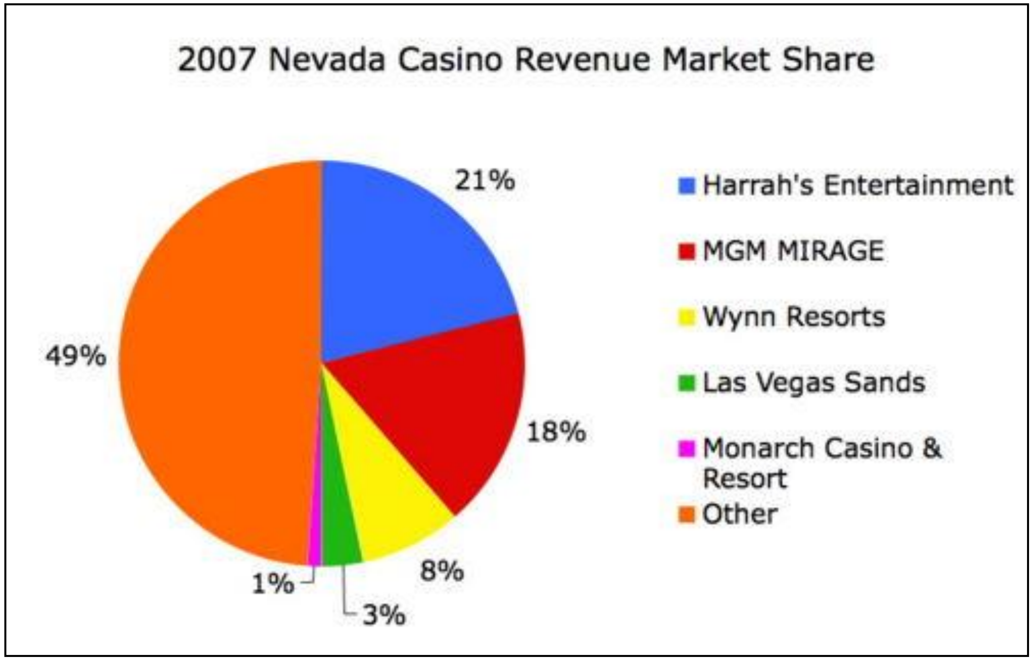
$$s_{itj} = Pr(y_{it} = j) = \frac{\exp(\beta_i' x_{tj} - \alpha_i p_{tj})}{1 + \sum_{k=1}^{J} \exp(\beta_i' x_{tk} - \alpha_i p_{tk})}$$

- Estimate using (simulated) maximum likelihood
- Usually observe large I, relatively short T_i, and small J

No Individual Data?

Aggregate Data

- Often consumer-level data is unavailable
- Instead, we have aggregate data on sales… e.g., market shares



We start with a microeconomic model of consumer behavior, then aggregate up to the population level.

-- Berry, Levinsohn, and Pakes (1995).

Overview of BLP (1995)

- Berry, Levinsohn, and Pakes (1995) or "BLP" consists of an economic model and a GMM estimator

- Demand estimation with many differentiated products
  - Product characteristics approach: products are bundles of characteristics and consumers have preferences over characteristics
  - Requires only aggregate market share data
  - Flexible substitution patterns / price elasticities
  - Controls for price endogeneity

Used extensively in marketing and industrial organization.

Overview of BLP (1995)

- Examples:
  - Infer nature of strategic interaction between manufacturers and retailers (Sudhir 2002)
  - New product introduction (Petrin 2002, Albuquerque & Bronnenberg 2009)
  - Effect of prices varying by store instead of by geographic zone (Chintagunta, Dubé & Singh 2003)
  - Retailer targeting strategies (Besanko, Dubé & Gupta 2003)
  - Optimal product launch and exit (Hitsch 2006)
  - Impact of advertising avoidance technology on TV stations and advertisers (Wilbur 2008)
  - Changes in product assortment and pricing, post merger (Draganska, Mazzeo, & Seim 2009)
  - Voter response to political advertising (Gordon & Hartmann 2010)

  - Anindya Ghose and Sang Pil Han, Estimating Demand for Mobile Applications in the New Economy, *Management Science*, forthcoming.
  - Beibei Li, Panos Ipeirotis and Anindya Ghose. Towards a Theory Model for Product Search. WWW2011 *Best Paper Award*.

Setup

- Markets/periods $t = 1, \ldots, T$
- Products $j = 0, 1, \ldots, J$, with $j = 0$ the outside good
- Consumer i makes choice $j \in J$ in market t
- Data
  - M is the market size
  - S_{tj} is *observed* market share, $S_{tj} = q_{tj}/M$, $S_{t0} = 1 - \sum_{j=1}^{J} S_{tj}$
  - x_{tj} is a $(K \times 1)$ vector of observed product characteristics
  - p_{tj} is the price of product j in market t
- Model
  - s_{tj} is *predicted* market share
  - β is a $(K \times 1)$ vector of unknown coefficients on the observed characteristics
  - α is the scalar unknown price coefficient
  - ε_{itj} is an i.i.d. unobserved random shock

Homogeneous Logit Model

- Consumer utility for $j = 1, \ldots, J$ inside products

$$U_{itj} = \underbrace{\beta' x_{tj} - \alpha p_{tj} + \xi_{tj}}_{\delta_{tj}} + \varepsilon_{itj}$$

$$U_{itj} = \delta_{tj} + \varepsilon_{itj}$$

$\Rightarrow$ Normalize $\delta_{t0} = 0$ such that $U_{it0} = \varepsilon_{it0}$

- Assuming ε_{itj} distributed i.i.d. Type I extreme value,
- Integrating out ε_{itj} yields:

$$s_{tj}(\delta_{t1}, \ldots, \delta_{tJ}) = \frac{\exp(\delta_{tj})}{1 + \sum_{k=1}^{J} \exp(\delta_{tk})}$$

$$\boxed{S_{tj} = s_{tj}\left(\beta, \alpha, \xi_{t1}, \ldots, \xi_{tJ}\right)}$$

Homogeneous Logit: Estimation (Berry 1994)

In market t, matching observed to predict shares:

$$\begin{aligned}
S_{t0} &= \frac{1}{1+\sum_{k=1}^{J}\exp(\delta_{tk})} \\
S_{t1} &= \frac{\exp(\delta_{t1})}{1+\sum_{k=1}^{J}\exp(\delta_{tk})} \\
\vdots \quad & \quad \vdots \\
S_{tJ} &= \frac{\exp(\delta_{tJ})}{1+\sum_{k=1}^{J}\exp(\delta_{tk})}
\end{aligned}$$

Takings logs yields a system of linear equations for δ_{tj}'s:

$$\begin{aligned}
\log(S_{t0}) &= 0-\log(\text{denominator}), \quad j=0 \\
\log(S_{tj}) &= \delta_{tj}-\log(\text{denominator}), \quad j=1,\ldots,J
\end{aligned}$$

$$\boxed{\Rightarrow \delta_{tj} = \log(S_{tj})-\log(S_{t0}) = \beta' x_{tj}-\alpha p_{tj}+\xi_{tj}}$$

2SLS

Problem with Homogeneous Logit Model

- Recall the elasticity in the homogeneous logit:

$$\eta_{tjk} = \frac{\partial s_{tj}}{\partial p_{tk}} \frac{p_{tk}}{s_{tj}} = \begin{cases} \alpha p_{tj}(1 - s_{tj}), & \text{if } j = k \\ -\alpha p_{tk} s_{tk}, & \text{otherwise} \end{cases}$$

- Own-price elasticities are proportional to own price: lower price implies lower elasticity, so a standard pricing model would predict higher markup for low-priced brands
- Cross-price elasticities imply substitution towards brands in proportion to shares regardless of characteristics. Cross-price elasticity of products j_1, j_2 w.r.t. product k will be identical regardless of differences in x_{j_1} and x_{j_2}

IIA Failure of MNL

Introducing Heterogeneity

- Random coefficients: ***i* represents an individual consumer *i***

$$U_{itj} = \beta_i' x_{tj} - \alpha_i p_{tj} + \xi_{tj} + \varepsilon_{itj}, \qquad \text{where } \begin{bmatrix} \beta_i \\ \alpha_i \end{bmatrix} \sim N\left(\begin{bmatrix} \bar{\beta} \\ \bar{\alpha} \end{bmatrix}, \Sigma\right)$$

- $(\beta_i, \alpha_i) \sim$ MVN with diagonal variance matrix Σ (can also allow for correlations)
- To simplify notation (and eventually estimation), easier to rewrite utility in terms of <u>mean utilities</u> and <u>deviations</u> from the mean

Mean Utility vs. Individual Deviations

- Rewrite utility be separating the means and deviations:

$$U_{itj} = \underbrace{\bar{\beta}' x_{tj} - \bar{\alpha} p_{tj} + \xi_{tj}}_{\delta_{tj}} + \underbrace{[x_{tj}, p_{tj}]' v_i \sigma}_{\mu_{itj}} + \varepsilon_{itj}, \text{ with } v_i \sim N(0,1)$$

$$U_{itj} = \underbrace{\delta_{tj}}_{\text{mean utility}} + \underbrace{\mu_{itj}(x_{tj}, p_{tj}, v_i; \theta_2) \quad + \varepsilon_{itj}}_{\text{individual deviations}}$$

- v_i – $(K+1)$ vector of unobserved individual characteristics
- Parameters to be estimated:
  - Linear: $\theta_1 = (\bar{\beta}, \bar{\alpha})$
  - Nonlinear: $\theta_2 = (\sigma)$ is a $(K+1)$ vector of characteristic-specific standard deviations

Incorporating Demographics

Homogeneous Logit:

$$U_{itj} = \beta' x_{tj} - \alpha p_{tj} + \xi_{tj} + \varepsilon_{itj}$$

Heterogeneous Logit

$$\begin{aligned} U_{itj} &= \bar{\beta}' x_{tj} - \bar{\alpha} p_{tj} + \xi_{tj} + [x_{tj}, p_{tj}]' v_i \sigma + \varepsilon_{itj} \\ & v_i \sim N(0, I) \end{aligned}$$

Heterogeneous Logit with Demographics

$$\begin{aligned} U_{itj} &= \bar{\beta}' x_{tj} - \bar{\alpha} p_{tj} + \xi_{tj} + [x_{tj}, p_{tj}]' v_i \sigma^u + [x_{tj}, p_{tj}]' d_i \sigma^o + \varepsilon_{itj} \\ & v_i \sim N(0, I), \; d_i \sim G(\cdot) \end{aligned}$$

- v_i are consumer-level *unobservable* variables
- d_i are consumer-level *observable* variables (e.g. demographics)

How exactly do we infer individual preferences from aggregate data?

BLP Identification

How do we infer individual preferences from aggregate observations?

What do we know?

- Demographic distributions!
- Differences in demographic distributions in different markets!
- Market shares in different markets!

Basic Idea: Monitor demand for similar products in different markets.

differences in demand → different demographics

BLP Identification - Example

Example: Breakfast Buffet

Rainbow Cereal vs. Wholegrain Cereal

Table A: 80% Kids, 20% Adults;
- Rainbow: 80% gone, Wholegrain: 20% gone.

Table B: 10% Kids, 90% Adults;
- Rainbow: 10% gone, Wholegrain: 90% gone.

→ **Kids favor rainbow cereal, and adults favor wholegrain!**

BLP: Aggregate Demand → Individual Preference

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- Nevo (2000)
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Overview of Estimation

$\boldsymbol{\theta_2} \rightarrow$ s $\rightarrow \delta \rightarrow \xi \rightarrow$ GMMobj

Nonlinear search over the nonlinear parameters θ_2 - will concentrate out θ_1 using the first-order conditions

Steps:

1. Given guesses for (δ, θ_2), calculate aggregate shares
2. Given θ_2, recover δ from share inversion
3. Given θ_2, compute residuals ξ, interact with Z, minimize GMM objective function

Instruments

We will see more details in Nevo (2000).

Nevo (2000, 2001)

- Nevo (2000). A Practitioner's Guide to Estimation of Random-Coefficients Logit Models of Demand. *Journal of Economics & Management Strategy*, 9(4), 513–548.

- Nevo (2001). Measuring Market Power in the Ready-to-Eat Cereal Industry. *Econometrica*, 69(2), 307-342.

Nevo (2000)

- **Data**: 24 brands of RTE cereal, 47 U.S. cities, 2 quarters (94 markets).
- **Challenges**: 1) No individual-level observations, only market-level data;
  2) Price endogeneity: price ~ brand-city-quarter demand shocks.

Nevo (2000, 2001)

TABLE A(I)

BRANDS USED FOR ESTIMATING DEMAND

All Family/ Basic Segment	Taste Enhanced Wholesome Segment	Simple Health Nutrition Segment	Kids Segment
K Corn Flakes	K Frosted Mini Wheats	K Special K	K Corn Pops
K Crispix	K Raisin Bran	GM Total	K Froot Loops
K Rice Krispies	GM Raisin Nut	P Grape Nuts	K Frosted Flakes
GM Cheerios	P Honey Bunches of Oats	N Shredded Wheat	GM Cinn Toast Crunch
GM Wheaties	P Raisin Bran		GM Honey Nut Cheerios
	Q 100% Natural		GM Kix
			GM Lucky Charms
			GM Trix
			Q CapN Crunch
			Q Life

Nevo (2000, 2001)

TABLE A(II)

SAMPLE STATISTICS

Description	Mean	Median	Std	Min	Max
Calories	137.6	120	36.32	110	220
Fat Calories (/100)	0.124	0.100	0.139	0	0.60
Sodium (% RDA/100)	0.087	0.090	0.042	0	0.150
Fiber (% RDA/100)	0.095	0.050	0.094	0	0.310
Sugar (g/100)	0.084	0.070	0.060	0	0.200
Mushy (= 1 if cereal gets soggy in milk)	0.35	—	—	0	1
Serving weight (g)	35.1	30	9.81	25	58
Income ($)	13,083	10,475	11,182	14	275,372
Age (years)	29.99	28	23.14	1	90
Child (= 1 if age < 16)	0.23	—	—	0	1

Source: Cereal boxes and samples from the CPS.

Nevo (2000, 2001)

- **Setup**:

✓ $t = 1,\ldots, T$ markets; $i = 1,\ldots, I_t$ consumers;

✓ Market definition: "*city-quarter*" combination;

✓ Market share (inside goods): Converting sales into number of servings; The potential denominator is assumed to be one serving per capita per day;

✓ Market share (outside good): One minus the sum of the inside goods market shares;

✓ Observable for a market *t*: aggregate quantities (market share), prices, and product characteristics.

Nevo (2000, 2001)

- **Utility**:

$$\boxed{\begin{array}{l} u_{ijt} = x_j \beta_i^* - \alpha_i^* p_{jt} + \xi_j + \Delta\xi_{jt} + \epsilon_{ijt}, \\ \quad i = 1, \ldots, I_t, \qquad j = 1, \ldots, J_t, \qquad t = 1, \ldots, T, \end{array}}$$

- ξ_j is brand-level mean unobservable (absorbed by brand dummy);
- $\Delta\xi_{jt}$ is a market (city-quarter) specific deviation from the mean;

$$\begin{pmatrix} \alpha_i^* \\ \beta_i^* \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \Pi D_i + \Sigma v_i, \qquad v_i \sim N(0, I_{K+1}),$$

- D_i is a *d* * 1 vector of observed demographic variables;
- Π is a *(K+1)* **d* matrix of coefficients that measure how taste characteristics vary with observed demographics;
- v_i is a *(K+1)* * 1 vector of unobserved demographic variables;
- Σ is a scaling matrix;
- v_i and D_i are independent;
- $\alpha, \beta, \Pi, \Sigma$ are the final estimates;

Nevo (2000, 2001)

$$\boxed{\begin{pmatrix} \alpha_i^* \\ \beta_i^* \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \Pi D_i + \Sigma v_i, \qquad v_i \sim N(0, I_{K+1}),}$$

$$D_i = \begin{bmatrix} D_i^{\,1} \\ D_i^{\,2} \\ \cdots \\ D_i^{\,d} \end{bmatrix} = \begin{bmatrix} Age_i \\ Income_i \\ Child_i \\ \cdots \end{bmatrix}$$

$$\Pi_{(K+1)*d} = \begin{array}{c} \\ x_1 \\ x_2 \\ \cdots \\ x_K \\ p \end{array} \begin{array}{c} \begin{array}{ccc} Age & Income & Child \ldots \end{array} \\ \begin{bmatrix} & & \\ & & \\ \vdots & \vdots & \vdots \\ & & \end{bmatrix} \end{array}$$

$$v_i = \begin{bmatrix} v^1{}_i \\ v^2{}_i \\ \cdots \\ \cdots \\ v^{K+1}{}_i \end{bmatrix} \sim N(0, I_{K+1})$$

$$\Sigma_{(K+1)*(K+1)} = \begin{bmatrix} & \cdots & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & & & 0 & & 0 \\ 0 & & 0 & & & 0 \\ 0 & \cdots & 0 & 0 & \cdots & \end{bmatrix}$$

Nevo (2000, 2001)

- **Re-write Utility (Mean + Deviation):**

$$u_{ijt} = \delta_{jt}(x_j, p_{jt}, \xi_j, \Delta\xi_{jt}; \theta_1) + \mu_{ijt}(x_j, p_{jt}, v_i, D_i; \theta_2) + \epsilon_{ijt},$$
$$\delta_{jt} = x_j\beta - \alpha p_{jt} + \xi_j + \Delta\xi_{jt}, \qquad \mu_{ijt} = [p_{jt}, x_j]' * (\Pi D_i + \Sigma v_i),$$

- δ_{jt} represents the (population) mean utility;
- $\mu_{ijt} + \epsilon_{ijt}$ a mean-zero individual-specific deviation from the mean utility;
- ϵ_{ijt} random error, i.i.d., Type I EV;

- Let $\theta = (\theta 1, \theta 2)$ be a vector containing all parameters;
- $\theta 1 = (\alpha, \beta)$ linear parameters;
- $\theta 2 = (\Pi, \Sigma)$ nonlinear parameters.

Nevo (2000, 2001)

- **Individual taste attributes:**

An individual is defined as a vector of observed and unobserved demographics and product-specific shocks, $(D_i, v_i, \epsilon_{i0t}, \ldots, \epsilon_{iJt})$.

- **Set of individuals who will choose brand *j* in market *t***

$$A_{jt}(x_{\cdot t}, p_{\cdot t}, \delta_{\cdot t}; \theta_2) = \left\{(D_i, v_i, \varepsilon_{i0t}, \ldots, \varepsilon_{iJt}) | u_{ijt} \geq u_{ilt}\ ,\ \forall l = 0, 1, \ldots, J\right\},$$

- **Computing (predicted) market share of brand *j* in market *t*:**

Integral over the mass of individual consumers in the region A_{jt}.

$$\begin{aligned} s_{jt}(x_{\cdot t}, p_{\cdot t}, \delta_{\cdot t}; \theta_2) &= \int_{A_{jt}} dP^*(D, v, \varepsilon) \\ &= \int_{A_{jt}} dP^*_{\varepsilon}(\varepsilon)\, dP^*_{v}(v)\, d\widehat{P}^*_{D}(D), \end{aligned}$$

Nevo (2000, 2001)

- **Computing (predicted) market share:**

$$s_{jt}(x_{\cdot t}, p_{\cdot t}, \delta_{\cdot t}; \theta_2) = \int_{A_{it}} dP^*(D, v, \varepsilon)$$
$$= \int_{A_{jt}} dP^*_\varepsilon(\varepsilon)\, dP^*_v(v)\, d\widehat{P}^*_D(D),$$

✓ Given assumptions on the distributions, we can compute the integral, either analytically or numerically.

✓ Simplest Assumption → Heterogeneity via only individual taste shock, ε_{ijt} with i.i.d., Type I EV assumption, we have compute market share analytically. This is the simple Logit Model:

$$s_{jt} = \frac{\exp(x_{jt}\beta - \alpha p_{jt} + \xi_{jt})}{1 + \sum_{k=1}^{J} \exp(x_{kt}\beta - \alpha p_{kt} + \xi_{kt})}.$$

$\boxed{\textbf{How to integrate over } D_i, v_i\textbf{?}}$

Nevo (2000, 2001)

- **Computing (predicted) market share:**

Predicted Market Share = Observed Market Share

$$s_{jt}(x_{\cdot t}, p_{\cdot t}, \delta_{\cdot t}; \theta_2) = \int_{A_{jt}} dP^*(D, v, \varepsilon) = \int_{A_{jt}} dP^*_\varepsilon(\varepsilon)\, dP^*_v(v)\, d\widehat{P}^*_D(D),$$

✓ Given assumptions on the distributions, we can compute the integral, either analytically or numerically.

✓ Further Assumption → Heterogeneity via D_i, v_i, ε_{ijt} , we can compute the market share numerically using Monte Carlo simulation.

Aggregate over simulated individuals

$$s_{jt}(p_{\cdot t}, x_{\cdot t}, \delta_{\cdot t}, P_{ns}; \theta_2) = \frac{1}{ns}\sum_{i=1}^{ns} s_{jti}$$

$$= \frac{1}{\mathrm{ns}}\sum_{i=1}^{\mathrm{ns}} \frac{\exp\left[\delta_{jt} + \sum_{k=1}^{K} x_{jt}^k(\sigma_k v_i^k + \pi_{k1}D_{i1} + \cdots + \pi_{kd}D_{id})\right]}{1 + \sum_{m=1}^{J}\exp\left[\delta_{mt} + \sum_{k=1}^{K} x_{mt}^k(\sigma_k v_i^k + \pi_{k1}D_{i1} + \cdots + \pi_{kd}D_{id})\right]},$$

where $(v_i^1, \ldots, v_i^K)$ and $(D_{i1}, \ldots, D_{id})$, $i = 1, \ldots, \mathrm{ns}$, are random draws.

Overview of Estimation

$\boldsymbol{\theta}_2 \rightarrow s \rightarrow \delta \rightarrow \xi \rightarrow$ GMMobj

Nonlinear search over the nonlinear parameters θ_2 - will concentrate out θ_1 using the first-order conditions

Steps:

1. Given guesses for (δ, θ_2), calculate aggregate shares
2. Given θ_2, recover δ from share inversion
3. Given θ_2, compute residuals ξ, interact with Z, minimize GMM objective function

Instruments

Step 1: Calculate Market Shares (Conditional on $\boldsymbol{\delta}_t$, $\boldsymbol{\theta}_2$)

For each market, given a guess of δ_t:

- Integrate out ε_{itj} analytically as before
- Integrate out random coefficients (β_i, α_i) numerically
- Easiest approach uses Monte Carlo integration

$$s_{tj}(\delta_t; \theta_2) \approxeq \frac{1}{NS} \sum_{ns=1}^{NS} \frac{\exp\left(\delta_{tj} + \mu_{itj}(v_{ns}; \theta_2)\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_{tk} + \mu_{itk}(v_{ns}; \theta_2)\right)}$$

where v_{ns} is $(K+1)$ vector of draws from $N(0, I)$

- See Ken Train's book for more details on simulation methods (http://elsa.berkeley.edu/books/choice2.html)

Step 2: Computing $\boldsymbol{\delta_t}$ (Conditional on $\boldsymbol{\theta_2}$)

For each market t:

- Compute the J-dimensional vector of mean valuations δ_t that equates observed and predicted shares

$$\boxed{S_t = s_t\left(\delta_t; \theta_2\right)}$$

- In homogeneous logit we calculated $\delta_{tj} = \log\left(S_{tj}\right) - \log\left(S_{t0}\right)$, but now δ_{tj} is inside an integral and cannot be separated

- Now numerically compute the inversion: $s_t^{-1}(S_t; \theta_2) = \delta_t$

- BLP propose a *contraction mapping*:
  - Given a guess of θ_2 and initial δ_t^0, iterate on

$$\delta_{tj}^{h+1}\left(\theta_2\right) = \delta_{tj}^{h}\left(\theta_2\right) + \underline{\log S_{tj} - \log s_{tj}\left(\delta_{tj}^{h}; \theta_2\right),} \quad j = 1, \ldots, J$$

  - Stop when $\left\|\delta_j^h - \delta_j^{h-1}\right\| \leq \epsilon_{\text{in}}$
  - Guaranteed to find a solution from any starting value of δ

Step 3: Moment Estimation
(Compute GMM Objective Function Conditional on $\boldsymbol{\delta_t}$, $\boldsymbol{\theta_2}$)

- Basis of estimation is the moment condition:

$$\mathbb{E}\left[\xi(\theta)'Z\right] = 0$$

- Minimize GMM objective function:

$$\min_{\theta} Q(\theta) = \xi(\theta)'ZA^{-1}Z'\xi(\theta)$$

IV for Price

where consistent estimate of weighting matrix is $A = \mathbb{E}\left[Z'\xi(\theta)\xi(\theta)'Z\right]$

- Given estimates of δ and a θ guess, could calculate:

$$\xi(\theta) = \delta\left(\theta_2\right) - \left(\bar{\beta}'x_j - \bar{\alpha}p_j\right)$$

Final Goal:
Minimize the objective function $Q(\boldsymbol{\theta_2})$ – search nonlinearly over $\boldsymbol{\theta_2}$.

IV for Price?

1. Cost shifters (Nevo 2000)

Variables that affect marginal cost (product, packaging, distribution costs); Cost proxies like city density for storage cost, salary for labor cost.

2. Price in other markets (Hausman 1996, Nevo 2001)

Assumes demand shocks uncorrelated across markets, but cost shocks are correlated across markets

3. Characteristics of competing products (BLP 1995)

Firm set price of a product based on characteristics of competing products from competitors (but these characteristics will not affect consumer valuation for the firm's own product).

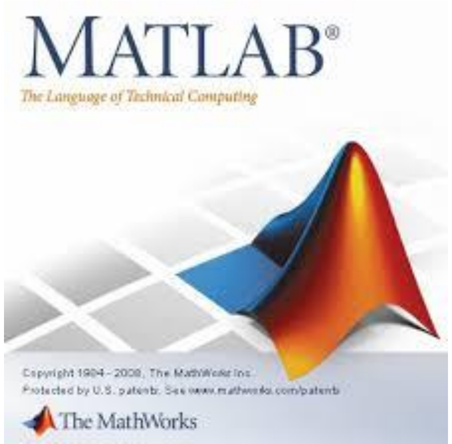
Estimation Summary

$\boldsymbol{\theta}_2 \rightarrow$ s $\rightarrow \delta \rightarrow \xi \rightarrow$ GMMobj

Steps:

1. Draw $u_{ns} \sim N(0, I)$, for $ns = 1, \ldots, NS$,
2. Compute initial δ from homogeneous model
3. Outer loop iteration $\ell = 1, 2...$, given value of θ_2^ℓ
    1. Inversion: Given a starting δ^0,
        1. Compute shares $s_{tj}\left(\delta_t^h; \theta_2^\ell\right)$ via simulation
        2. Iterate on $\delta_t^{h+1} = \delta_t^h + \log S_t - \log s_{tj}\left(\delta_t^h; \theta_2^\ell\right)$ until convergence
    2. Use $\delta(\theta_2)$ to calculate $\theta_1 = (\bar{\beta}, \bar{\alpha})$
    3. Get residuals $\xi_{tj} = \delta_{tj}(\theta_2) - \left(\bar{\beta}' x_{tj} - \bar{\alpha} p_{tj}\right)$
    4. Evaluate objective function $Q(\theta_2)$
4. Iterate minimization of $Q(\theta_2)$ until convergence

Class Exercise – Nevo (2000)



Matlab Code

Aviv Nevo's Original BLP Code + Data:
http://faculty.wcas.northwestern.edu/~ane686/supplements/rc_dc_code.htm
(Some issues due to Matlab version update…)

Eric Rasmusen's revision, Indiana U (partial success…):
http://www.rasmusen.org/zg604/lectures/blp/frontpage.htm

Bronwyn H. Hall's revision, UC Berkeley (Matlab 7):
http://eml.berkeley.edu/~bhhall/e220c/rc_dc_code.htm

A recent version (with minor changes) available to download:
http://www.andrew.cmu.edu/user/beibeili/BLPdemo_SMART.rar

Optimization: fminunc (option quasi Newton's Method, derivitive-based)
fminsearch (simplex method, random walk on a convex polytope)

Data Files: Original Excel

Original Excel Spreadsheets:

data_cereal.xlsx contains 2256 observations on id, brand, firm, city, quarter, share, price, sugar, mushy, and the 20 instruments in ***iv***, called ***z1-z20***.

data_demog.xlsx contains the demographic draws for each market. There are 94 markets (47 cities by 2 quarters) and 80 variables (20 individuals * 4 demographic variables on "Income" "Income^2" "Age" "Child").

data_v.xlsx contains the unobserved individual iid normal draws for each market. There are 94 markets (47 cities by 2 quarters) and 80 variables (20 individuals * 4 variables, for each individual there is a different draw for each brand-level variable on "Constant" "Price" "Sugar" "Mushy").

Matlab Data Inputs: ps2.mat

id - an id variable in the format bbbbccyyq, where bbbb is a unique 4 digit identifier for each brand (the first digit is company and last 3 are brand, i.e., 1006 is K Raisin Bran and 3006 is Post Raisin Bran), cc is a city code, yy is year (=88 for all observations is this data set) and q is quarter. All the other variables are sorted by date city brand.

s_jt - the market shares of brand *j* in market *t*. Each row corresponds to the equivalent row in ***id***.

x1 - the variables that enter the linear part of the estimation. Here this consists of a price variable (first column) and 24 brand dummy variables. Each row corresponds to the equivalent row in ***id***. This matrix is saved as a sparse matrix.

x2 - the variables that enter the non-linear part of the estimation (i.e., individual deviation). Here this consists of a constant, price, sugar content and a mushy dummy, respectively. Each row corresponds to the equivalent row in ***id***.

Matlab Data Inputs: ps2.mat

id_demo - an id variable for the random draws and the demographic variables, of the format ccyyq. Since these variables do not vary by brand they are not repeated. The first observation here corresponds to the first market, the second to the next 24 and so forth.

v - random draws given for the estimation. For each market 80 iid normal draws are provided. They correspond to 20 "individuals", where for each individual there is a different draw for each column of ***x2***. The ordering is given by ***id_demo***.

demogr - draws of demographic variables from the CPS for 20 individuals in each market. The first 20 columns give the income, the next 20 columns the income squared, columns 41 through 60 are age and 61 through 80 are a child dummy variable (=1 if age <= 16). Each of the variables has been demeaned (i.e. the mean of each set of 20 columns over the 94 rows is 0). The ordering is given by ***id_demo***.

Matlab Code

rc_dc.m - A script file that reads in the data and calls the other functions;

gmmobjg.m - This function computes the GMM objective function;

meanval.m - This function computes the mean utility level;

mufunc.m - This function computes the individual deviation of the utility (mu_ijt);

mktsh.m - This function computes the market share for each product;

ind_sh.m - This function computes the "individual" probabilities of choosing each brand;

jacob.m - This function computes the Jacobian of the implicit function that defines the mean utility;

var_cov.m - This function computes the VCov matrix of the estimates.

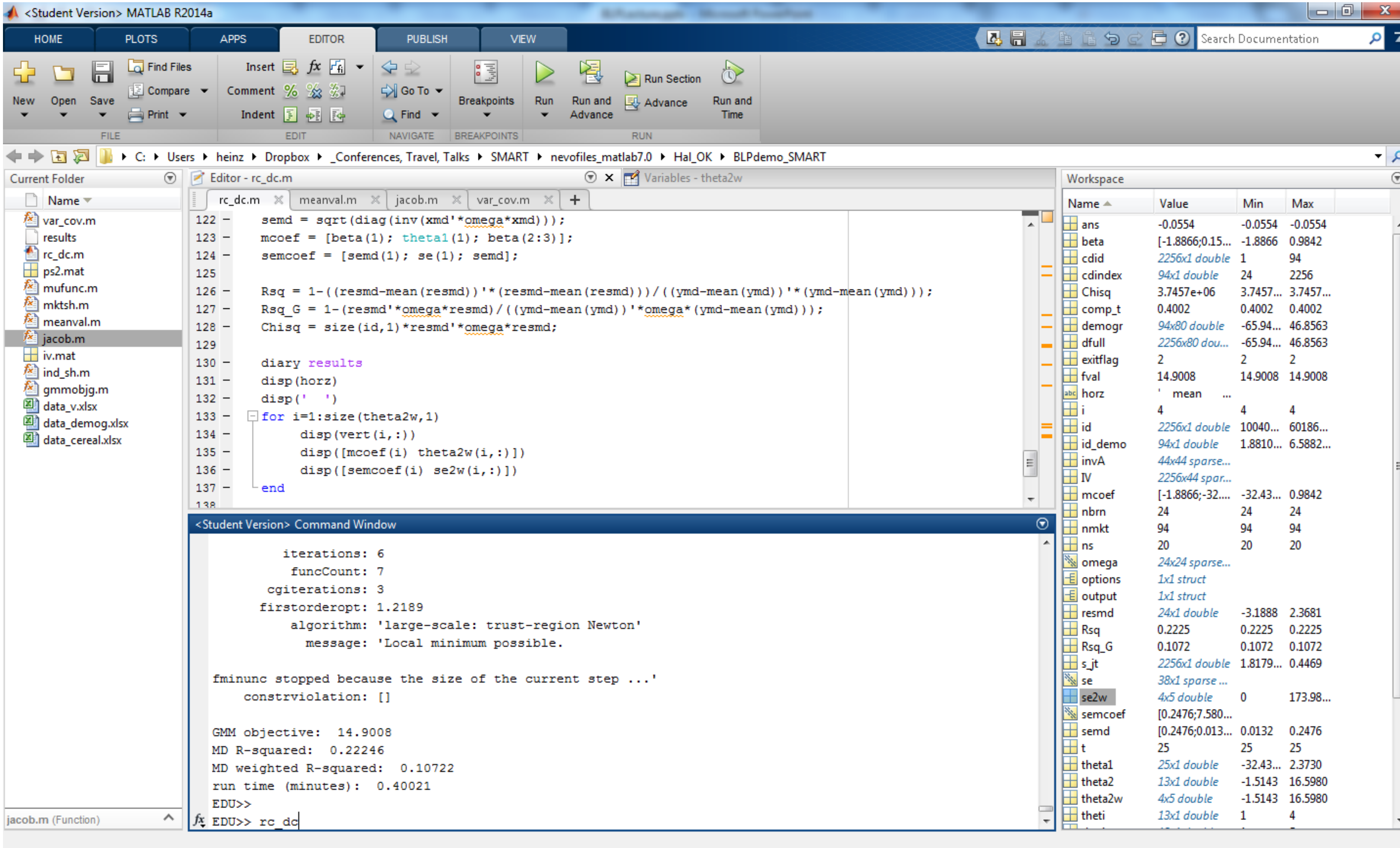
Estimation – Matlab Code

$\boldsymbol{\theta_2} \rightarrow s \rightarrow \delta \rightarrow \xi \rightarrow \text{GMMobj}$

Steps:

1. Draw $u_{ns} \sim N(0, I)$, for $ns = 1, \ldots, NS$, **Preprocessing**
2. Compute initial δ from homogeneous model **Preprocessing**
3. Outer loop iteration $\ell = 1, 2 \ldots$, given value of θ_2^ℓ
    1. Inversion: Given a starting δ^0,
        1. Compute shares $s_{tj}\left(\delta_t^h; \theta_2^\ell\right)$ via simulation **mktsh.m, ind_sh.m**
        2. Iterate on $\delta_t^{h+1} = \delta_t^h + \log S_t - \log s_{tj}\left(\delta_t^h; \theta_2^\ell\right)$ until convergence **meanval.m**
    2. Use $\delta(\theta_2)$ to calculate $\theta_1 = (\bar{\beta}, \bar{\alpha})$ **gmmobjg.m**
    3. Get residuals $\xi_{tj} = \delta_{tj}(\theta_2) - \left(\bar{\beta}' x_{tj} - \bar{\alpha} p_{tj}\right)$ **gmmobjg.m**
    4. Evaluate objective function $Q(\theta_2)$ **gmmobjg.m**
4. Iterate minimization of $Q(\theta_2)$ until convergence **rc_dc.m**

Matlab Demo



<Student Version> MATLAB R2014a
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Current Folder
Name
var_cov.m
results
rc_dc.m
ps2.mat
mufunc.m
mktsh.m
meanval.m
jacob.m
iv.mat
ind_sh.m
gmmobjg.m
data_v.xlsx
data_demog.xlsx
data_cereal.xlsx
jacob.m (Function)
Editor - rc_dc.m
Variables - theta2w
rc_dc.m meanval.m jacob.m var_cov.m
semd = sqrt(diag(inv(xmd'*omega*xmd)));
mcoef = [beta(1); theta1(1); beta(2:3)];
semcoef = [semd(1); se(1); semd];
Rsq = 1-((resmd-mean(resmd))'*(resmd-mean(resmd)))/((ymd-mean(ymd))'*(ymd-mean(ymd)));
Rsq_G = 1-(resmd'*omega*resmd)/((ymd-mean(ymd))'*omega*(ymd-mean(ymd)));
Chisq = size(id,1)*resmd'*omega*resmd;
diary results
disp(horz)
disp(' ')
for i=1:size(theta2w,1)
disp(vert(i,:))
disp([mcoef(i) theta2w(i,:)])
disp([semcoef(i) se2w(i,:)])
end
<Student Version> Command Window
iterations: 6
funcCount: 7
cgiterations: 3
firstorderopt: 1.2189
algorithm: 'large-scale: trust-region Newton'
message: 'Local minimum possible.
fminunc stopped because the size of the current step ...'
constrviolation: []
GMM objective: 14.9008
MD R-squared: 0.22246
MD weighted R-squared: 0.10722
run time (minutes): 0.40021
EDU>>
EDU>> rc_dc
Workspace
Name Value Min Max
ans -0.0554 -0.0554 -0.0554
beta [-1.8866;0.15... -1.8866 0.9842
cdid 2256x1 double 1 94
cdindex 94x1 double 24 2256
Chisq 3.7457e+06 3.7457... 3.7457...
comp_t 0.4002 0.4002 0.4002
demogr 94x80 double -65.94... 46.8563
dfull 2256x80 dou... -65.94... 46.8563
exitflag 2 2 2
fval 14.9008 14.9008 14.9008
horz ' mean ...
i 4 4 4
id 2256x1 double 10040... 60186...
id_demo 94x1 double 1.8810... 6.5882...
invA 44x44 sparse...
IV 2256x44 spar...
mcoef [-1.8866;-32.... -32.43... 0.9842
nbrn 24 24 24
nmkt 94 94 94
ns 20 20 20
omega 24x24 sparse...
options 1x1 struct
output 1x1 struct
resmd 24x1 double -3.1888 2.3681
Rsq 0.2225 0.2225 0.2225
Rsq_G 0.1072 0.1072 0.1072
s_jt 2256x1 double 1.8179... 0.4469
se 38x1 sparse ...
se2w 4x5 double 0 173.98...
semcoef [0.2476;7.580...
semd [0.2476;0.013... 0.0132 0.2476
t 25 25 25
theta1 25x1 double -32.43... 2.3730
theta2 13x1 double -1.5143 16.5980
theta2w 4x5 double -1.5143 16.5980
theti 13x1 double 1 4

Main Results

Theta1 - Mean Effect

Price	1	-32.4374
Brand Dummies	2	-3.4939
	3	0.7485
	4	-1.3989
	5	-1.7301
	6	2.3730
	7	0.1387
	8	-1.5570
	9	-1.4858
	10	-0.2475
	11	-0.8883
	12	-0.0332
	13	-3.4607
	14	-1.1772
	15	0.0706
	16	0.5026
	17	1.8213
	18	-1.0325
	19	-1.5984
	20	0.2314
	21	0.7887
	22	1.0814
	23	-1.6915
	24	-0.4042
	25	-0.7128

Theta2 – Interaction Effect

		Sigma	Income	Income2	Age	Child
		1	2	3	4	5
Constant	1	0.3772	3.0888	0	1.1859	0
Price	2	1.8480	16.5980	-0.6590	0	11.6245
Sugar	3	-0.0035	-0.1925	0	0.0296	0
Mushy	4	0.0810	1.4684	0	-1.5143	0

Recap: Why do we need random coefficients?

- Avoid IIA Failure & Flexible Substitution Pattern:

Without random coefficient, homogeneous Logit model indicates that a consumer who substitutes away from a given product will tend to substitute toward other popular products (popularity measured by market shares), not to other similar products.

- Heterogeneous Tastes (interacted with product features):

We can estimate the distribution of the individual taste → Random coefficients allow individual i to have a specific substitution pattern that appeals to her.

Additional References

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