Efficient Financial Crises[*]

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ABSTRACT _____

We analyze the optimal capital structure and investment strategy of banks and other financial institutions. We develop conditions under which banks optimally choose a fragile capital structure that is subject to runs. We show that when bank depositors have limited ability to commit to long-term lending arrangements, they strictly prefer to lend to banks using short-term debt rather than with long-term debt or equity. We argue that when there are multiple banks, the same limited commitment of depositors leads them to prefer a financial system in which banks pursue correlated, risky investments as opposed to one in which banks pursue independent, less risky investments. The optimal financial system features occasional crises in which all banks are subject to ex-post inefficient liquidations, and in this sense, financial crises are efficient.

1. Introduction

Banks and other financial firms typically rely heavily on short-term debt to finance their assets. The short-term debt heavy capital structure of banks and other financial firms naturally exposes them to runs or other panic-like phenomena as in Diamond and Dybvig (1983) for example and suggests their capital structure is *fragile*. Such crises are pervasive: Reinhart and Rogoff (2008) document the occurrence of 435 banking crises in 70 developed and developing economies since 1800. Given that banking crises are frequent and associated with economic recessions, it is important to understand the reasons why banks and other financial firms rely so heavily on short-term debt.

In this paper, we analyze the incentives of banks to adopt a fragile capital structure. We develop conditions under which a single bank chooses to finance its investments with primarily short-term debt which is subject to runs. The bank's reliance on short-term debt leads to occasional terminations of the bank's activity which are inefficient and, in this sense, resemble bank runs.

We then use the model to analyze the efficiency of crises by extending the model to feature multiple banks. We show that a financial system in which bank returns are correlated and all banks fail at the same time is more efficient than a system in which bank returns are independent and at most a single bank fails at any point in time. Equilibrium outcomes with a fragile financial system feature occasional crises. These outcomes are efficient in the sense that any alternative arrangement leads to lower welfare. We conclude that in the absence of other spillover effects, financial crises are efficient.

We begin by analyzing the optimal capital structure of a single bank. Following the literature in corporate finance, we develop a model in which the capital structure of banks is optimally designed to solve incentive problems (see Biais et al. (2007) for an example). Specifically, we develop a model in which depositors to a bank must design compensation contracts for a bank manager, or *banker*, to provide incentives to exert effort in a dynamic environment. The banker is protected by limited liability. In the model, the banker's effort affects the distribution of future project outcomes. High effort implies that good outcomes are likely, and low effort implies that poor outcomes are likely (see Hölmstrom (1979) for an example of this kind of incentive problem). One way of providing incentives to exert high effort is to commit to dismiss the banker and terminate the bank if project outcomes are poor. Such dismissal, which we call *liquidation* henceforth, typically, is costly not just for the banker but for the depositors as well.

Consider more specifically the tradeoffs involved with liquidating the bank after poor outcomes. In any history in which the depositors continue the project, the banker receives strictly positive expected value (net of effort costs), or a "rent," due to the combination

of moral hazard and limited liability. When the depositors dismiss the banker, the banker does not receive the rents involved with continuing the project. By dismissing the banker after poor outcomes, depositors align the banker's ex-ante incentives with their own, and, therefore, can save on how much they must compensate the banker after good outcomes and signals in order to provide appropriate incentives for effort. The benefit to depositors from such a liquidation strategy then is cost savings in terms of providing the banker with incentives to exert effort. These costs savings must be compared to the direct costs to depositors of liquidation, which is forgone profits earned from the project. When the likelihood of poor project outcomes is low if the banker exerts high effort, we show it is efficient to liquidate the bank following poor project outcomes in spite of the costs of such dismissal to the depositors. We then demonstrate that this result implies that it is efficient for depositors in a bank to lend via uninsured, short-term debt or deposit contracts.

We then explain how the optimal provision of the banker's incentives can be used to explain why banks and financial institutions rely heavily on short-term debt. To allow for a meaningful distinction between short and long-term debt, we analyze the same environment under an assumption that depositors cannot commit to the entirety of their long-term contract. A consequence of assuming that depositors lack full commitment, however, is that implementing a contract which calls for liquidation after poor outcomes may not be feasible. The reason is that after such outcomes, both the depositors and the banker stand to gain by renegotiating their contract and allowing the banker to continue. If the banker and the depositors expect such renegotiation, then the banker rationally chooses a low level of effort (ex ante). Thus, if bankers and depositors cannot commit to carrying out their contracts, outcomes on average are worse than with commitment. In this sense, lack of commitment creates a *time inconsistency problem* (see Kydland and Prescott (1977) for an example of this problem).

The main theoretical contribution in this paper is to show that the use of short-term debt introduces a coordination problem among depositors that can help solve the time inconsistency problem. In particular, if an agreement to renegotiate the contract requires all or a substantial fraction of depositors to agree to a renegotiation, I show that the time inconsistency problem can be resolved. The basic idea is that such an agreement to renegotiate creates incentives on the part of each depositor to threaten to disagree unless that depositor is paid a large fraction of the bank's future earnings. Such incentives make it difficult for depositors to renegotiate the terms of the contract and help ensure that the original contract is implemented even when it is undesirable from the perspective of the collective interests of the depositors. Since the banker anticipates the likelihood of such disagreement, the banker expects the original contract to be implemented and rationally chooses to exert high effort

to reduce the likelihood of poor outcomes.

Coordination problems of the kind studied here are well known in the literature on the problem of providing public goods which serve common purposes such as military defense or pollution control (see Rob (1989) or Mailath and Postlewaite (1990) for examples). This literature has emphasized that requiring all or most citizens to agree to an appropriate level of defense or pollution control is difficult and has emphasized that government action might be desirable in such circumstances. The theoretical result, that coordination problems can be used to resolve time inconsistency problems demonstrates how such coordination problems can actually serve a desirable social role.

Short-term debt plays a desirable social role by introducing a coordination problem among depositors that allow them to, in effect, commit to dismiss the banker after poor project outcomes. As a result, it implies that in some cases (when the project yields a poor outcome) along the equilibrium path, outcomes trigger actions that look like they could not be part of an ex ante efficient contract – e.g., bank runs. Each depositor refuses to roll-over their debt even though it is in the collective interest of the depositors to do so.

The usefulness of short-term debt in this model relies on the lack of outside investors with deep pockets who are willing to invest in the bank when (some of) the original depositors are unwilling to do so. This raises a concern about the usefulness of short-term debt if there are multiple banks or other depositors with funds they are willing to invest. We address this concern by extending the model to allow for multiple banks. We show that the same motive that leads a single bank to use short-term debt will lead multiple banks to optimally undertake investments with correlated returns that are riskier than those they may obtain with independent returns.

To understand this result, consider first possible outcomes when bank project returns are independent. Given the independence of the banks, it is not surprising that commitment outcomes will mimic the single bank outcomes bank by bank. If high project outcomes are likely when bankers exert high effort, then when a single bank realizes low project outcomes, it is likely that the other bank has realized high project outcomes. As a consequence, even if the depositors in the bank with low project outcomes would like to liquidate the bank, the depositors in banks with high project returns may be willing to lend the returns from their bank to both banks since even the bank with low project returns has a profitable investment opportunity. If depositors are unable to commit to prevent dilution of deposits, then it is likely if a single bank realizes low project returns that the project will be continued. Again, if the banker and the depositors expect such dilution to occur, then the banker rationally chooses a low level of effort (ex ante). Thus, if bankers and depositors cannot commit to prevent rollover from outside depositors, outcomes on average are worse than

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with commitment.

Consider next outcomes when bank project returns are perfectly correlated across banks. In this case, if one bank realizes low returns, then all banks realize low returns. If low returns are sufficiently small, then all of the depositors will find it difficult to agree to finance even a single bank. We show that while under full commitment depositors would be indifferent between a financial sector in which bank returns are independent and have low volatility and one with correlated returns and high volatility, when depositors and bankers lack full commitment, they strictly prefer a sector with correlated and highly volatile returns. In this sense, facing limited commitment, the efficient financial sector features banks financed with short-term debt that undertake correlated, volatile investments. Along the outcome path, all banks will earn low returns and be liquidated.

An efficient financial sector, then, in this model is fragile and susceptible to crises. In the model, such fragility and susceptibility serves an important purpose by providing bankers of financial institutions the incentives needed to achieve good outcomes. Equilibrium outcomes in the model are efficient in the sense that no planner confronted with the same informational structures as other agents could achieve a better outcome. In this sense, government interventions can only be harmful. Of course, there may be other spillover costs associated with bank runs or bank liquidations in which case equilibrium outcomes in this model are likely to be inefficient. An extended version of the model her could be useful for analyzing the best way to mitigate the probability of financial crises and to address them appropriately when they do occur.

Related Literature

This paper is related to an extensive literature on bank runs and the role of demand deposits or short-term debt (see Cole and Kehoe (2000) and Diamond and Dybvig (1983)). The theoretical results on the use of short-term debt as a commitment device are closest in nature to those found in Diamond and Rajan (2001), Diamond and Rajan (2000), Calomiris and Kahn (1991), and Bolton and Scharfstein (1990), and we view our results as an important generalization of the ones in these papers. Specifically, in Diamond and Rajan (2001), bank runs do not occur along the equilibrium path; in Diamond and Rajan (2000), inefficient dismissal of the banker is not a feature of the optimal contract; and in Calomiris and Kahn (1991), when the optimal contract is, in their terminology, "short-term debt," dismissal of the banker is desirable from the perspective of the collective interest of the depositors. Bolton and Scharfstein (1990) consider an arbitrary ex-post coordination game between two lenders. When terminating the firm is costly from the collective interests of the lenders, the lenders have strong incentives to coordinate and roll over their debt. In this paper, we

allow the lenders to coordinate to develop arbitrary incentive-feasible contracts that induce the lenders as a whole to roll their debt over and demonstrate that even when the option to do so exists, short-term debt prevents them from doing so. Generalizing these results to the case where bank runs occur in equilibrium and are a feature of the optimal contract is a necessary first step in building a framework to analyze the effects of regulatory policy on the capital structure of banks.

The idea that bank runs may be a feature of optimal lending arrangements is related to results in Allen and Gale (1998) and Allen and Gale (2004). In these papers, when intermediaries are restricted to offer demand deposits, bank default or crises allow intermediaries to share risk and effectively offer fully state-contingent contracts. In this paper, I analyze general optimal contracts and show under what conditions particular frictions of moral hazard on the part of the bank manager and incomplete information regarding depositors' liquidity shocks give rise to crises as a feature of optimal lending arrangements.

The idea that a coordination problem can resolve a time inconsistency problem is related to the results in Laffont and Tirole (1988) and Netzer and Scheuer (2010). In their environments, a risk-neutral principal wants to provide both incentives for effort and insurance to a risk-averse principal. Under commitment, the principal provides incentives by delivering less than full insurance to the agent. In both of these papers, when the principal (or markets) lacks commitment, the optimal contract introduces an adverse selection problem ex-post, which limits the ability of the principal to provide full insurance after effort has been provided. This adverse selection problem allows the principal to commit to deliver less than full insurance and is the efficient way to provide ex-ante incentives. In this paper, because the agent or banker is risk-neutral, a different type of ex-post informational problem is necessary for the principal to commit to deliver the appropriate incentives.

Additionally, this paper provides new results regarding the optimality of short-term contracts in long term agency relationships. Fudenberg et al. (1990) develop conditions under which spot contracts implement optimal commitment outcomes in a long-run relationship. One key condition for their result is that the utility frontier describing payoffs of the principal and payoffs of the agent must be decreasing. In other words, after each history, continuation utilities for the principal and the agent lie on the set of efficient continuation allocations. The main result in this paper demonstrates that short-term contracts may implement long-run commitment outcomes even when long-run commitment outcomes feature histories where continuation outcomes are ex-post inefficient. In this sense, our results differ from those found in Brunnermeier and Oehmke (2010), where a lack of commitment causes short-term contracts to deliver worse outcomes than long-run commitment outcomes.

Lastly, this paper is related to an extensive literature on the optimal maturity struc-

ture of firm debt (see Diamond (1991), Flannery (1986), Myers (1977) for examples). Each of these papers is primarily concerned with variation of maturity across non-financial firms, whereas I focus on differences in maturity structure between financial and non-financial firms.

The remainder of the paper is organized as follows. Section 2 contains a benchmark moral hazard problem between a single banker and and a large number of depositors. This sections develops conditions under which a fragile capital structure is optimal and demonstrates the optimality of short-term debt. Section 3 extends the benchmark model to one with multiple banks. This section demonstrates why a fragile financial system is optimal. Section 4 concludes.

2. The One-Bank Economy

We begin with a version of our economy with only a single bank and N depositors. We develop conditions under which, in this economy, optimal contracts under full commitment feature events along the outcome path that resemble bank runs. Under full commitment, the timing of payments to depositors which attains the optimum is indeterminate. Motivated by this indeterminacy, we introduce a limited commitment constraint and show that with limited commitment, optimal contracts feature early payouts to depositors and bank runs. We develop a simple decentralization of this benchmark economy and show that short-term debt with the possibility of bank runs dominates long-term debt or equity contracts.

Consider a three period environment with $N + 1$ agents. Let the periods be indexed by $t = 0, 1, 2$. We will call the $N + 1$st agent a *banker* and the remaining agents *depositors*. The production technology requires both resources and effort of the banker in periods 0 and 1. Formally, we assume the banker has access to a project which requires I units of resources in period 0 and effort of the banker $e_0 \in \{\pi_l, \pi_h\}$. The project yields gross output $I + y_1 \in \{y_l, y_h\}$ in period 1 where $y_1 = y_h$ with probability e_0 and $y_1 = y_l$ with prob. $1 - e_0$. To conserve notation, we have indexed the banker's effort choice by the probability of high returns following that effort choice and we will scale the cost of the banker's effort appropriately. If the project is continued from period 1 to 2, it requires resource inputs I again and additional effort of the banker e_1. The project then yields gross output $I + \rho y_1 + z_2, z_2 \in \{z_l, z_h\}$ with $z_h > z_l$, $\rho > 0$, and $z_2 = z_h$ with probability e_1. Since $\rho > 0$, high and low gross output in period 2 following a high return in period 1 are both larger than following a low return in period 1. In this sense, there is persistence in project returns which is independent of the banker's previous effort level. IF the project is not continued, we will say the project has been *liquidated*. For simplicity, we will consider the case where $y_l = z_l = 0$.

The banker is endowed with zero physical resources but does have the abilities needed

e_0 $\qquad\qquad$ y_1, v $\qquad\qquad\qquad\qquad$ I_1, e_1 $\qquad\qquad\qquad$ y_2

$t=0$	$t=1$	$t=1$	$t=2$
initial effort	project returns & discount factor shocks	Investment cost & effort if continued	project returns if continued

Figure 1: Timeline for the model described in Section 2.

operate the project technology. Each depositor i is endowed with $k_0^i = I/N$ in period 0, may choose how much to invest with the banker, and may store the remainder at a one-for-one rate. Since the project requires I resources, investment will require the participation of each depositor.[1] The banker is risk neutral and has preferences over streams of consumption c_t and effort e_t given by

$$c_0 - q(e_0) + c_1 - q(e_1) + \beta c_2$$

where β is the banker's discount factor from period 1 to period 2 and $q(e_t)$ is the cost of effort in period t. For simplicity, we will normalize $q(\pi_l) = 0$ and simply denote $q(\pi_h) = q$.

Each depositor is also risk neutral and values streams of consumption according to $U(c_0, c_1, c_2) = c_0 + c_1 + v_i c_2$. We assume that v_i is an i.i.d. preference shock realized at the beginning of period 1 with $v_i \sim G_i(v_i)$ having support $[\underline{v}, \bar{v}]$. Let G denote the joint distribution over v. The depositor's preference shocks can be thought of as liquidity shocks to the depositors, causing them to have a stronger preference for period 1 consumption when they realize lower values of v_i as in Diamond and Dybvig (1983). Additionally, the preference shocks are privately known by each individual depositor with $\beta < \underline{v} < \bar{v} < 1$.

2A. The Case of Full Commitment

We start by analyzing the benchmark economy under full commitment to contracts by the banker and the depositors. The time-line of the physical attributes of the environment are as follows

We begin by describing direct mechanisms, or contracts which specify recommended effort levels, transfers to the banker and depositors, and a period 1 continuation of whether to continue the project or not, all as functions of the relevant history. We will focus on *investment contracts* which call for investment in period 0 and later ensure that investment is superior to autarky from the ex-ante perspective of each of the depositors.

An investment contract allows for a general timing of payments to depositors which may be made in period 1 after project returns are realized and both before and after the

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[1]The assumption that exactly N depositors are required to undertake the investment can be relaxed by appropriately modifying some of the later assumptions.

depositors realize their private discount factor. We let p_t^i denote a payment to depositor i in period t. Let P^d denote the set of all payments to depositors so that

$$P^d = \left\{ \left(p_0^i, p_{1c}^i(y_1, v), p_{1n}^i(y_1, v), p_2^i(y_1, z_2, v) \right)_{i \in \{1, \ldots, N\}} \right\}$$

where p_{1c}^i denotes a payment by depositor i if the vector of reported discount factors is v and the realized return is y_1 and the project is continued while p_{1n}^i is a similar payment which occurs if the project is discontinued. Similarly, we will let p_t^b denote the payment to the banker in period t. Then, $P^b = \left\{ p_1^b(y_1), p_2^b(y_1, z_2, v) \right\}$. An investment contract is then a contract is then a collection of functions, $\left\{ p^d, p^b, x(y_1, v) \right\}$ where $p^d \in P^d$, $p^b \in P^b$ and $x(y_1, v) \in [0, 1]$ where x represents a continuation decision.

We now discuss the constraints a direct mechanism must satisfy. First, the contract must satisfy nonnegativity constraints so that in each period consumption of the banker and depositors are positive. These constraints require $p_{1c}^i, p_{1n}^i, p_2^i, c_t \geq 0$.

Second, the mechanism must be *resource feasible*. That is, we require period 1 payments to be less than period 1 resources in the event of continuation

$$p_1^b(y_1) + \sum_{i=1}^{N} p_{1c}^i(y_1, v) \leq y_1$$

and in the event of liquidation

$$p_1^b(y_1) + \sum_{i=1}^{N} p_{1n}^i(y_1, v) \leq y_1 + I.$$

We will write these two constraints compactly using the continuation rule $x(y_1, v)$ as

$$p_1^b(y_1) + \sum_{i=1}^{N} \left[x(y_1, v) p_{1c}^i(y_1, v) + (1 - x(y_1, v)) p_{1n}^i(y_1, v) \right] \leq y_1 + I - I x(y_1, v). \qquad (1)$$

The contract must also be feasible in period 2 so that

$$p_2^b(y_1, z_2, v) + \sum_{i=1}^{N} p_2^i(y_1, z_2, v) \leq I + \rho y_1 + z_2.$$

Will write this constraint, after taking an expectation across period 2 output under the

recommended effort level of the banker as

$$E_{e_1(y_1,v)} \sum_{i=1}^{N} p_2^i(y_1, z_2, v) \leq I + \rho y_1 + E_{e_1(y_1,v)} \left(z_2 - p_2^b(y_1, z_2, v) \right). \tag{2}$$

Third, the investment contract must satisfy two types of incentive compatibility conditions. The contract must induce the banker to choose the recommended level of effort in each period and the contract must induce truth-telling of the depositors. We will say a contract is *incentive compatible for the banker* if for all y_1 and v,

$$\beta e_1(y_1, v) p_2^b(y_1, z_h, v) + \beta(1 - e_1(y_1, v)) p_2^b(y_1, z_l, v) - q(e_1(y_1, v)) \tag{3}$$
$$\geq \max_{e'} \beta e' p_2^b(y_1, z_h, v) + \beta(1 - e') p_2^b(y_1, z_l, v) - q(e')$$

and

$$E_{e_0} \left[p_1^b(y_1) + \int_v U_1(y_1, v) dG(v) \right] - q(e_0) \geq \max_{e'} E_{e'} \left[p_1^b(y_1) + \int_v U_1(y_1, v) dG(v) \right] - q(e') \tag{4}$$

where $U_1(y_1, v) = x(y_1, v) \left[\beta E_{e_1(y_1,v)} p_2^b(y_1, z_2, v) - q(e_1(y_1, v)) \right]$. Constraint (3) requires that the banker prefers the stream of payments from period 1 to period 2 expected under effort level $e_1(y_1, v)$ net of the disutility of effort to that which could be obtained under an alternative effort level e'. Constraint (4) is similar but provides the banker incentives to choose effort level e_0 in period 0 taking as given period 1 consumption and the continuation utility the banker will receive in each state of the world.

To define incentive compatibility, it is useful to define the continuation utility of a depositor conditional on realizing a preference shock v_i and reporting preference shock $\hat{v}_i$, which we will denote by $w_i(y_1, \hat{v}_i, v_i)$ and with a slight abuse of notation is given by

$$w_i(y_1, \hat{v}_i, v_i) = \int_{v_{-i}} x(y_1, \hat{v}_i, v_{-i}) \left(p_{1c}^i(y_1, \hat{v}_i, v_{-i}) + v_i p_2^i(y_1, \hat{v}_i, v_{-i}) \right) dG_{-i}(v_{-i})$$
$$+ \int_{v_{-i}} (1 - x(y_1, \hat{v}_i, v_{-i})) p_{1n}^i(y_1, \hat{v}_i, v_{-i}) dG_{-i}(v_{-i}).$$

In evaluating this continuation utility, the depositor assumes the other depositors will report truthfully in which case with probability $x(y_1, \hat{v}_i, v_{-i})$ the project will be continued and the depositor will receive payment $p_{1c}^i(y_1, \hat{v}_i, v_{-i})$ in period 1 and, with a slight abuse of notation, a payment $p_2^i(y_1, \hat{v}_i, v_{-i})$ in period 2. We will say a contract is *incentive compatible for the*

depositors if for each y_1 and v_i,

$$w_i(y_1, v_i, v_i) \geq \max_{\hat{v}_i \in [\underline{v}, \bar{v}]} w_i(y_1, \hat{v}_i, v_i). \tag{5}$$

Lastly, because each depositor is free to store the endowment which earns a rate of return of 1 and since $\bar{v} < 1$, if the investment contract is superior to autarky, it must satisfy the following *voluntary participation constraint*

$$E_{e_0}\left[\int_{v_i} w_i(y_1, v_i, v_i)dG_i(v_i)\right] \geq \frac{I}{N}. \tag{6}$$

As well, the investment contract must satisfy the participation constraint of the banker, however, this constraint is necessarily slack as we will argue below.

Because we wish to consider decentralized outcomes in which bankers compete ex-ante for depositors, we will focus on contracts which maximize the ex-ante expected welfare of the depositors. Ex-ante welfare of the depositors is given by

$$E_{e_0}\left[\sum_i \int_{v_i} w_i(y_1, v_i, v_i)dG_i(v_i)\right]. \tag{7}$$

Then, an *optimal investment contract* maximizes (7) among all investment contracts which satisfy the nonnegativity constraints, the resource feasibility constraints (1) and (2), the incentive constraints (3), (4), and (5), and the participation constraint (6).

We now turn to characterizing the optimal investment contract under the assumption that it is always optimal to induce high effort (in Appendix ****, we provide conditions such that this is the case). We begin by providing a sharp characterization of optimal payments to the banker (for any set of payments and continuation rules). As in most moral hazard models, the principal, who here can be thought of as representing the coalition of depositors, would like to use the payments to the banker to align the banker's incentives to exert high effort with her own. This implies making zero payments to the banker following a realization of low output in either period 1 or period 2. Moreover, following low period 1 output, the banker's incentive constraint must bind as must the banker's period 0 incentive constraint. We then have the following Lemma.

LEMMA 1. Without loss of generality, we may restrict attention to contracts which feature zero payment to the banker following low output in either period 1 or period 2 and (3) holds with equality when $y_1 = y_l$. Moreover, the banker's incentive constraint in period 0 (4) also holds with equality.

Substituting for $p_2^b(y_1, y_l, v) = 0$ and $p_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$ into (3), we find that in the optimal contract, the period 2 payment to the banker is given by $p_2^b(y_1, y_h, v) = \bar{q}/\beta(\pi_h - \pi_l)$. Then, the banker's continuation utility in period 1 satisfies

$$U_1(y_1, v) = x(y_1, v)[\beta \pi_h p_2^b(y_1, x(y_1, v) - \bar{q}] \geq x(y_1, v) \frac{\pi_l \bar{q}}{\pi_h - \pi_l}.$$

The fact that the banker receives a strictly positive rent in the second period arises from the assumptions of risk-neutrality and non-negativity of the banker's consumption. Using Lemma 1 and the fact that $p_1^b(y_l) = 0$ allows us to simplify (4) as

$$p_1^b(y_h) = \frac{\bar{q}}{\pi_h - \pi_l} + \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x(y_l, v) G(dv) - \int_v x(y_h, v) \left[\beta \pi_h p_2^b(y_h, z_h, v) - \bar{q} \right] G(dv) \quad (8)$$

An important feature of the banker's period 0 incentive constraint is that by choosing to continue projects after high output is realized in period 1 relaxes the banker's incentive constraint and allows the principal to reduce the payment $p_1^b(y_h)$ to the banker. By choosing to continue projects after low output is realized in period 1, however, the principal actually tightens the incentive constraint and requires the principal to make a larger payment to the banker in period 1 after high output is realized to preserve the banker's incentives to exert effort in period 0. We summarize this discussion in the following lemma.

LEMMA 2. The optimal payment to the banker in period 1 following a realization of high output is decreasing in the probability of continuation after high output and increasing in the probability of continuation after low output.

We now use the above results to develop conditions under which the optimal contract under commitment features events that resemble bank runs. In particular, we will ask whether the optimal contract calls for liquidation in period 1 after some output realizations when this liquidation is (at least under full information of the depositors' discount factors) inefficient. We begin by developing sufficient conditions to ensure that liquidation is optimal for all reported discount factors following a realization of low output in period 1. Towards this end, we will re-write the depositors' ex-ante welfare by substituting the resource constraints and using Lemma 1 as

$$I + \pi_h \left[y_h - p_1^b(y_h) + \int_v x(y_h, v) \left[-I + \sum_i v_i p_2^i(y_h, v) \right] dG(v) \right]$$

$$+ (1 - \pi_h) \int_v x(y_l, v) \left[-I + \sum_i v_i p_2^i(y_l, v) \right] dG(v). \quad (9)$$

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From Lemma 2, we know that $p_1^b(y_h)$ is increasing in $\int x(y_l, v) \, dG(v)$. Thus, in states where $-I + \sum_i v_i p_2^i(y_l, v)$ is positive, there is a tradeoff between incentive provision and future returns. When agents realize high values of v_i, liquidation after low output in period 1 is realized is very costly (even considering the future rents the banker will obtain), but such liquidation relaxes the incentive constraint and reduces the payment to the banker after high output is realized in period 1. Consider then the benefit from reducing $\int_v x(y_l, v) dG(v)$. Using equation (8), the benefit from relaxing the incentive constraint is $\pi_h \pi_l \bar{q} / (\pi_h - \pi_l)$. The cost of such a reduction, in ex-ante terms, is bounded above by

$$(1 - \pi_h) \left[-I + \bar{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) \right]$$

which is the cost in terms of forgone future returns (net of the banker's rent) of reducing the continuation probability if all depositors have the highest discount factor. As long as the benefit exceeds this maximum cost, or,

$$\frac{\pi_h \pi_l \bar{q}}{\pi_h - \pi_l} - (1 - \pi_h) \left[-I + \bar{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) \right] > 0 \tag{10}$$

then the optimal contract will satisfy $x(y_l, v) = 0$ for all v.

Consider next the optimal continuation rule following a realization of high output in period 1. From Lemma 2, the payment to the banker, $p_1^b(y_h)$ is decreasing in the continuation probability $x(y_h, v)$. This suggests that as long as projects are profitable following high first period output, then continuing projects improves surplus available to pay depositors and relaxes the banker's incentive constraint thereby increasing welfare. Formally, substituting equation (8) into depositor's welfare (equation (9), we see that depositors' welfare depends on the continuation rule according to

$$\int_v x(y_h, v) \left[-I + \sum_i v_i p_2^i(y_h, v) + \beta \pi_h p_2^b(y_h, z_h, v) - \bar{q} \right] G(dv)$$

Note that under full information, depositors' welfare is maximized by choosing $x(y_h, v) = 1$ if and only if

$$\sum_i v_i p_2^i(y_h, v) + \beta \pi_h p_2^b(y_h, z_h, v) \geq I + \bar{q}.$$

Since aggregate resources (in expectation) in period 2 following high first period output are given by $I + \rho y_h + \pi_h z_h$ and the banker is more impatient than all of the depositors,

straightforward algebra can be used to show that if

$$\beta\left[I + \rho y_h + \pi_h z_h\right] \geq I + \bar{q} \tag{11}$$

then under full information depositor welfare is maximized when $x(y_h, v) = 1$ for all v. Moreover, even when depositors' discount factors are unobservable, since for all possible realizations of depositors' discount factors, continuing the project is ex-post efficient, one can show that depositors' welfare is maximized by choosing $x(y_h, v) = 1$. Note that equation (11) asserts that even under the banker's discount factor, the project yields positive present value net of the both the resource costs and the banker's effort costs.

We have developed conditions such that the continuation rule which maximizes depositor welfare has the property that $x(y_l, v) = 0$ and $x(y_h, v) = 1$ for all v. We summarize these results in the following proposition.

PROPOSITION 1. *Suppose the banker is more impatient than all of the depositors, the expected incentive benefits of liquidation exceed the expected costs (or equation (10) is satisfied), and continuing the project yields positive net present value following high first period output (equation (11) is satisfied. Then the optimal investment contract calls for continuation after a high output realization in period 1 and liquidation after a low output realization in period 2.*

The above proposition illustrates conditions under which liquidation of projects in the interim period is ex-ante optimal. A consequence of the proposition is that each depositor receives an equal share of their original investment, I/N in period 1 after low output is realized and the project is liquidated. This result follows from the fact that without the promise of future transfers, no other transfer scheme is incentive compatible for the depositors.

To argue that such interim period liquidations are inefficient, we will appeal to a notion of ex-post efficiency in which the depositors' discount factors are observable but the effort of the banker is unobservable. Formally, we will say a continuation rule is ex-post efficient if there is no other continuation rule and associated payments to the depositors which is incentive compatible for the banker, is feasible, and weakly increases the utility of each depositor and strictly increases at least one depositor's utility.

We now evaluate the ex-post efficiency of the optimal contract under commitment. It is straightforward to show that the continuation of the optimal contract following high period 1 output is ex-post efficient, so we focus on the continuation outcome following a low first period ouput realization. Along this outcome path, each depositor receives a transfer of I/N and the project is liquidated. We now ask whether an alternative continuation contract

which calls for continuation of the project can improve depositor welfare. Such an alternative continuation contract would necessarily satisfy the resource constraint

$$\sum_i \hat{p}_2^i(v) = I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}$$

and to improve welfare for each depositor would require $\hat{p}_2^i(v) \geq I/(v_i N)$. Such a scheme exists if

$$I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} > \frac{I}{N} \sum_i \frac{1}{v_i}. \tag{12}$$

Hence, the optimal contract is ex-post efficient only when the inequality in (12) is violated. As long as there are realizations of the vector of discount factors v that violate the inequality in (12), for example if $\bar{v} \left[I + \pi_h z_h - \pi_h \bar{q}/\beta(\pi_h - \pi_l) \right] > I$, then the optimal contract features ex-post inefficient liquidations. In order to make a more precise statement about limiting results as the number of depositor grows, we must make an additional assumption as in the following proposition.

PROPOSITION 2. *As the number of depositors tends to infinity, if the discount factor of depositors is such that*

$$IE \left[\frac{1}{v_i} \right] < I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)},$$

then the probability that the optimal continuation rule following a low first period output realization is ex-post efficient tends to zero.

Propositions 1 and 2 imply that the optimal ex-ante contract necessarily features ex-post inefficiencies. Such features arise in many environments with informational frictions (see Phelan and Townsend (1991) and Yared (2010) among many others for examples). In this benchmark model, the ex-post inefficiencies take the form of forgone surplus. Along the outcome path of the optimal contract, a large number of depositors refuse to roll-over their investment and liquidate the bank. Because both the banker and the depositors could be made better off (with arbitrarily high probability as $N \to \infty$) by continuing the project following a low period 1 output realization and because this information is publicly known, these outcomes would resemble bank runs to an outside policymaker. As a result, we view our benchmark economy when the average depositor is patient and liquidations are ex-post inefficient as a complement to the model in Calomiris and Kahn (1991) when such liquidations are ex-post efficient.

As is clear, however, this version of the benchmark model cannot be immediately applied to discuss the optimal maturity structure of banks. In the benchmark model, de-

positors and the banker are able to write long-term state-contingent contracts and are able to commit to the future terms of the contract. We now turn to a modified version of our benchmark economy in which we introduce a form of limited commitment and show how such limited

2B. The Case of Limited Commitment

Here we describe our notion of limited commitment and characterize optimal contracts. We show that while the limited commitment constraint does not bind in the sense that it does not strictly reduce the attainable level of ex-ante depositors' welfare, it does restrict set of contracts which can yield the highest level of depositors' welfare. In particular, we will show that short-term debt contracts approximate this optimal contract while long-term debt or equity contracts do not.

We will assume that the depositors and the banker are free to re-negotiate certain features of the contract after first period output and the depositors discount factors have been realized. To allow for some, limited form of commitment, we divide the middle period (period 1) and allow for payments to be made after output is realized but before the discount factors have been realized. We denote this payment $p_1^i(y_1)$. An individual depositor's payoffs from an investment contract are now written as

$$E_{e_0}\left[p_1^i(y_1) + \int_v \left[x(v) \left(p_{1c}^i(v) + v_i p_2^i(y_1, v) \right) + (1 - x(v)) p_{1n}^i(v) \right] dG(v) \right].$$

We assume that the banker and depositors can commit to these "early" payments as well as those to the banker in period 1, but may freely choose to alter the remaining components of the contract.

The only remaining restrictions on the new, re-negotiated contract are that it cannot deliver negative consumption to any agent in period 1 and no agent can be coerced into participating. The first constraint limits any single depositor (or a small block of depositors) from fully financing the investment project in period 1. The second constraint serves to implicitly define the outside option of any individual depositor, which is simply $p_1^i(y_1)$. Importantly, we allow for the depositors to choose new contracts which may be worse for individual depositors than the status quo. This distinction will be important in explaining why the model features an optimal maturity.

To be precise, let us define a *continuation contract* as the following collection of functions,

$$\left\{ \left(\hat{p}_{1c}^i(v), \hat{p}_{1n}^i(v), \hat{p}_2^i(v) \right)_{i \in \{1,...,N\}}, \hat{x}(v), \hat{e}_1(v), \hat{p}_2^b(z_2, v) \right\}.$$

Facing any arbitrary, status quo contract, a possible continuation contract, denoted with ˆ's

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must be incentive compatible for the depositors so that if

$$\hat{w}_i(\hat{v}_i, v_i) = \int_{v_{-i}} \left[x(\hat{v}_i, v_{-i}) \left(\hat{p}^i_{1c}(\hat{v}_i, v_{-i}) + v_i \hat{p}^i_2(\hat{v}_i, v_{-i}) \right) + (1 - x(\hat{v}_i, v_{-i})) \hat{p}^i_{1n}(\hat{v}_i, v_{-i}) \right] dG_{-i}(v_{-i})$$

and represents the continuation utility a depositor with discount factor v_i would receive from the alternative continuation contract when reporting $\hat{v}_i$, then

$$\hat{w}^i(v_i, v_i) \geq \max_{\hat{v}_i \in [\underline{v}, \bar{v}]} \hat{w}_i(\hat{v}_i, v_i).$$

Since depositors cannot be coerced to participate, we require $\hat{w}^i(v_i, v_i) \geq 0$.

The contract must also be incentive compatible for the banker so that

$$\hat{e}_1(v) \hat{p}^b_2(z_h, v) + (1 - \hat{e}_1(v)) \hat{p}^b_2(z_l, v) - q(\hat{e}_1(v))$$
$$\geq \max_{e'} e' \hat{p}^b_2(z_h, v) + (1 - e') \hat{p}^b_2(z_l, v) - q(e').$$

The banker's incentive constraint places (weakly) tighter bounds on the continuation utility of the banker than the status quo contracts incentive constraints, so that the banker will also choose to participate in any re-negotiation.

The continuation contract must be feasible so that

$$\sum_{i=1}^N \hat{p}^i_2(v) \leq I + \rho y_1 E_{\hat{e}_1(v)} \left[z_2 - \hat{p}^b_2(z_2, v) \right]$$

and

$$\sum_{i=1}^N \hat{p}^i_{1c}(v) \leq y_1 - \sum_{i=1}^N p^i_1(y_1) - p^b_1(y_1),$$
$$\sum_{i=1}^N \hat{p}^i_{1n}(v) \leq I + y_1 - \sum_{i=1}^N p^i_1(y_1) - p^b_1(y_1).$$

Again, we will combine these later constraints into a single, ex-ante constraint as

$$\sum_{i=1}^N \int_v \left[\hat{x}(v) \hat{p}^i_{1c}(v) + (1 - \hat{x}(v)) \hat{p}^i_{1n}(v) \right] dG(v)$$
$$\leq y_1 + I - \sum_{i=1}^N p^i_1(y_1) - p^b_1(y_1) - I \int_v \hat{x}(v) dG(v). \tag{13}$$

Lastly, the continuation contract must satisfy the nonnegativity constraints for each depos-

itor so that

$$p_1^i(y_1) + \hat{x}(v)\hat{p}_{1c}^i(v) + (1 - \hat{x}(v))\hat{p}_{1n}^i(v) \geq 0. \tag{14}$$

Given the above constraints, we will say a contract is enforceable if there is no continuation contract which satisfies depositor and banker incentive compatibility, satisfies non-negative consumption of depositors and yields strictly greater utility to the depositors in ex-ante terms than the status quo contract, or,

$$\sum_i \int_{v_i} \hat{w}^i(v_i, v_i) dG_i(v_i) > \sum_i \int_v \left[x(v) \left(p_{1c}^i(v) + v_i p_2^i(y_1, v) \right) + (1 - x(v)) p_{1n}^i(v) \right] dG(v). \tag{15}$$

Observe that elements of the status quo contract appear only in the objective (equation (15)), the resource constraint (equation (13)), and the nonnegativity constraint (equation (14)). This notion of enforceability makes clear the distinction between period 1 transfers made before depositors realize their preference shock (early transfers) and period 1 transfers made after depositors realize their type (late transfers). Because we assume that early transfers are made before new continuation contracts can be designed, these transfers affect the set of feasible continuation contracts. By allocating positive early transfers in period 1, the limited liability constraints of the depositors and the resource constraints in any continuation contract become more stringent. If early transfers are all equal to zero, then the limited liability constraints are weak – any contract (including contracts that call for continuation) are feasible as long as they deliver at least 0 transfers to each agent.

We now argue that the welfare obtained by the optimal contract under commitment is attainable with limited commitment so that in terms of welfare, it appears the limited commitment constraint does not bind. However, we will also show that the timing of contracts is determined and that to obtain the commitment outcome, the principal must use the early payments. Formally, we will show that if $p_1^i(y_1) = (I + y_1 - p_1^b(y_1))/N$ for $y_1 = y_l$ and $y_1 = y_h$ then the optimal contract under commitment is enforceable. Moreover, if $p_1^i(y_l) = 0$ and the contract calls for liquidation when $y_1 = y_l$, then the contract is not enforceable. We conclude that early payments to depositors are a necessary feature of optimal contracts. To establish these results, we will make use of the following assumptions.

Assumption 1. The support of the depositors' discount factors satisfy

$$\underline{v}\left[I + \rho y_l + \pi_h \left(z_h - \frac{\bar{q}}{\beta (\pi_h - \pi_l)} \right) \right] < I < \underline{v}\left[I + \rho y_h + \pi_h \left(z_h - \frac{\bar{q}}{\beta (\pi_h - \pi_l)} \right) \right].$$

Assumption 1 plays a key role in characterizing the nature of the coordination problem that arises in period 1 following low or high outcomes. Indeed, the left-hand side inequality asserts

that if all of the depositors have the lowest rate of time preference, then it would be ex-post inefficient for them to continue the project following a low period 1 outcome. Of course, as the number of agents becomes large, the probability of this outcome becomes arbitrarily small under the assumption of Proposition 2. Nonetheless the fact that it is possible for all of the depositors to have a preference shock of $\underline{v}$ implies that each individual depositor must be provided with incentives to reveal their type truthfully, and these incentives do not become arbitrarily small as N to ∞.

The incentive problem in renegotiated contracts facing the depositors following high period 1 outcomes is different however, due primarily to the right-hand inequality in Assumption 1. This inequality asserts that if all of the depositors have the lowest rate of time preference, then it is ex-post efficient to continue the project after high period 1 outcomes. As a result, even with a probability of continuation equal to 1, there exist transfers satisfying voluntary participation and incentive compatibility of the depositors. For example, a constant transfer scheme will satisfy these constraints. As a result, the depositors will always efficiently continue the project after high period 1 project outcomes.

We also impose the following regularity assumptions on the distribution of depositor discount factors.

Assumption 2. The distribution $G_i(v_i)$ is such that $(1 - G_i(v_i))\,/(v_i^2 g_i(v_i))$ is decreasing in v_i, there exists $\kappa > 0$ such that $g_i^n(v_i) > \kappa$, and $\bar{v}$ is finite.

We are now ready to demonstrate how under limited commitment the depositors may attain the commitment outcome. Given the nature of the time inconsistency, we focus on enforcing the continuation rule $x(y_l, v) = 0$ following low first period output. The simplest contract that attains the commitment outcome sets early payments equal to a pro-rata share of output, or $p_1^i(y_l) = I/N$.

Consider now the renegotiation problem facing depositors. We will ask whether the depositors can design renegotiation contracts which call for continuation. First, observe that by the nonnegativity constraint, any such renegotiation contract must satisfy $p_{1c}^i = -I/N$ since the depositors must roll over at least I resources. Using this fact, we can simplify the constraints of the renegotiation problem to

$$\hat{w}_i(\hat{v}_i, v_i) = \int_{v_{-i}} \left[\hat{x}(\hat{v}_i, v_{-i}) \left(\frac{-I}{N} + v_i \hat{p}_2^i(\hat{v}_i, v_{-i}) \right) \right] G_{-i}(dv_{-i}) \tag{16}$$

$$\hat{w}_i(v_i, v_i) \geq \hat{w}_i(\hat{v}_i, v_i) \tag{17}$$

$$\hat{w}_i(v_i, v_i) \geq 0 \tag{18}$$

$$\sum_i \hat{p}_2^i(v) \leq I + \rho y_l + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \tag{19}$$

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Notice, the banker's incentive constraint is nested in the resource constraint in period 2.

To further simplify these constraints, we appeal to results from Myerson (1981) and Myerson and Satterthwaite (1983) which allow us to characterize the global incentive constraints as local constraints and allow us to eliminate period 2 transfers from the problem (albeit under a weaker, ex-ante version of the period 2 resource constraint). These results are stated in the following lemmas.

LEMMA 3. A contract satisfies depositor incentive compatibility if and only if the function $\rho_i(v_i)$ defined by

$$\rho_i(v_i) = \int_{v_{-i}} x(v_{-i}, v_i) dG_{-i}(v_{-i}) \text{test}$$

is increasing in v_i for all i, v_i, y_1 and

$$u^i(v_i) \equiv w^i(v_i, v_i) = v_i \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N} \int_{\underline{v}}^{v_i} \frac{1}{z^2} \rho_i(z) dz \right].$$

Moreover, the contract satisfies voluntary participation if and only if $u^i(\underline{v}) \geq 0$.

The proof is in the appendix. Combining this lemma with the ex-ante version of the period 2 resource constraint, we have the following result.

LEMMA 4. Suppose that x is such that ρ^i is increasing in v_i. There exist payments p_2^i such that (p_2^i, x) satisfy depositors' incentive compatibility, voluntary participation, and the period 2 resource constraint if and only if

$$\int_v x(v) \left[I + \rho y_1 + \pi_h(z_h - p_2^b(y_1, z_h, v)) - \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \right] G(dv) \geq 0. \qquad (20)$$

Using this lemma, following a result in Mailath and Postlewaite (1990), it is straightforward to demonstrate that as as $N \to \infty$, the maximal probability of continuation for which there exist transfers so that the continuation rule and transfers satisfy the constraints of the renegotiation problem converges to zero (following low period 1 outcomes). We state this result in the following proposition, which makes use of the conditions in Assumption 1 as well as the regularity conditions of Assumption 2.

LEMMA 5. Following a low period 1 outcome, the maximum probability that the project is continued, or $\hat{x}(n)$ satisfying

$$\hat{x}(n) = sup \left\{ Ex(v) : \exists (p_2^i) suchthat(x, p_2^i) \text{ satisfies depositor IC, participation} \right.$$
$$\left. , \text{ and the RC in the n-agent economy (16)-(19)} \right\}$$

converges to 0 as n goes to infinity. Furthermore, the probability that it is ex-post efficient to continue the project goes to 1.

To understand the result, consider the tradeoffs a single depositor faces when determining which discount rate to report, $\hat{v}_i$. note that the participation constraints for impatient depositors are tighter than for patient depositors so that payments in period 2 must be higher for impatient depositors. By under-reporting her discount factor, a depositor can attain a higher payment in period 2 if the project is continued. The cost to the depositor of under-reporting her discount factor is that the probability of continuation declines (since ρ_i is increasing).

The key idea behind the limiting result, as in Mailath and Postlewaite (1990), is that as $N \to \infty$, the cost of under-reporting shrinks to zero since the likelihood that a single depositor is pivotal, and thus by under-reporting v_i would cause the investment not to be undertaken, becomes arbitrarily small. On the other hand, the costs of providing incentives do not shrink to zero since depositors can always gain a strictly higher payment by under-reporting. As a result, in the limit the incentive costs are so large as to make the value of continuation, net of incentive costs, equivalent to that which would occur in a full information economy where all depositors had the lowest discount factor; at this rate, under Assumption 1 the depositors prefer not to undertake the investment.

This result can be seen from (20). Consider maximizing the probability of continuation subject to (20). The maximal rule would set $x(v) = 1$ whenever

$$I + \rho y_1 + \pi_h(z_h - p_2^b(y_1, z_h, v)) - \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \geq 0. \tag{21}$$

As N becomes large, the term involving v_i's, which Myerson (1981) terms virtual utility, tends to $1/\underline{v}$. By Assumption 1, this inequality is never satisfied for N sufficiently large.

We have argued that a contract with $p_1^i(y_l) = I/N$ and a continuation rule $x(y_l, v) = 0$ for all v is enforceable. It remains to prove that the remaining features of the optimal contract with the limited commitment coincide with those under full commitment. Conditional on the continuation rule and the banker's consumption, the optimal transfer scheme with limited commitment must coincide with that under full commitment (up to the indeterminacy between p_1^i and p_{1c}^i and p_{1n}^i). Moreover, optimality of transfers under full commitment ensures that these transfers are enforceable with limited commitment (were they not enforceable, then there would exist superior transfers under full commitment). As a result, we have proved that the solutions to the commitment and limited commitment problem coincide (as $N \to \infty$).We summarize this discussion in the following proposition.

PROPOSITION 3. *As the number of depositors becomes large, if $p_1^i(y_l) = I/N$, then the depositors can attain their commitment value when contracts are subject to limited commitment.*

Proposition 3 illustrates that the limited enforcement of contracts, in this environment, does not lead to welfare losses. However, it also illustrates which types of contracts can enforce the optimal commitment outcome. These contracts resemble short-term debt because they require a payment (or demandable claim to a payment) roughly equal to the remaining resources of the bank. Indeed, below we show in a limiting case that non-contingent short-term debt can exactly implement the optimal commitment outcome. The benefit of short-term debt contracts in this environment is that such contracts make the continuation decision (or the debt-rollover decision) resemble a problem of providing public goods. Such contracts introduce ex-post public goods problems which make renegotiation difficult; however, by doing so, such contracts limit the extent of renegotiation which is beneficial from the perspective of providing ex-ante incentives to the banker.

The constraints also make plain why by using all long-term debt (i.e. $t_1^i = 0$), even when depositors have the option to walk away from the contract (but cannot individually force liquidation of the project) cannot implement the optimal continuation rule. With this contract, an individual depositor's payoff (from walking away) is simply 0. Thus, any contract that delivers 0 transfers in period 1 and constant transfers in period 2 will be incentive compatible, satisfy the limited liability constraints and the participation constraints. In particular, if the depositors maximize the ex-ante value of all the depositors, it will be optimal for the depositors to continue the project after low outcomes.

Notice then that the form of limited commitment studied here does not require renegotiated contracts to yield pareto improvements. This assumption is critical for obtaining a determinate maturity structure. The assumption is that early payments yield formal property rights to agents whereas promises to late payments (such as equity claims) do not. This result suggests an important feature of short-term versus long-term debt is not the timing of payments per se but the allocation of property rights over liquidation decisions.

2C. A Decentralization with Short-Term Debt

In this section, we formalize the idea that optimal contracts can be attained by way of short term debt contracts by offering a decentralized version of the benchmark economy. Technology and preferences are identical to the benchmark economy. We allow the banker to offer short-term contracts in period 0 and period 1. These contracts offer a gross rate of return $R_1(y_1)/I$ in period 1 and $R_2(y_1, z_2)/I$ in period 2. In the special case we consider below, we will show $R_1(y_1)$ is independent of y_1 but $R_2(y_1, z_2)$ will depend on the project returns in period 1.

The timing is as follows. The banker offers contracts for sale in period 0. Each depositor chooses how many period 0 contracts to purchase. If the banker sells I contracts, then the banker undertakes investment and chooses an effort level e_0. If I contracts are not sold, any purchased contracts are rebated to depositors (this last assumption rules out the possibility of no-investment equilibria when investment outcomes are feasible). In period 1, output y_1 is realized and the short term returns, $R_1(y_1)/I$, are paid to the depositors. Each depositor then realizes their private discount factor v_i. Next, the banker offers new contracts with gross rate of return $R_2(y_1, z_2)/I$. Again, each depositor chooses how many contracts to purchase and, if the banker sells I contracts, then investment is undertaken and the banker chooses an effort level e_1. In period 2, output $I + \rho y_1 + z_2$ is realized and returns are paid to depositors.

We focus on equilibria in which the banker chooses the terms of the contract to maximize ex-ante depositor value. This focus is motivated by the idea that there are multiple bankers in time 0 competing for depositors. In period 1, however, the banker in period 0 is the only banker which can generate returns from period 1 to 2, so we must assume that the banker can commit to a sequence of short-term interest rates. A *competitive equilibrium* is a set of time- and state-contingent returns $(R_1(y_1), R_2(y_1, y_2))$ and a number of contracts purchased by each depositor such that no alternative contract earns more ex-ante value for investors and investors purchases are optimal.

We now show that competitive equilibrium with short-term debt contracts necessarily feature liquidations after low first period returns. We go on to show, under a stronger assumption on the technologies, that the competitive equilibrium with short-term debt exactly replicates optimal commitment outcomes.

First, consider the best debt contracts the banker can offer depositors when projects yields low returns in period 1 and assume first that $R_1(y_l) = I/N$, The banker chooses rates of return R_h and R_l to maximize the payout to depositors under the expectation of their future valuation subject to a constraint that the banker must want to choose high effort and a resource constraint. That is, the banker solves the following problem

$$\max_{R_l, R_h} \frac{\tilde{v}}{I} \left(\pi_h R_h + (1 - \pi_h) R_l \right)$$

subject to

$$\beta \pi_h \left(z_h - R_h \right) + \beta (1 - \pi_h) \left(z_l - R_l \right) - \bar{q} \geq \beta \pi_l \left(z_h - R_h \right) + \beta (1 - \pi_l) \left(z_l - R_l \right)$$

and

$$R_i \leq I + z_i.$$

Given $z_l = 0$, the incentive constraint can be simplified to

$$z_h - R_h \geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)} - R_l.$$

Clearly, then, the banker chooses $R_l = I$ and $R_h = I + z_h - \bar{q}/(\beta(\pi_h - \pi_l))$.

We now study the depositor's decision of how many such contracts to purchase and show that under Assumption 1, the banker will not raise sufficient resources to continue the project. Let d denote the number of debt contracts an individual depositor purchases. The problem of a depositor can then be written as

$$\max_{c_1, c_{2h}, c_{2l}} c_1 + v_i (\pi_h c_{2h} + (1 - \pi_h)c_{2l})$$

subject to $c_1 + d \leq I/N$ and $c_{2j} = dR_j/I$.

$$
\begin{aligned}
c_1 + d &\leq I/N \\
c_{2j} &= \frac{d}{I}R_j
\end{aligned}
$$

Substituting the constraints, it is immediate that the depositor will choose $d = I/N$ and purchase I/N contracts if and only if

$$v_i \left[I + \pi_h \left(z_h - \frac{\bar{q}}{\beta(\pi_h - \pi_l)} \right) \right] > I. \tag{22}$$

Since continuation requires all depositors to roll their debt over, for a large number of depositors, the probability that at least one depositor has a valuation which violates 22 is high. Indeed, as $N \to \infty$, the probability that a depositor has a low enough discount factor such that the depositor would not purchase this contract tends to 1 which implies such contracts feature liquidation with probability 1. To simplify the problem, we now make one additional assumption on technologies.

Assumption 3. The project returns in period 1 satisfy $y_h = q(1 - \pi_l)/(\pi_h - \pi_l)$.

This assumption implies that there are no excess returns in period 1 following a realization of high output after compensating the banker. In other words, in the optimal contract with commitment, the payout to depositors in period 1 is zero for each depositor. The primary advantage of this formulation of the problem is to simplify the incentive constraints of the depositors following high output in period 1. In this case, since the continuation contract has zero payments in period 1 to depositors, they incentive compatibility of the depositors requires payments in period 2 to be independent of their reported discount factor. That is,

the optimal contract has the feature that $p_{1c}^i(y_h, v) = 0$ and $p_2^i(y_h, v)$ is constant. Under limited commitment, this implies the optimal contract has $p_1^i(y_1) = I/N$ for $y_1 = y_l$ and $y_1 = y_h$. Under constant transfers to depositors following high returns, it is immediate that the decentralized economy with short-term debt yields identical outcomes to the optimal contract under commitment. The equilibrium returns in this economy satisfy $R_1(y_h) = R_1(y_l) = I/N$ and $R_2(y_h, z_l) = I + \rho y_h$, $R_2(y_h, z_h) = I + \rho y_h + z_h - \bar{q}/(\beta(\pi_h - \pi_l))$. Given this structure of returns, it is clear the optimum can be implemented with non-contingent debt contracts as long as depositors may claim an equal share of the returns if low output is realized in period 2. We have the proved the following proposition.

PROPOSITION 4. *Under Assumptions 1-3, as the number of depositors becomes large, the optimal contract is implemented with short-term debt contracts. Short-term debt is non-contingent, is rolled over if period 1 returns are high and is not rolled over if period 1 project returns are low. When debt is not rolled over, such liquidation is ex-post inefficient.*

This proposition makes clear the value of short-term debt. Short-term debt provides a commitment to inefficiently liquidate projects. We have already argued that contracts, such as long-term debt without forced liquidation rights, do not yield liquidation along the equilibrium path. As such, short-term debt strictly dominates long-term debt claims. Moreover, liquidation of projects occurs after project returns are low even though repayment on existing debt has been made in full (the promised return is I/N regardless of outcomes).

3. A Multiple Bank Model

The above analysis rested heavily on the idea that all of the depositors were needed to roll over the debt of a single bank and that no individual depositor, or one external to the analysis, had deep pockets which could be used to rollover the banker's debt. In this section, we relax this assumption by allowing for multiple banks borrowing from their own sets of depositors. In this model, when one bank yields low returns in period 1, its depositors may wish to force the bank into liquidation as in the single bank benchmark model. However, the depositors of other banks may have excess resources which they would like to use to invest in the failing bank since its project is productive. We analyze this possibility and demonstrate that when the returns to bank projects are independent, such an outcome is indeed possible and limits the ability of depositors of all banks to provide their own banker with incentives to exert effort. We go on to show that if bankers undertake projects with correlated, riskier returns, depositors can credibly commit to liquidate banks after low project returns and thus provide better ex-ante incentives to bankers. We conclude that in this framework, depositors (and bankers) prefer their bankers to undertake projects with risky, correlated returns.

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For simplicity, we analyze a version of our benchmark model with precisely two banks or bankers, each of which is initially paired with its own set of N depositors. In all, then, there are $2N + 2$ agents in this model. Preferences are identical to those described above and each investor is endowed with I/N. The production technology across banks may now be correlated, in which case project returns may depend on the joint effort of both bankers. We let the probability over joint bank outcomes be denoted by π which is a mapping from the effort level of each banker, (e^1, e^2) and output from each bank (y^1, y^2). We will begin by analyzing the case where projects are uncorrelated and returns to one bank are independent of the effort choice of and returns to the other bank. Below we analyze a case where project returns are perfectly correlated.

3A. The Case of Independent Bankers

When bankers' projects are independent, we will assume that banker j's effort level has no effect on the distribution of returns obtained by banker $-j$. Similarly, conditional on the effort choices, project outcomes are independent. In terms of maximizing resources available to depositors (or minimizing the cost of providing both bankers with incentives to exert effort), the complete independence of the two banks implies that optimal payments to the bankers and the optimal continuation rules are identical to the case of a single banker. That is, it is optimal to continue projects in bank 1 if and only if bank 1 earns high returns in period 1 and similarly for bank 2. We will continue to consider the case studied in Section 2 under Assumption 3. That is, regardless of the project returns to any bank, the net resources available to depositors after compensating the banker is I.

Here, we restrict our attention to asking whether it is feasible for the depositors to implement the commitment continuation rule when there is limited commitment, as they were able in the single bank context. Since the renegotiation constraints are tightest when all of the returns are paid out in period one, we focus on determining whether choosing $p_1^i(y_l) = I/N$ for both banks can implement the commitment outcome of liquidate either project if either project yields low first period returns. We will consider outcomes when only one bank realizes low returns and when both banks realize low returns.

Suppose first only one bank, say bank 1, realizes low returns. In this case, the optimal contract calls for bank 1 to be liquidated and bank 2 to continue. We show that there do not exist payments to depositors which satisfy the renegotiation constraints and continue both projects. In this sense, when only one bank realizes low returns, depositors are able to commit to liquidate that bank. To see this result, note that a continuation contract in which both projects are continued requires each depositor to pay $\hat{p}_{1c}^i = -I/N$. The problem is then identical to the renegotiation problem studied under a single bank (assuming transfers

between depositors at both banks are feasible), except there are twice as many depositors and the aggregate resources available, in expectation, are given by

$$2I + \rho y_h + \rho y_l + 2 \left[E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta (\pi_h - \pi_l)} \right].$$

Much as in the one bank model, as long as

$$\underline{v} \left[2I + \rho y_l + \rho y_h + 2\pi_h \left(z_h - \frac{\bar{q}}{\beta (\pi_h - \pi_l)} \right) \right] < 2I \tag{23}$$

then the incentives of depositors to mis-report their discount factor are too strong for each depositor to agree to participate in the renegotiation. As a result, we have the following lemma.

LEMMA 6. With independent bank projects, if (23) holds, then if bank j realizes low output and bank $-j$ realizes high output, then at most one bank's project is continued. Optimally, bank $-j$ is continued.

Consider next outcomes when both banks realize low returns in period 1. In this case, again aggregate resources available to depositors is $2I$. By the same argument as above, both bank projects cannot be continued. However, under a condition on the asymptotic behavior of the median depositor, it is feasible for the depositors to continue one bank. Specifically, we assume that

$$G_i \left(\frac{I}{I + \pi_h z_h - \pi_h \bar{q}/(\beta(\pi_h - \pi_l))} \right) < \frac{1}{2}$$

so that the median depositor would strictly prefer to continue the investment project following low first period returns. In this case, among the $2N$ depositors, the N most patient depositors would be willing to continue the project even if they receive only an equal share of the future returns. We assume that the depositors randomize equally between banks when they continue one of the two. As a result, we have the following lemma.

LEMMA 7. With independent bank projects, if both banks realize low output, then each bank is continued with probability $1/2$. In expectation, the banker expects to receive a rent $(1/2)\pi_l \bar{q}/ (\pi_h - \pi_l)$.

As a consequence of Lemma 7, the optimal commitment outcome is not consistent with outcomes under limited commitment. The reason is that the depositors cannot commit to liquidate the bank with probability 1 when both banks realize low output. This tightens the incentive constraint and implies that even with short-term debt, the depositors cannot obtain commitment outcomes.

3B. The Case of Correlated Bankers

We now construct an example in which bankers undertake correlated investment projects which are riskier than those considered in the independent case and show that in such an environment, the depositors can obtain the same welfare as in the independent case under full commitment. To be precise, we construct the example so that under full commitment, the depositors are indifferent between the independent and the correlated case; under limited commitment, depositors have a strict preference for correlated outcomes.

We alter the production technology to allow for perfectly correlated returns. Specifically, we assume that for any effort levels of the bankers, $\pi(y_h, y_l; e^1, e^2) = 0$. Moreover, we assume that $\pi(y_h, y_h; e^1, e^2) = \min\{e^1, e^2\}$ so that if either banker chooses low effort, the probability of high output for both banks is low. This assumption ensures that depositors face the same difficulties in providing bankers with incentives to exert effort as in the independent case.

We further assume that projects are riskier in the sense that there is more spread between y_h and y_l than in the independent case. In particular, we assume that $y_l = -I/2$ so that if both banks realize low returns, aggregate resources available to depositors is I. We then choose y_h so that under full commitment, the depositors are indifferent between the independent bankers and the correlated bankers cases.

In this correlated returns environment, it is immediate that when both bankers realize low returns, neither project can be continued. Each of the $2N$ depositors is needed to continue even just one of the banks, but then the set of $2N$ depositors face exactly the renegotiation problem which a single group of depositors faced in the single bank model. As such, depositors can clearly commit to the commitment outcomes. We have then proved the following proposition.

PROPOSITION 5. *If returns are perfectly correlated, sufficiently risky, and have the same mean return as the independent projects, then depositors strictly prefer a financial system which features perfectly correlated returns across banks.*

The above proposition implies that optimal equilibrium outcomes feature a financial system which is subject to crises. With probability $(1 - \pi_h)$, both banks realize low returns, and, although there are sufficient resources to finance at least one of the banks and such financing is ex-post efficient, none of the banks receive financing. We interpret this result as illustrating the optimality of a fragile financial system.

4. Conclusion

We have argued that banks find it optimal to use a fragile capital structure to finance their investments. Fragility for a single bank serves a useful purpose in providing bank managers appropriate incentives to exert effort to yield a superior distribution of returns. When depositors and bankers have limited ability to commit to long-term, state contingent outcomes, short-term debt can allow depositors to in effect commit to liquidate banks when it is ex-post inefficient to do so. This commitment is beneficial from an ex-ante perspective because it allows depositors to obtain greater ex-ante returns from the bank. With multiple banks, short-term debt may not be sufficient to allow depositors to commit to the optimal liquidation strategy. Instead, depositors find it optimal for their banks to engage in riskier, correlated investment strategies to limit their ability to finance banks in the midst of a crisis. We interpret this finding as illustrating the appropriateness of modeling the financial system as a representative, fragile bank.

This paper suggests that the same incentives that lead banks to seek out fragile capital structures should lead such banks to engage in correlated, risky investment strategies with significant downside risk. With more evidence on the distribution of banks, one could formally test these predictions by comparing the correlation of stock returns among banks with a high amount of uninsured short-term debt versus those banks with a low amount of uninsured short-term debt.

Appendix

A1. Figures

Figure 2: Empirical Cumulative Distributions of Short-Term Debt to Total Assets for Financial and Non-Financial Firms.
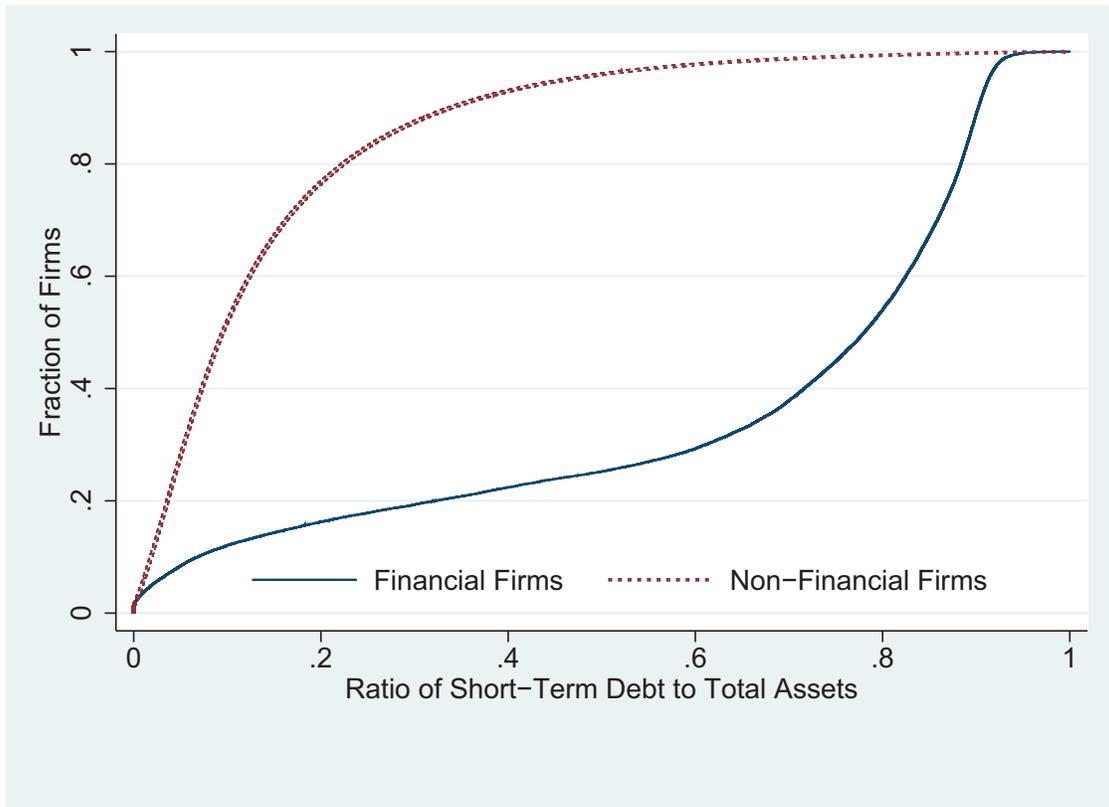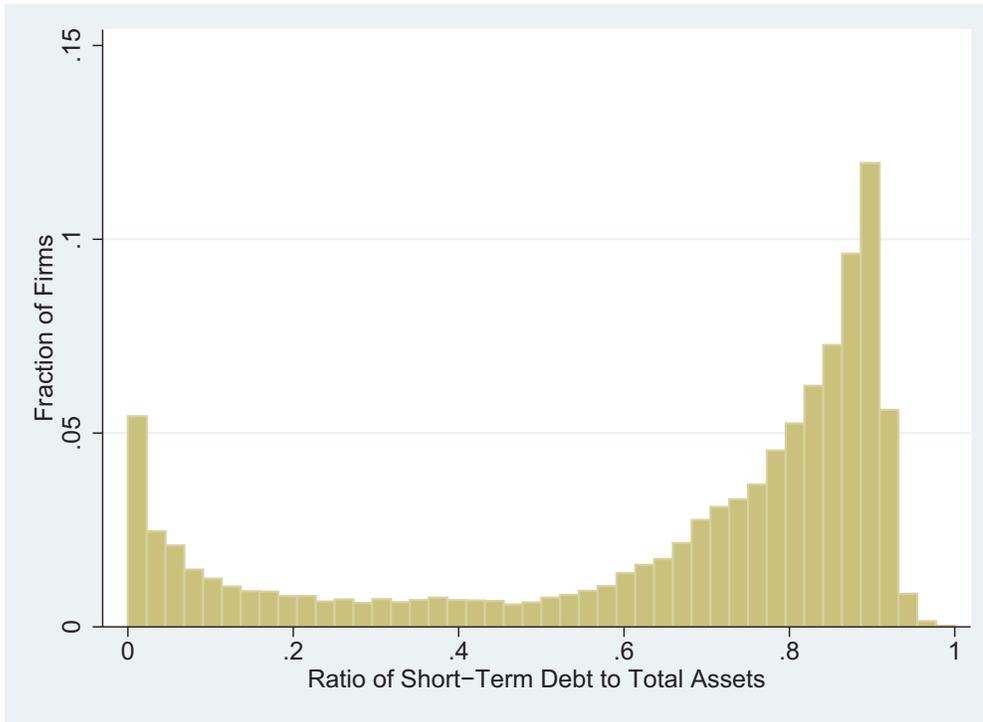
Figure 3: Short-Term Debt to Total Assets Ratios.
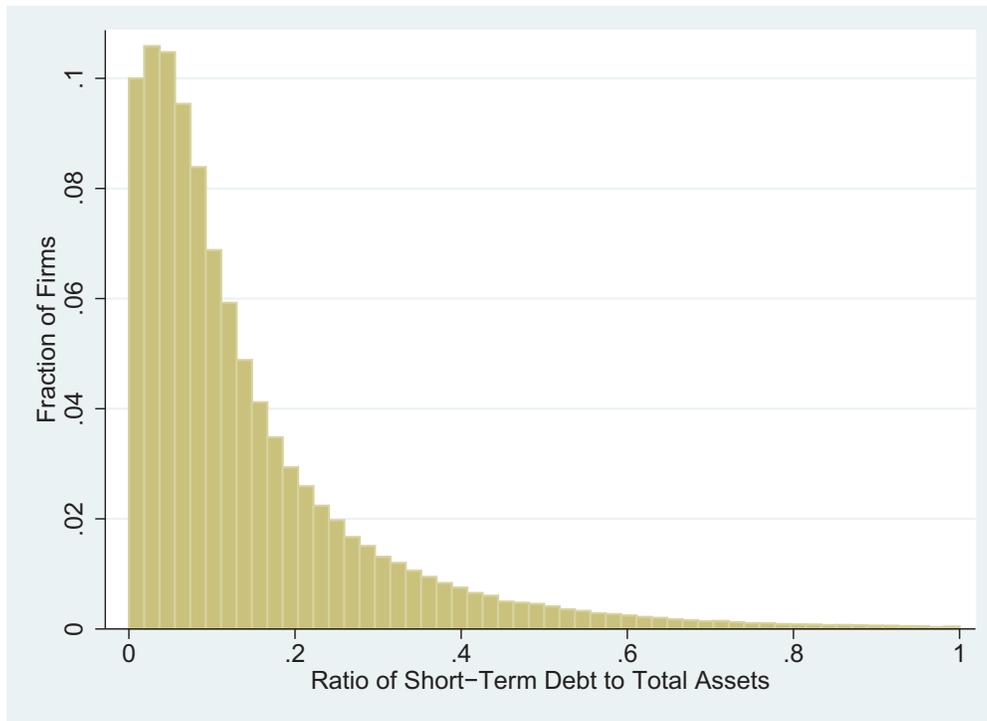
(a) Financial Firms
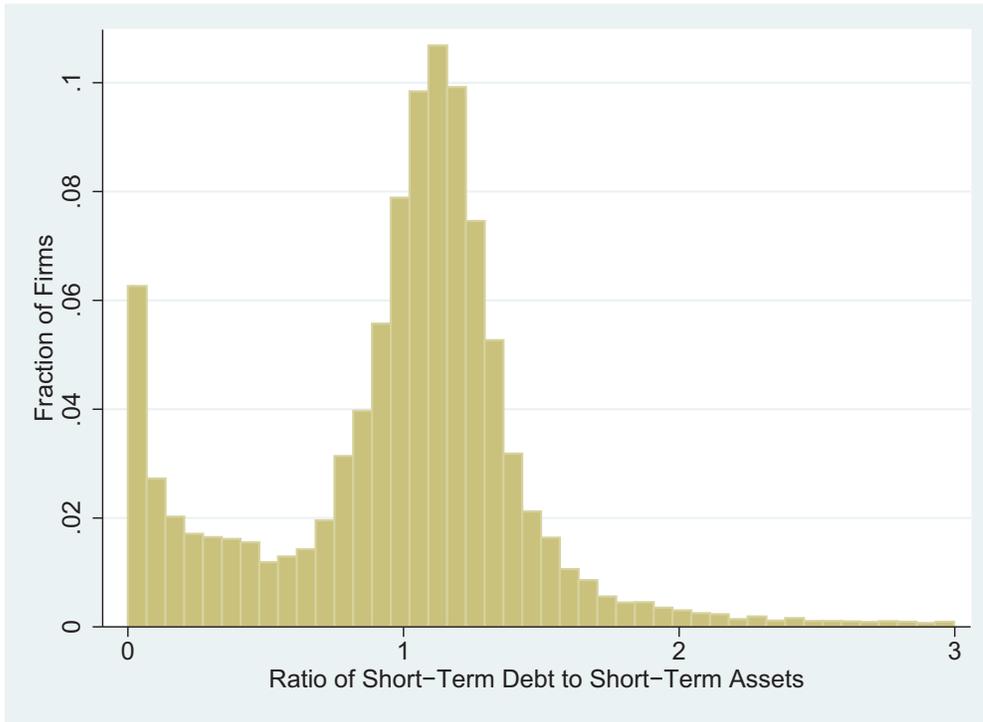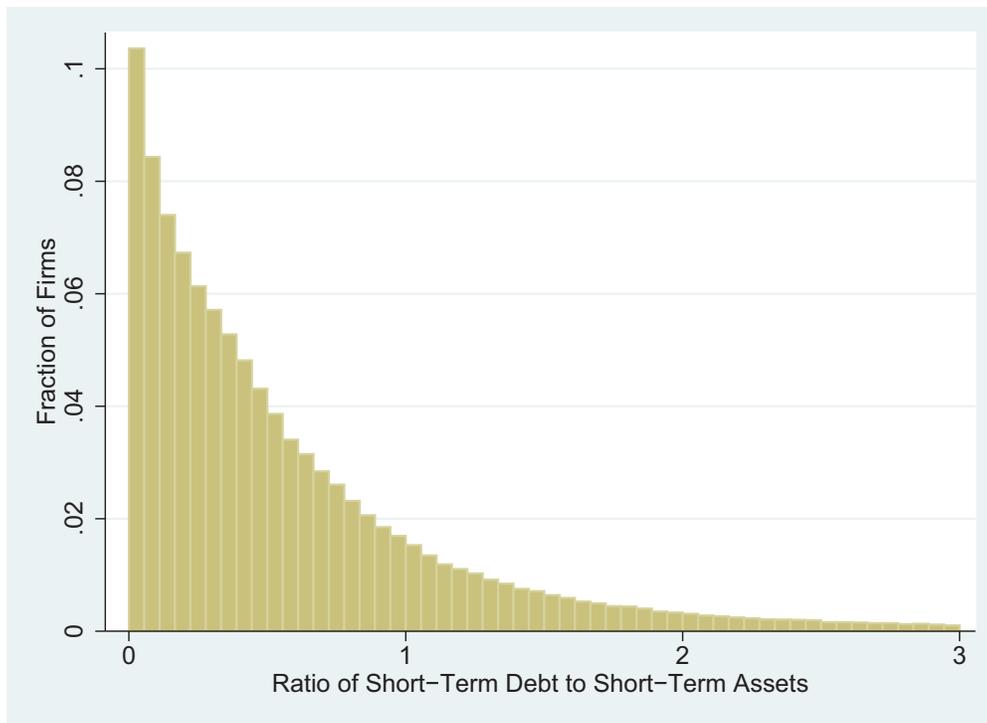

(b) Non-Financial Firms

Figure 4: Short-Term Debt to Short-Term Assets, Ratios.

(a) Financial Firms

(b) Non-Financial Firms

A2. Proofs

Proof of Lemma 1

Here, we prove the following features of optimal contracts under commitment:

1. $p_1^b(y_l) = 0$,
2. $p_2^b(y_l, z_l, v) = p_2^b(y_h, y_l, v) = 0$,
3. $p_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$,
4. If the banker's period 0 outside option is sufficiently low, then

$$p_1^b(y_h) + \int_v U_1(y_1, v)G(dv) = \frac{\bar{q}}{\pi_h - \pi_l} + \int_v U_1(y_l, v)G(dv).$$

We show these results in turn.

1. $p_1^b(y_l) = 0$. Since the banker must be provided incentives to exert high effort, equation (4) can be written more simply as

$$p_1^b(y_h) + \int_v x(y_h, v) \left[E_{\pi_h} p_2^b(y_h, z_2, v) - \bar{q}\right] G(dv)$$
$$\geq \frac{\bar{q}}{\pi_h - \pi_l} + p_1^b(y_l) + \int_v x(y_l, v) \left[E_{\pi_h} p_2^b(y_l, z_2, v) - \bar{q}\right] G(dv) \quad \text{(A1)}$$

   Clearly, if $p_1^b(y_l) > 0$, then by setting this payment, $p_1^b(y_l) = 0$ relaxes the incentive constraint and frees up resources which can be paid to depositors in period 1. These payments to depositors must be constructed to be incentive compatible and hence must increase payments uniformly in v. This can be done in a straightforward fashion so that the term

$$x(y_l, v_i, v_{-i})p_{1c}^i(y_l, v_i, v_{-i}) + (1 - x(y_l, v_i, v_{-i}))p_{1n}^i(y_l, v_i, v_{-i})$$

   increases by a constant and preserves depositors' incentives.

2. $p_2^b(y_l, z_l, v) = p_2^b(y_h, z_l, v) = 0$. Exactly as above, the first term, $p_2^b(y_l, z_l, v)$ enters there right hand side of (A1) and so reducing this term relaxes this constraint. Hence, the value $p_1^b(y_h)$ can be reduced and paid to depositors after high first period output has been realized in an incentive compatible way. The second term, $p_2^b(y_h, z_l, v)$ enters the right hand side of the period 1 incentive constraint for the banker (equation (3)) but the left hand side of the period 0 incentive constraint, so it is not immediate that this term can be set to zero. However, since both the banker and the depositor are risk-neutral, we may, without loss of generality set $p_2^b(y_h, z_l, v) = 0$.[2]

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[2] For any contract with $p_2^b(y_h, z_l, v) > 0$, there is another incentive compatible contract with $p_2^b(y_h, z_l, v) =$

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3. $p_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$. As in the previous claim, if $p_2^b(y_l, y_h, v) > \bar{q}/(\beta(\pi_h - \pi_l))$, then this value can be reduced while respecting the period 1 banker's incentive constraint. Doing so also relaxes the period 0 incentive constraint which allows more resources to be paid to depositors in period 1 following a realization of high first period output.

4. Constraint (A1) holds with equality. Ignoring the banker's participation constraint, if equation (A1) is slack, then payments to depositors can be increased in an incentive compatible way following high first period output. As a consequence, in an optimal contract, this equation must hold with equality.

Proof of Proposition 1

Here, we demonstrate that optimal contracts under commitment have the feature that $x(y_l, v) = 0$ and $x(y_h, v) = 1$. To see that $x(y_l, v) = 0$, suppose instead that the optimal contract had $x(y_l, v) > 0$ for v in some positive measure set, B. We construct an alternative contract which is incentive compatible and yields strictly higher welfare, ex-ante, for depositors. The alternative contract has $\hat{x}(y_l, v) = 0$ for all v and has payment to the banker in period one following a realization of high output of

$$\hat{p}_1^b(y_h) = p_1^b(y_h) - \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_{v \in B} x(y_l, v) dG(v).$$

The savings is paid out equally to all depositors in period 1 by increasing p_{1c}^i and p_{1n}^i for all reports, v. If equation (10) is satisfied, then this perturbation must strictly increase welfare.

Here we show that that optimal contracts under commitment satisfy $x(y_h, v) = 1$ for all v. **TO BE COMPLETED.**

Proof of Proposition 2

We prove that as $N \to \infty$, the probability that $x(y_l, v) = 1$ is ex-post efficient tends to 1. Consider the problem of maximizing depositor welfare given the incentive constraint of the banker. This problem is given by

$$\max \sum_i \int_v x(v) \left[-I + v_i p_2^i(v) \right] dG(v)$$

subject to

$$\sum_i p_2^i(v) \leq I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}$$

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0 which gives each depositor the exact same utility and respects the banker's incentives.

and $p_2^i(v) \geq I/(v_i N)$ where we have nested the banker's incentive constraint in the above resource constraint. Clearly, efficiency dictates that $x(v) = 1$ if and only if

$$\frac{I}{N} \sum_i \frac{1}{v_i} \leq I + \pi_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}.$$

Under the assumption of the proposition that

$$IE\left[\frac{1}{v_i}\right] < I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)},$$

by a law of large numbers, the result follows.

Proof of Lemma 3

Define

$$\zeta_i(v_i) = \int_{v_{-i}} x(v_i, v_{-i}) p_2^i(v_i, v_{-i}) G_{-i}(dv_{-i})$$

$$\rho_i(v_i) = \int_{v_{-i}} x(v_i, v_{-i}) G_{-i}(dv_{-i})$$

Then

$$u_i(v_i) = -\frac{I}{N} \rho_i(v_i) + v_i \zeta_i(v_i). \tag{A2}$$

Adding and subtracting $\rho_i(\hat{v}_i) I/(N\hat{v}_i)$ to the incentive constraint implies

$$\frac{1}{v_i} u_i(v_i) \geq \frac{1}{\hat{v}_i} u_i(\hat{v}_i) + \rho_i(\hat{v}_i) \frac{I}{N} \left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right]$$

and similarly

$$\frac{1}{\hat{v}_i} u_i(\hat{v}_i) \geq \frac{1}{v_i} u_i(v_i) + \rho_i(v_i) \frac{I}{N} \left[\frac{1}{v_i} - \frac{1}{\hat{v}_i}\right].$$

Combing these inequalities, we obtain

$$\rho_i(v_i) \frac{I}{N} \frac{v_i - \hat{v}_i}{\hat{v}_i v_i} \geq \frac{1}{v_i} u_i(v_i) - \frac{1}{\hat{v}_i} u_i(\hat{v}_i) \geq \rho_i(\hat{v}_i) \frac{I}{N} \frac{v_i - \hat{v}_i}{\hat{v}_i v_i}$$

$$\frac{\rho_i(v_i)}{\hat{v}_i v_i} \frac{I}{N} \geq \frac{\frac{1}{v_i} u_i(v_i) - \frac{1}{\hat{v}_i} u_i(\hat{v}_i)}{v_i - \hat{v}_i} \geq \frac{\rho_i(\hat{v}_i)}{\hat{v}_i v_i} \frac{I}{N}$$

For $v_i > \hat{v}_i$, this implies that $\rho_i(v_i)$ is increasing in v_i. Taking limits as $\hat{v}_i \to v_i$, we have

$$\frac{1}{v_i} \rho_i(v_i) \frac{I}{N} = u_i'(v_i) - \frac{1}{v_i} u_i(v_i).$$

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By solving the differential equation we, obtain the integral form of the local incentive constraint given

$$u_i(v_i) = v_i \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N} \int_{\underline{v}}^{v_i} \frac{1}{z^2} \rho_i(z) dz \right] \tag{A3}$$

This concludes the "If" portion of the proof. The "Only if" portion follows standard arguments.

Proof of Lemma 4 We prove the following "If" statement: Suppose (p_2^i, x) satisfy the depositor's incentive constraints, depositors participation constraints, and the period 2 resource constraint. Then (21) is satisfied and $\rho_i(v_i)$ is increasing for all i. To see this result, recall by Lemma 3 that we obtain immediately that ρ_i is increasing. Next, recall the resource constraint

$$\sum_i \int_v x(v) p_2^i(v) G(dv) \leq \left[I + \rho y_l + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right] \int_v x(v) G(dv)$$

Using the definition of $\zeta_i(v_i)$, we can express expected payments to depositor i in period 2 as $\int_v x(v) p_2^i(v) G(dv) = \int_{v_i} \zeta_i(v_i) G_i(dv_i)$. We now expand the term $\zeta_i(v_i)$ using results from the proof of Lemma 3. First, note from equation (A2), the expected value of ζ_i can be written as

$$\int_{v_i} \zeta_i(v_i) G_i(dv_i) = \int_{v_i} \left[\frac{u_i(v_i)}{v_i} + \frac{I}{N v_i} \rho_i(v_i) \right] G_i(dv_i).$$

Expanding $u_i(v_i)/v_i$ using equation (A3), we then have

$$\int_{v_i} \zeta_i(v_i) G_i(dv_i) = \int_{v_i} \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N} \int_{\underline{v}}^{v_i} \frac{1}{z^2} \rho_i(z) dz + \frac{I}{N v_i} \rho_i(v_i) \right] G_i(dv_i).$$

The above expression can be simplified with straightforward calculus to

$$\int_{v_i} \zeta_i(v_i) G_i(dv_i) = \frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N} \left[\int_{v_i} \rho_i(v_i) \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] G_i(dv_i) \right].$$

Summing over i and combining with the resource constraint yields the desired result.

The "Only if" can be demonstrated using a transfer scheme similar to that considered in Mailath and Postlewaite (1990).

Proof of Lemma 5 The general idea is to consider the following auxiliary problem:

$$\max \int_v x(v) G(dv)$$

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subject to

$$\int_v x(v) \left[I + \pi_h z_h - \pi_h \frac{\bar{q}}{\beta(\pi_h - \pi_l)} - \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \right] G(dv) \geq 0$$

Then, the optimal continuation rule has the property that

$$x(v) = 1 \Leftrightarrow I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \geq \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right]$$

In any case, forming the lagrangian, we have that $x(v) = 1$ if and only if the condition above modified by incorporating the inverse of the lagrange multiplier on the implementability constraint. An argument from Mailath and Postlewaite (1990) ensures that the lagrange muliplier converges to ∞ as $N \to \infty$ so that this term vanishes in the limit. Then, the term multiplying I, by a law of large numbers, converges to

$$E \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] = \frac{1}{\underline{v}}$$

Thus, as $N \to \infty$, the RHS converges to $\frac{I}{\underline{v}}$ and $\underline{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) < I$. Therefore, $x(v) \to 0$ for all v. This completes the proof.

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