External Financing and the Role of Financial Frictions over the Business Cycle: Measurement and Theory∗

Ariel Zetlin-Jones  Ali Shourideh

CMU Wharton

azj@andrew.cmu.edu shouride@wharton.upenn.edu

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Abstract

We investigate the importance of financial shocks which disrupt firms’ access to external funds over the business cycle. Empirically, we measure how much external funds firms use for investment. We find significant heterogeneity in firms’ use of external funds in the cross-section: roughly 80% of investment by privately-held firms is financed externally compared to 20% for publicly-traded firms. We develop a model with financial constraints consistent with the observed financing patterns. Quantitatively, we find that financial shocks induce significant fluctuations in output but may not be able to account for particularly large fluctuations such as the Great Recession.

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1 Introduction

Financial markets have long been thought to play an important role in business cycle fluctuations. Recently, many authors have identified shocks to financial markets as one source of disruptions in economic activity. The conventional view is that firms use external funds for investment and shocks to financial markets disrupt the flow of funds among firms and households, thereby interrupting production. In this paper, we investigate the importance of this channel both empirically and theoretically. Empirically, we measure how much external funds firms use both in the aggregate and in the cross-section. We show that the use of external funds for investment is particularly high among privately-held firms – around 80% of investment – compared to 20% for publicly-traded firms. Inspired by this evidence, we develop a model of public and private firms consistent with the observed financing patterns and in which financial market shocks disrupt the flow of funds among firms and induce recessions. Our model illustrates the quantitative importance of non-financial linkages that allow financial shocks to transmit even to those firms that do not use external funds for investment. We find that privately-held firms play an important role in the transmission of financial shocks to the aggregate economy. In our quantitative model, we find that a financial shock calibrated to induce an average fall in aggregate leverage generates a 0.4% decline in GDP – roughly of the same order of magnitude as a average aggregate productivity shock. However, when the size of the financial shocks is calibrated to generate a decline in aggregate leverage observed in the U.S. during the 2007-2009 recession, the model implies a decline of only about 1% in GDP. In this sense, we argue that financial shocks may play a sizable role over the business cycle, but they may face challenges in rationalizing particularly large economic recessions.

We focus on studying the importance of the role financial markets play in allocating capital across households and firms. Specifically, we think of financial markets as channeling funds to those firms that require external funds to finance their investment. We think of shocks to financial markets as shocks that limit the amount of funds those firms can borrow to finance investment. Understanding the quantitative importance of this role of financial markets requires an understanding of how much firms actually borrow to finance investment.

Our approach is to analyze the flow of funds into and out of firms. In particular, our measure of external financing is the difference between a firm’s revenues after paying for interest, taxes, and payments to labor and other businesses, which we refer to as Available Funds, and that firm’s capital expenditures, or Investment. A firm with available funds less than its investment needs must necessarily use external funds. A firm with available
funds in excess of its investment needs generates an outflow of funds. This relationship
gives us a sense of how much firms use external funds, and through the lens of a model
with financial frictions, is informative about the effect of shocks to financial markets on
aggregate outcomes.

Our evidence on the use of external funds is based on data from the U.S. and the U.K.
Evidence from aggregate data in both countries shows that available funds are almost
always higher than investment. In this sense, in the aggregate, firms do not use any
external funds to finance investment, or put differently, in most periods, funds flow from
firms to households not from households to firms. This suggests that disruptions in flow
of funds between households and firms may not play an important role in aggregate
fluctuations. However, it is possible that at times some particular firms rely heavily on
external funds even though in the aggregate they do not. To investigate this possibility,
we turn to cross-sectional evidence.

Evidence from the cross-section demonstrates two important sources of heterogeneity.
First, we find that in both the U.S. and the U.K., there are indeed firms that use external
funds for investment in certain periods and others that generate funds in excess of their
investment needs. Second, the extent to which firms use external funds for investment
is very different among firms of different listing statuses. Specifically, in any year for
publicly-traded firms, which we refer to as public firms, the total inflow of funds to firms
that use external funds is roughly 20% of total public firms’ investment. For privately-
held firms, which we refer to as private firms, the analogous number is roughly 80%. This
difference between public and private firms holds even when controlling for industry
and size. In particular, while there is significant heterogeneity in the use of external funds
across industries and size, within each category we consider, external financing is much
higher among private firms. These observations suggest that disruptions in flow of funds
among firms and particularly among private firms may play an important role in aggre-
aggregate fluctuations.

To study these fluctuations, we develop a general equilibrium model where financial
shocks disrupt the flow of funds among firms. Our model features both public and pri-
ivate firms and is consistent with our evidence on the different financing needs of these
firms. In our model, financial shocks might have an impact on the decision of the firms
who do not use external funds through investment, thereby amplifying the consequences
of these shocks.

We obtain these features by extending an otherwise standard model of firm dynamics
with financial frictions.\footnote{Examples of such models include Albuquerque and Hopenhayn (2004) and Buera et al. (2011) among}
are subject to idiosyncratic productivity shocks. These idiosyncratic shocks create a mo-
tive for capital to flow from low productivity, high net worth firms to high productivity,
low net worth firms. The latter firms require inflows of funds. Second, we introduce fi-
nancial frictions in the form of collateral constraints. These constraints limit the ability of
firms to borrow to finance investment and determine the equilibrium amount of external
funds the high productivity, low net worth firms use for investment. Financial shocks, in
the form of aggregate shocks to the collateral constraints, disrupt the flow of funds among
firms and distort the investment decisions of constrained firms.

Third, we partition firms exogenously into two groups of public and private firms.
The key difference between these firms is their ability to diversify certain forms of id-
iosyncratic risk. In particular, we assume that public firms are owned by diversified
households and may diversify their idiosyncratic productivity risk. Private firms, on the
other hand, are owned by non-diversified entrepreneurs who are unable to issue equity
and may only borrow to finance investment using debt. This distinction is motivated by
evidence from Vissing-Jorgensen and Moskowitz (2002) who show that owners of pri-
ate equity are less diversified than owners of public equity. The differences between
public and private firms in our model lead private firms to use more external funds than
public firms in equilibrium. Furthermore, these differences lead private firms to be more
sensitive to financial shocks than public firms. To the extent that private firms are more
sensitive to financial shocks, the aggregate response of our model economy to shocks to
the collateral constraint depends on both how much external funds they use for invest-
ment and how they are linked to the rest of the economy.

Fourth, we introduce non-financial trade linkages across firms. In particular, we as-
sume that each firm, both public and private, produces a differentiated good that is used
in the production of a final good. Moreover, we assume that there is monopolistic com-
petition between firms as in Dixit and Stiglitz (1977). Additionally, all firms use the final
good produced in the economy as an intermediate input. We illustrate how these non-
financial linkages may allow shocks that only directly affect a subset of (constrained)
firms to transmit to other (unconstrained) firms in the economy.

Consider next the qualitative consequence of an aggregate financial shock in our model.
This financial shock tightens the collateral constraints faced by all firms and directly im-
pacts the production and investment decisions among constrained firms. Not surpris-
ingly, constrained firms are typically productive, private firms with low net worth, and
the financial shock limits their demand for capital, labor, and intermediate inputs. Hence,
the financial shock induces more misallocation of capital across firms leading to a decline
many others.
in aggregate productivity and output.

How does a financial shock impact the production decisions of unconstrained firms? We show that general equilibrium effects have opposing forces on the production decisions of unconstrained firms – most of which are public firms in our model. The reduced demand for capital and labor by constrained firms induces a decline in the capital rental rate and the wage rate. Downward pressure on the costs of production tends to raise the capital and labor demand as well as the output of unconstrained firms. However, the decline in aggregate income as well as the decline in demand for intermediate inputs by constrained firms induces a decline in aggregate demand of goods produced by all firms in the economy. Because of monopolistic competition, this decline in aggregate demand tends to lower the output of unconstrained firms.

Theoretically, we derive conditions such that in response to a financial shock, all firms – both constrained and unconstrained firms – reduce their output. Our quantitative results illustrate that while initially, public firms may increase their production in response to the financial shock, over time public firms produce less output and undertake less investment leading to a persistent recession. Consequently, our results establish that under a plausible quantitative specification of the non-financial trade linkages among firms, even unconstrained firms will reduce their production and investment following a financial shock, at least in the medium run.

In our model, a shock that tightens the collateral constraints is likely to lead to a larger decline in output when more firms have binding collateral constraints. Hence, disciplining the extent to which firms face binding collateral constraint is critical for the behavior of the economy in response to financial shocks. We use our evidence from the cross-sectional data on financial flows to discipline the tightness of the collateral constraints in the steady state of our model.

We then examine the effect of a shock to collateral constraint, which generates a one standard deviation decline in the aggregate debt-to-asset ratio (upon impact). We let this shock be partially persistent and slowly dissipate. We find that this financial shock causes a decline in aggregate GDP of roughly 0.4% on impact. The response of the economy to such a shock is similar in order of magnitude to average aggregate productivity shocks,\(^2\) emphasizing the importance of financial shocks.

Finally, we consider imposing a larger financial shock which generates the decline in aggregate debt-to-assets observed following the Great Recession in the U.S since 2007. We show that the response of the economy to such a large financial shock is roughly a 1% decline in the Solow Residual causes a 0.9% decline in GDP on impact.

\(^2\)We illustrate below that in our model, an aggregate shock which induces a one standard deviation decline in the Solow Residual causes a 0.9% decline in GDP on impact.
decline in aggregate GDP. We conclude that while financial shocks may play a sizable role over the business cycle, in our model financial shocks face challenges in generating the kind of fluctuations in output and investment experienced by the U.S. economy during the Great Recession.

The nature of our exercise is closest to that of Jermann and Quadrini (2012) and Khan and Thomas (2013). In a seminal paper, Jermann and Quadrini (2012) document the behavior of aggregate flows in the form of debt and equity over the business cycle between households and firms. Complementing their exercise, we instead focus on the cross-section of firms and identify the factors affecting external financing. More recently, Khan and Thomas (2013) show that in an economy with firm heterogeneity and irreversibilities, a large financial shock can generate a large recession of the magnitude observed in the Great Recession. While our model has a similar flavor, there are two key differences that lead to contrasting results. First, disciplined by external financing data, our model includes public firms who are less constrained and can dampen the effect of the financial shock – the presence of trade linkages mitigates some of this effect. Second, we differ in the data we use to calibrate the magnitude of the financial shock. If we calibrate our financial shock in the same fashion as Khan and Thomas (2013), we obtain larger effects – roughly a 2% decline in GDP – but still substantially smaller than the observed decline during the Great Recession. In other words, once our model matches patterns of external financing in the data, our model can generate at best a mild recession. In this sense, we conclude that it is a challenge for financial shocks to rationalize particularly large recessions in models which are consistent with patterns of external financing.

1.1 Related Literature

Our paper is related to an extensive literature on the effect of financial frictions in macroeconomics, starting from Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke et al. (1999), and more recently Kiyotaki and Moore (2008) and Jermann and Quadrini (2012). The common goal is to identify and understand the channels through which financial market disruptions affect economic activity and their quantitative importance. Our goal in this paper is the same while our approach differs since we use data on external financing to discipline the importance of these channels.

Our empirical work on external financing needs is related to a literature in corporate finance that attempts to identify the extent to which firms face constraints in financing their investment (see Fazzari et al. (1988), and Gilchrist and Himmelberg (1995) among
The approach in this literature is to test the implications of models with financing constraints, namely that Tobin’s Q as well as cash-flow would have a significant effect on investment. Our approach, however, differs from theirs in that our measurement approach emphasizes the role of financial markets in firms’ financing decision, i.e., how much of their investment is financed using external funds. In this regard, our approach is closer to the one taken by Rajan and Zingales (1998) and Buera et al. (2011). Additionally, this measurement approach allows us to abstract from which firms in particular face binding financing constraints while disciplining the importance of financial markets for investment.

It is worth pointing out that while many of the papers in the literature on interaction of financial markets and the real economy take a strong stand on financial flows among firms and households, they do not try to use observation about use of external funds to discipline their models. For example, Bernanke et al. (1999) take a stand on the distribution of financial inflows in the data by assuming that all the investing firms use external funds (and are subject to constraints) while exiting firms generate external funds. Alternatively, Jermann and Quadrini (2012) assume that a representative firm faces adjustment costs to dividends. As a consequence, even though their representative firm generates external funds as implied by our measure, not all of these funds are available for investment since part of it has to be paid out as dividends. However, since their model is a representative agent model, they cannot address the issue of the heterogeneity observed in the use of external funds. To the best of our knowledge, our paper is the first paper that tries to use evidence on the use of external funds to discipline models of financial frictions.

From a modeling perspective, our model of financial frictions is a natural extension of Evans and Jovanovic (1989) to dynamic environments. The basic structure of the model is very similar to Gomes (2001). While he models financial frictions as additional costs to external financing, we follow Evans and Jovanovic (1989) and assume that investment is bound by a factor of net worth. Our model is also related to an extensive series of papers on the effect of idiosyncratic investment risk on firm dynamics and its financial structure, including Cooley and Quadrini (2001), Hennessy and Whited (2005), and Angeletos (2007) to name a few. More recently, Eisfeldt and Muir (2014) use an approach similar to that of Gomes (2001) in a model where firms can invest in low-yield liquid assets to measure the cost of external financing.

Our modeling approach is also similar to an extensive literature that analyzes the effects of financial frictions on misallocation and Total Factor Productivity, such as Midrigan and Xu (2014), Buera et al. (2011), and Moll (2014). While our basic model is very

3Kaplan and Zingales (1997) question the validity of this approach.
similar, aside from inclusion of monopolistic competition and intermediate goods as inputs, we focus on short-term dynamics of the model. Moreover, because of our focus on business cycle frequency fluctuations and the importance of the role played by financial markets, our calibration is somewhat different. In particular, we calibrate the model to match evidence on external financing as well as the variance of debt-to-asset ratios while the papers mentioned above focus on the dynamics of firm size. While these measures are correlated, our focus on financial flows and external financing makes our paper different from theirs. Moreover, our firm level employment dynamics closely resemble the evidence documented by Davis et al. (2007).

As we mentioned above, our quantitative exercise is most closely related to Jermann and Quadrini (2012) where shocks to financial constraints cause fluctuations in economic activity. Our analysis, however, is substantially different from theirs. They focus on a model with a representative firm that is financially constrained and which faces an exogenously specified cost of reducing dividends to finance investment. One way of thinking about our paper is that we develop a model in which the cost of reducing dividends is endogenous, in the sense that for private firms, dividend reductions affect the marginal utility of consumption of entrepreneurs. We think of their paper as a first step toward developing a workhorse model for analysis of shocks to financial markets and the effect of such shocks on real activity. In this paper, we take the next step and show that there is a great deal of heterogeneity in firms’ reliance on external financing. This helps us impose further discipline on our model of financial frictions and firm heterogeneity and the mechanisms that translate financial shocks to the real economy. Our paper is also related to a number of studies that focus on how presence of financial frictions amplify and propagate the effect of productivity shocks to the economy, such as in Nezafat and Slavik (2013). Furthermore, our transitional dynamics exercise is very similar to Guerrieri and Lorenzoni (2011), however, they focus on changes in households’ borrowing opportunities while we focus on the production side of the economy.

The paper is organized as follows: Section 2 documents evidence on firms’ external financing behavior; Section 3 contains our model and theoretical analysis; Section 4 contains our main quantitative exercise; and Section 5 is our conclusion.

2 Evidence on the Use of External Financing

In this section, we present evidence on the use of external funds for investment in the aggregate and at the firm level in the United States and in the United Kingdom. We establish the following stylized facts:

1. In the aggregate, firms can self-finance the entirety of their investments. In other
words, total payouts from the non-financial corporate businesses to debt and equity holders are positive most of the times – except 12 quarters in the entire period from 1952-2014.

2. In the cross-section of firms, private firms use external funds much more than public firms.

3. The difference between use of external financing between public and private firms cannot be attributed to other characteristics such as size, industry, or profitability.

We begin this section by discussing our measure of a firm’s use of external financing. We then describe our data sources and outline our key facts on external financing.

2.1 Conceptual Measurement of Financial Flows

We start by constructing our empirical measure of external financing. We say an individual, or a representative firm, uses external funds if its capital expenditure exceeds its net cash flows from operations. The use of external funds is defined as the difference between capital expenditure and net cash flows. We define the total use of external funds in the cross-section for any subset of firms in any year as the total amount of external funds used by firms whose capital expenditure exceed their net cash flows. We normalize total use of external funds by aggregate investment by the subset of firms in order to provide a sense of scale.

Put differently, our measure of the use of external funds is simply the net financial inflow to a firm and can be illustrated by the following budget constraint:

\[ Div_t + \Delta FA_t + Int_t + X_t = \Pi_t + IFA_t + \Delta B_t + \Delta EQ_t. \]

\( Div_t \) represents dividends paid to equity holders; \( \Delta FA_t \) is the change in the stock of financial assets; \( Int_t \) is total interest payments on debt; \( X_t \) is physical investment; \( \Pi_t \) is revenues from operations less cost of goods sold and taxes; \( IFA_t \) is income from financial investments; \( \Delta B_t \) is the change in debt; and \( \Delta EQ_t \) is the change in equity. We rearrange this expression as

\[ Div_t + \Delta FA_t - \Delta B_t - \Delta EQ_t s = \Pi_t + IFA_t - Int_t - X_t. \]  

The term \( \Pi_t + IFA_t - Int_t \) is net cash flow from operations which we will refer to as \textit{Available Funds}. The left hand side of equation (1) represents the net financial flow into or out of the firm. If the right hand side of equation (1) is positive so that the firm’s investment is less than available funds, then funds flow out of the firm and we say the firm generates external funds. If the right hand side of equation (1) is negative, then funds must flow into the firm as the firm’s investment is greater than available funds and we say the firm
uses external funds. Note that by netting interest payments, \( \text{Int}_t \), out of available funds, but leaving dividend payments, \( \text{Div}_t \), on the left hand side, our measure treats dividends as financial flows as opposed to costs of operation. Conceptually, this is consistent with the view that the firm can adjust its dividends contemporaneously while its interest payments are fixed in advance.

If a firm uses external funds, it finances its capital expenditure either through issuing debt or equity, or via reducing its holding of financial assets. While financial inflows measure use of external funds, an inflow or an outflow for a firm does not necessarily imply that the firm is financially constrained. For example, a firm might be using external funds by simply selling some of its financial assets which does not necessarily imply that the firm is constrained in any way. On the other hand, a firm’s external funds may come from issuing debt which may or may not be subject to constraints. Our approach here is to simply measure the use of external financing by firms, and later we use a stylized model consistent with these measures to obtain implications on how binding financial constraints may be.

When we study the aggregate or representative firm, our use of external funds is the difference between aggregate investment and aggregate available funds. Furthermore, our preferred statistic of how individual firms rely on external funds for investment is given by

\[
\frac{1}{T} \sum_{t=1}^{T} \frac{\sum_{i \in J} (X_{it} - AF_{it}) \mathbf{1}_{[X_{it} \geq AF_{it}]} }{\sum_{i \in J} X_{it}}
\]

(2)

where \( J \) is the subset of firms with a given characteristic. The statistic in equation (2) represents the time series average of the total financial inflow to firms whose investment is greater than their available funds as a fraction of total investment. The statistic in equation (1) informs us about what fraction of aggregate investment must be financed externally among a subset of firms and, therefore, depends on well-functioning financial markets.

### 2.2 Data Description

We base our analysis on data from the U.S. and the U.K. Our data sources for the U.S. include the Flow of Funds and Compustat. Our data sources for the U.K. include the U.K. Economic Accounts, Compustat Global, and Amadeus. We now describe each data set and how we measure available funds and investment in each source. Details of these constructions are in the Appendix.

**Aggregate Data**—Aggregate U.S. data are from the Flow of Funds. Our measure of ag-
aggregate available funds in the U.S. is the sum of Internal Funds and Dividends of all non-financial corporate businesses. Note that we add dividends here because the definition of Internal Funds has excluded them. Our measure of aggregate investment is given by Capital Expenditures of non-financial corporate businesses.

Our source of aggregate data in the U.K. is the National Economic Accounts. We measure available funds as the sum of gross disposable income and dividends of private non-financial corporations. Again, we add dividend payments because they have been previously excluded in the construction of gross disposable income. We measure investment as the sum of gross fixed capital formation, change in inventories, and acquisitions less disposals of valuables and non-financial, non-produced assets.

**Firm-Level Data**—Our source of firm-level data on public firms in the U.S. and the U.K. is Compustat. In the U.S., we focus on data from 1974 to 2013 for non-financial firms headquartered in the U.S. In the U.K., we focus on data from 1992 to 2013 for non-financial firms headquartered in the U.K. After undertaking standard data cleaning, our Compustat sample in the U.S. consists of about 51,000 firm-year observations, with roughly 1,400 firm level observations in a typical year. Our Compustat U.K. sample consists of roughly 10,000 firm-year observations with 550 firm level observations in a typical year. We measure available funds as Net Cash Flow from Operating Activities. Since we do not want to distinguish between physical investment in existing assets or acquisition of new assets, we define investment as the sum of capital expenditures and acquisitions less sale of property, plant, and equipment.

Our source of firm-level data on private firms is the Amadeus database. Amadeus contains financial information on over 18 million private and public firms in Europe with a focus on private firms. We restrict attention to the sample of private limited firms which are not quoted on a stock exchange and are located in the United Kingdom from 2005 to 2012. Our sample consists of over 980,000 firm-year observations with roughly 100,000 firm-level observations in each year. We focus on data from the balance sheet and profit and loss account. Our measure of available funds is profit per period plus depreciation; our measure of capital expenditures is the change in tangible fixed assets plus depreciation.

### 2.3 Facts about Financial Flows and External Financing

We now describe the facts on financial flows and external financing. We begin by describing the lessons from aggregate data and then discuss the evidence from the cross-section.
Aggregate Facts on Financial Flows—Figures 1 depicts available funds and investment for the U.S. normalized by non-financial corporate business GDP (see the analogous Figure 9 for the U.K.). On average, available funds are roughly 1.25 times as large as investment in the U.S. and 1.6 times as large in the U.K. Moreover, in the U.S., available funds exceed investment by roughly 3% on average over the entire sample. In this sense, the aggregate firm in the U.S. and the U.K. does not rely on outside financing to fund investment.  

Figure 1: U.S. Available Funds and Investment Normalized by Non-Financial Corporate Business GDP.

This observation has important implications for the way shocks to financial markets may affect the macroeconomy. In particular, this finding suggests that shocks to financial markets do not disrupt the economy by limiting the flow of funds to firms in the aggregate. Obviously, it is possible that particular firms use external funds in particular periods and shocks to financial markets may directly impact investment choices of these firms. Next, we attempt to understand the extent to which individual firms use external funds as well as the key characteristics of those firms that do use external funds.

Firm-Level Facts on Financial Flows—In the cross section, the use of external funds by public firms is 22% of aggregate investment (by public firms) in the U.S., and 18% in the U.K. In contrast, use of external funds by private firms is 82% of aggregate investment

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4In the Appendix, we also plot these figures with a measure of Internal Funds, which is our measure of available funds less dividends. Even when we subtract dividends from available funds, in most periods we find that Internal Funds are larger than investment in the aggregate. In this sense, even if aggregate dividends cannot be adjusted contemporaneously by firms, firms may not rely extensively on financial markets for external funds.

5The average reported for public firms in the U.K. excludes the years 1999 and 2000 because of a lack of
by private firms in the U.K. In Figure 2, we plot the annual use of external funds in our data samples.\textsuperscript{6} This figure shows that the different use of external funds by public and private firms is persistent over time.

![Figure 2: Use of External Funds by Listing Status, and Country](image)

The above evidence points to two types of heterogeneity in use of external funds in the cross-section. First, there are indeed firms, both public and private, which use external funds for investment. Note that this is in contrast to the fact that in all samples, in the aggregate, firms generate funds in excess of what they use for investment in all periods. Second, our evidence also establishes that the extent to which firms use external funds for investment is very different across public and private firms.

Since private firms finance a large fraction of investment using external funds and since private firms account for a substantial fraction of investment in our sample (61.3% of U.K. sample investment on average from 2005-2012), the response of private firms to financial market shocks may contribute significantly to the aggregate consequences of financial market shocks. Our estimate of the importance of investment by private firms is consistent with estimates from Asker et al. (2011) who find that U.S. private firms account for 54.5% of investment as well as employment estimates from Davis et al. (2007) who find that private firm employment accounts for more than 70% of aggregate employment.

We now decompose our measure of use of external funds for the entire public or private sample. Our aim is twofold. First, we provide additional guidance on which types of data availability for acquisitions. Alternatively, if we ignore acquisitions, then over the entire public U.K. sample, roughly 8% of investment is externally financed.

\textsuperscript{6}We have experimented with winsorising our data sample at the 0.1% and 1% levels and found little change in terms of the differential importance in the use of external funds for investment by private versus public firms. These experiments are available on request.
public or private firms within a given sample use a substantial amount of external funds. Second, we illustrate that even controlling for industry or size, differences in the use of external funds by public and private firms are robust. As a result, we conclude that dividing our sample between public and private firms is not simply capturing differences in the nature of business activities or the scale of operations of these firms. As a result, we interpret the difference in the use of external funds for investment between public and private firms as arising from the possibility that public firms have greater access to financial markets than private firms. In all of the results that follow, we restrict attention to our sample of private firms in the U.K. from Amadeus and public firms in the U.K. from Compustat Global.

The first source of heterogeneity we consider is the difference in industry composition. One possibility is that private firms are concentrated more heavily in high external financing industries. This is not the case in our sample. While we do find differences in the industry composition of our public and private sample, we also find that within each industry, private firms use more external funds as a share of private firm investment than do public firms.

Table 1 reports each industry’s average share of investment and use of external funds by company type, either private or public. Note that patterns of investment shares by industry are broadly similar between public and private firms. Not surprisingly, the services sector accounts for a greater deal of investment in private firms while the manufacturing sector accounts for a larger share of investment by public firms.

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<th>Industry</th>
<th>Investment Share</th>
<th>Use of Ext. Fin.</th>
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<tr>
<td></td>
<td>Private</td>
<td>Public</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.58%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Construction</td>
<td>-1.32%</td>
<td>0.17%</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>19.53%</td>
<td>34.71%</td>
</tr>
<tr>
<td>Mining</td>
<td>17.68%</td>
<td>2.21%</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>10.31%</td>
<td>18.74%</td>
</tr>
<tr>
<td>Services</td>
<td>30.64%</td>
<td>8.85%</td>
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<tr>
<td>Transportation</td>
<td>17.39%</td>
<td>35.19%</td>
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<tr>
<td>Wholesale Trade</td>
<td>5.20%</td>
<td>1.03%</td>
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<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
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Table 1: Industry Share of Public or Private Investment in the United Kingdom.

Table 1 clearly illustrates that total use of external funds by private firms in each industry as a share of total private investment is larger than the analogous number for public
firms. In other words, the contribution to the total use of external funds by private firms from each industry is substantially larger than that from public firms at the broad industry classification level. Figures 11 and 12 show the time series variation in investment shares and use of external funds by industry and firm type. These figures show that investment shares of each industry are roughly stable and the differences in use of external funds by public and private firms are persistent.

Additionally, Table 1 reveals that public firms’ use of external funds in the manufacturing sector is higher than that of the services sector. This evidence is in line with similar findings in Buera et al. (2011). Interestingly, this relationship is reversed for private firms; the services sector externally finances a higher fraction of total investment among private firms. One reason for this difference is simply scale – service sector firms account for a substantially higher share of investment compared to the manufacturing sector firms. If we normalize use of external funds in these industries by industry specific investment, then we find use of external funds by these sectors are somewhat comparable (roughly 70% for manufacturing and 88% for services).

The second source of heterogeneity we consider is size. It is possible that dividing our sample into public and private firms is simply capturing differences in size because we expect those firms that typically become public are among the larger private firms. While public firms are typically larger than private firms, we show that large private firms use more external funds (as a share of total private investment) than do comparably sized large public firms.

Table 2 reports the time series average of the cross-sectional average and median of various measures of size for public and private firms. As expected, public firms are significantly larger in terms of assets, investment, and sales.

<table>
<thead>
<tr>
<th>Company Type</th>
<th>Total Assets</th>
<th>Investment</th>
<th>Sales</th>
<th>I/A</th>
<th>AF/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-Sectional Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>19.57</td>
<td>0.66</td>
<td>18.80</td>
<td>4.33</td>
<td>43.44</td>
</tr>
<tr>
<td>Public</td>
<td>1,191.48</td>
<td>53.15</td>
<td>1026.25</td>
<td>4.21</td>
<td>4.91</td>
</tr>
<tr>
<td>Cross-Sectional Median</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>0.24</td>
<td>0.002</td>
<td>0.38</td>
<td>1.23</td>
<td>14.99</td>
</tr>
<tr>
<td>Public</td>
<td>115.86</td>
<td>2.66</td>
<td>126.71</td>
<td>3.07</td>
<td>10.42</td>
</tr>
</tbody>
</table>

Table 2: Time Series Average of Sample Mean and Median measures for Public and Private firms in the United Kingdom. Assets, Investment, and Sales reported in millions of pounds. I/A is Investment-to-Total Assets ratio. AF/A is cash generated from operations (available funds) relative to lagged total assets.
Table 2 also reveals that the distinction between public and private firms in our sample does not necessarily capture simple differences in profitability or investment activity. Because Tobin’s q cannot be constructed for private firms, we have compared investment-to-asset ratios as well as cash return on assets for public and private firms. Table 2 reports the time series average of these statistics.

We see that on median, private firms undertake investment relative to their total assets on the same order of magnitude as public firms. Moreover, on median, the cash generated from operations relative to (lagged) total assets for public and private firms is roughly comparable. That is, relative to their size and over the entire sample, public and private firms undertake investment and generate available funds of roughly the same magnitude. This evidence suggests that private firms’ extensive use of external funds does not reflect a lack of investment opportunities.

With the understanding that private firms are typically smaller than public firms, we now seek to construct comparable samples by size and compare the use of external firms of similarly sized public and private firms. To do so, in each year we partition our sample of public firms into quartiles by total assets. This partition yields 3 thresholds per year which we then use to partition our sample of privately held firms. As a result, if a single private company is in the largest quartile, for example, and if that single firm were to become publicly traded, it would be in the top quartile among public firms. Within each public or private sample, for each quartile, we measure total use of external funds and normalize by total sample investment in that year. Thus, for each quartile, we find the contribution of that quartile’s use of external funds to the overall sample’s use of external financing.

Table 5 in Appendix B reports sample statistics for each quartile across public and private firms. Within each quartile, we do find some discrepancy in average firm size, sales, and investment between the largest privately and largest public firms, although the gap is much smaller than in the sample average. In the Appendix, we also consider smaller bin sizes (using deciles) which allow us to construct more directly comparable samples at the top of the firm size distribution. There, we argue that similar findings to those we show next in terms of use of external funds still hold.

Table 3 reports the time series average investment shares and use of external funds for each quartile. Figure 13 displays these measures for each quartile over time.

We draw three conclusions from Table 3. First, for both public and private firms, it is the largest group of firms that do the majority of investment for these samples and contribute the most to the entire sample’s use of external funds. Second, private firms at all asset classes use substantially more external funds, as a share of aggregate private
Table 3: Time Series Average of Use of External Funds, Investment Share, and Investment Share of Firms using External Funds for Public and Private firms by Public Firm Asset Quartiles in the United Kingdom

investment, than do the largest public firms. Third, among private firms, those firms that use external funds also undertake the majority of investment. For example, in the largest quartile, firms using external funds undertake over 50% of all private firm investment while all firms in the largest quartile account for roughly 62% of aggregate private firm investment. Figure 14 demonstrates that over entire the sample period where both public and private firm data are available, the differences in use of external funds by size are prevalent and sizable.

3 A Dynamic Model of Public and Private Firms

In this section, we develop a dynamic model of publicly-traded and privately-held firms and define a symmetric stationary equilibrium. In our model, all firms face constraints in accumulating capital, have a monopoly in producing differentiated goods and require a bundle of goods produced by other firms as an input to production. Public firms are owned by diversified households; private firms are owned by individual entrepreneurs. One can view our model as incorporating features of a model of firm dynamics under financial constraints (as in Albuquerque and Hopenhayn (2004), Buera et al. (2011), and Kiyotaki and Moore (2008) ) which incorporates novel ingredients in the form of multiple productive sectors (public and private) and trade linkages (monopolistic competition and the input-output structure).

3.1 Model and Equilibrium Definition

Environment—Time is discrete, lasts forever, and is indexed by $t = 1, 2, \ldots$. Agents in the model include households, final good producers, and intermediate good producers. There are two types of households: entrepreneurs and workers. There is a single consumption good in the economy which is a composite good produced by a sector of competitive final good producers. We normalize the price of the final good to be 1 in each
period. Final good producers aggregate the output of the intermediate good producers.

Each intermediate good producer has a monopoly in producing a differentiated output. There are two classes of intermediate good producers: private and public firms. We normalize the total measure of intermediate good producing firms to be 1, and we assume there is a fixed measure of private firms, \( s \). A firm’s type is exogenously given and fixed for the lifetime of the firm. Let firms \( i \in [0, s] \) denote the names of the \( s \) private firms and \( i \in [s, 1] \) denote the names of the \( 1 - s \) public firms in any period. Firms exogenously exit at rate \( \zeta \). Upon exit, firms are replaced by an otherwise identical firm endowed with the existing firm’s assets.

In our model, we distinguish between public and private firms by assuming that public firms are owned by and rebate dividends to diversified households. Private firms are owned by individual entrepreneurs for whom the cost of delayed dividend payments is forgone consumption.

In each period, a firm of either type can produce output according to the constant returns to scale production function

\[
y_{it} = z_{it} \left( k_{it}^{\alpha} l_{it}^{1-\alpha} \right)^{\eta} I_{it}^{1-\eta}
\]

where \( z_{it}, k_{it}, l_{it}, I_{it} \) are firm \( i \)'s productivity, capital input, labor input, and intermediate input in period \( t \), respectively. We assume that intermediate inputs are perfect substitute for the final good.\(^7\) (See Basu (1995) for an example of this type of input-output production structure). We assume that the firm-level process for productivity follows a first order Markov process governed by transition probability distribution, \( \Psi(z_{it}|z_{it-1}) \).

We now describe the problem and constraints of each type of agent in turn.

**Final Good Producers**—There are a large number of competitive final good producers. Each of these producers can combine the output of the intermediate producers to produce a composite final good according to the production function

\[
Q_t = \left[ \int q_{it}^{1-\frac{1}{\rho}} \, di \right]^\frac{\rho}{\rho-1}
\]

where \( q_{it} \) is the input of firm \( i \) in period \( t \), and \( \rho \) is the elasticity of substitution across all goods in the economy. Perfect competition among final good producers ensures that we

\(^7\) An equivalent formulation is to assume that there are no final good producers while households can consume a bundle of good produced by intermediate good producers. In addition, each intermediate good producer can also purchase a bundle of other intermediate good producers where elasticity of substitution for households and each intermediate good producers is the same.
can focus on a representative firm that solves

$$\max_{Q, q_{it}} Q - \int p_{it} q_{it} di$$

where $Q$ is given by the production function (3) in each period. The final good producer’s problem gives rise to an inverse demand curve for each intermediate good as a function of prices:

$$p_{it} = \frac{1}{Q_t^p} \frac{1}{q_{it}^p}.$$ (4)

**Financing and Timing of Production among Intermediate Good Producers**—Intermediate good producers are local monopolists who face an inverse demand function in (4). We assume that all intermediate good producers have access to competitive labor markets; however, their access to capital markets is limited. This limit is proportional to their financial asset holding or net worth. In particular, capital rented, $k_{it}$, must satisfy

$$k_{it} \leq \lambda a_{it}$$ (5)

where $a_{it}$ is the total financial assets held by the firm. The financial assets held by the firm must satisfy the following budget constraint:

$$d_{it} + a_{it+1} \leq p_{it} z_{it} \left( k_{it}^{\alpha} l_{it}^{1-\alpha} \right)^{\eta} l_{it}^{1-\eta} - w_{l} l_{it} - I_{it} - (r_{t} + \delta) k_{it} + (1 + r_{t}) a_{it}.$$ (6)

The key idea behind financial friction is that firms can post their financial assets as collateral in order to rent capital. The variable $\lambda$ determines the ease with which a firm can obtain capital using their financial assets as collateral and represents the ability of the financial markets to reallocate capital across different firms.

**Private Firms**—Each entrepreneur owns a private firm. We assume that these entrepreneurs face incomplete markets in the sense that ownership of a private firm is concentrated and entrepreneurs cannot issue equity. Furthermore, we assume that bankruptcy is an extreme event in which a firm’s entire stock of financial assets is reduced to zero. Critically,

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8See Zetlin-Jones (2012) for a version of the model with a slight change in timing of production where capital is owned by firms.

9One way to rationalize the use of such collateral constraint is to assume that firms have access to competitive financial intermediaries who receive deposits and rent capital to firms, while lending to firms is not perfectly enforceable. After production, firms can choose to default on their loan from the financial intermediary and, with probability $1/\lambda$ they are able to retain their undepreciated capital stock. If a firm defaults, the financial intermediary seizes the financial wealth of the firm, but the loss of financial wealth is the only punishment firms face from defaulting on their debt obligations. This implies that the intermediary only lets the firm borrow at most $\lambda a_{it}$. 

---
this bankruptcy risk is fully non-diversifiable. As a result, entrepreneurs’ preferences are given by
\[ E_j \sum_{t \geq j} (\beta(1 - \zeta))^{t-j} \log(d_{it}). \]
for \( i \in [0, s] \). As discussed before, in making their production and financing choices, entrepreneurs are subject to the budget constraint described in (6), the collateral constraint (5), and the inverse demand (4) for their production. The solution to the private firms’ problem gives rise to allocations as a function of the history shocks.

**Public Firms**—Public firms have the same production opportunities as a private firm. The main difference between public and private firms arises from their ability to diversify bankruptcy risk as well as differences in ownership. Since public firms are owned by diversified households, these firms can fully diversify the bankruptcy risk. As a result, public firms maximize the expected present discounted value of their dividends under the pricing kernel derived from household preferences, which, in the economy with no aggregate risk is given by \( M_t = \beta^t U_t \). The objective of a public firm is given by the expected present discounted value of dividends, or
\[ E_0 \sum_t M_t d_{it}. \]
Similar to private firms, the owners of the public firms are subject to a budget constraint, a collateral constraint, and an inverse demand function. Furthermore, we assume that they cannot issue any equity, i.e., \( d_{it} \geq 0 \).

**Bankruptcy and Entry**—As mentioned above, one key difference between private and public firms is the ability of the owners of public firms to diversify bankruptcy risk associated with a given firm. This implies that at each period a fraction \( \zeta \) of private firms exit. We assume that in each period an equal measure of firms are born and draw assets and productivities from the joint stationary distribution of assets and productivities. That is, we assume that there are no social losses in the value of the assets from bankruptcy.

**Households**—In each period, households decide how much to work, how much to consume and how much to save in the risk-free bond. They maximize lifetime expected utility
\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_{ht}, L_{ht}) \]
subject to the sequence of budget constraints

\[ C_{ht} + A^h_{t+1} \leq w_t L_{ht} + (1 + r_t) A^h_t + \int_s^1 d_i dt. \]

We assume that \( U(C, L) = u(C - v(L)) \) where \( v(L) = \psi t^{1+1/\epsilon} / (1 + 1/\epsilon) \) and \( u'(0) = \infty \) as in Greenwood et al. (1988). Note that since households face no constraint on their saving, their Euler equation implies that the pricing kernel is given by \( M_t = \Pi_{s=0}^{t-1} \frac{1}{1+r_s} \). In what follows, we will use this in formulating the problem of a public firm.

**Equilibrium Definition**—The aggregate state of the economy in any period can be summarized by the distribution over net worth, \( a \), and productivity, \( z \), of private firms which we denote by \( G_{u,t}(a, z) \), of public firms which we denote by \( G_{t,t}(a, z) \) and household assets \( A^h_t \). Thus a stationary recursive competitive equilibrium is defined as follows:

**Definition 1** A stationary recursive competitive equilibrium consists of prices, \( p_t(a, z) \), \( w, w \), aggregate output \( Q \), distributions \( G_t(a, z), G_{u,t}(a, z), \) value functions \( V_t(a, z), V_u(a, z), \) policy functions for firms \( (d_t(a, z), a'_t(a, z), k_t(a, z), l_t(b, k, z), I_t(b, k, z)) \) \( i \in I,u \), households’ asset holding \( A^h_t \), policy functions for households, \( C(h, A), L(h, A), A'(A) \) such that

1. **Given aggregate output, the wage, the interest rate, and the inverse demand curve of the final good producer,** \( (d_u(a, z), a'_u(a, z), k(a, z), l_u(a, z)) \) and \( V_u(a, z) \) solves the problem of a private firm given by

\[
V_u(a, z) = \max_{d,a',k,l} \log(d) + \beta (1 - \zeta) \int_{z'} V_u(a', z') d\Psi(z'|z) \tag{7}
\]

subject to

\[
d + a' \leq p_u(a, z) z \left(k^\alpha t^{1-\alpha}\right)^\eta I^{1-\eta} - w l - I - (r + \delta) k \]

\[
+ (1 + r) a
\]

\[
k \leq \lambda a
\]

\[
p_u(a, z) = Q^{\frac{1}{\rho}} \left(z \left(k^\alpha t^{1-\alpha}\right)^\eta I^{1-\eta}\right)^{\frac{1}{\rho}}
\]

2. **Given aggregate output, the wage, the interest rate, and the inverse demand curve of the final good producer,** \( (d_t(a, z), a'_t(a, z), k_t(a, z), l_t(a, z), I_t(a, z)) \) and \( V_t(a, z) \) solves the problem

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\(^{10}\)In formulating the households’ problem, we have assumed that households only have access to risk-free bond and cannot trade shares. Alternatively, we can let households trade shares of the public intermediate good firms via an intermediary.
of a public firm given by

\[
V_l(a, z) = \max_{d, a', k, l, i} d + \frac{1}{1 + r} E_z V_l(a', z')
\]  
subject to

\[
d + a' \leq \rho z(\frac{k^\alpha l^{1-\alpha}}{1})^\eta l^{1-n} - w l - I - (r + \delta) k + (1 + r) a
\]  

\[
k \leq \lambda a
\]

\[
p_l(a, z) = Q^\frac{1}{\rho} \left( z \left( \frac{k^\alpha l^{1-\alpha}}{1} \right)^\eta l^{1-n} \right)^{-\frac{1}{\rho}}
\]

subject to (4), (5), and (6);

3. Given dividend payments of public firms and the wage rate, household consumption and labor supply solve

\[
V^h(A) = \max U(C_h, L_h) + \beta V^h(A')
\]  
subject to

\[
C_h + A' = w L_h + (1 - s) \int d(a, z) G_l(da, dz) + (1 + r) A
\]

4. Markets clear:

- **Final goods:**

\[
C_h + \sum_{i=l, u} \int_{a, z} d_u(a, z) G_u(da, dz) + K - (1 - \delta) K = Q - \sum_{i=l, u} \int_{a, z} I_i(a, z) G_i(da, dz).
\]  

where \(n_i\) is the measure of firms of type \(i\) firms (either public or private) and \(K\) is the aggregate capital stock given by

\[
K = \sum_{i=l, u} n_i \int_{a, z} k_i(a, z) G_i(da, dz)
\]

- **Intermediate goods:**

\[
q_i(a, z) = z \left( k_i(a, z)^\alpha l_i(a, z)^{1-\alpha} \right)^\eta l_i(a, z)^{1-n}, \text{ for } i = l, n
\]
• **Labor:**

\[ L_h = \sum_{i=L,U} n_i \int_{a,z} l_i(a, z)G_i(da, dz). \]  

(13)

5. The distributions \( G_L \) and \( G_U \) are stationary:

\[ G_i(A, Z) = \int_{a,z} 1\{a_i'(a, z) \in A\} \Psi(z|z) G_i(da, dz) \]  

(14)

In Appendix C.1, we provide a partial characterization of the optimal decision rules of firms. By choosing to model all debt as intra-period, the optimal capital and labor decisions are purely static. As a result, the problem of a private firm can be simplified by two-stage budgeting. In any period, the firm chooses capital and labor; after making these decisions, the firm then decides how much to consume and save. The resulting consumption-savings decision is essentially the same as the one studied in Huggett (1993) or Aiyagari (1994). The key difference from those models is that savings affect future profits of the firm by potentially relaxing the collateral constraint in future periods.

Before discussing theoretical and quantitative results from our model, we find it useful to demonstrate how we compute external financing in the model. Mechanically, we think of the model as generating simulated balance sheets and income statements for each firm that we analyze exactly as we do in the data. To see this process, let us rewrite the budget constraint in terms of physical capital \( k_t \) and debt \( b_t = k_t - a_t \). With this notation, the budget constraint of the firm can be written as

\[ d_t + x_t = p_t y_t(z_t, k_t, l_t) - wL_t - I_t + b_{t+1} - b_t - r_t b_t \]

At this point, we can define available funds and investment as we do in the data. We have \( AF_t = p_t y_t - wL_t - I_t - r_t b_t \) and \( x_t = k_{t+1} - (1 - \delta)k_t \). Since the state at \( t \) is given by \((a_t, z_t)\), we may define investment \( x_t \) as a function only of \((a_t, z_t)\), and \( z_{t+1} \). We can then define external financing in the model exactly as in the data (with a slight abuse of notation defining the distribution over \( a, z, z' \)):

\[ \int_{a,z,z'} (x_t(a, z, z') - AF_t(a, z)) 1_{[x_t \geq AF_t]} dH(a, z, z') \]

\[ \int_{a,z,z'} x_t(a, z, z') dH(a, z, z') \].

### 3.2 Theoretical Results

In this section, we describe theoretical results that shed light on the use of external funds by public firms in our model and the general equilibrium effects that arise from shocks to
the collateral constraints and impact the choices of public firms.

Equilibrium External Financing by Public Firms—Here, we argue that because public firms discount at the same rate as household, in any equilibrium in which household consumption is stationary, public firms never face binding collateral constraints. Consequently, the amount of external funds used by public firms in our model is indeterminate.

To see this, note that if in a certain period, a public firm is constrained (faces a binding collateral constraint) or it will be constrained with positive probability in the future, it is optimal to not pay dividends. This is because the value of funds inside the firm is more than the value of funds outside the firm. Furthermore, stationarity of households’ consumption implies that \( \beta (1 + r) = 1 \) or \( r > 0 \). Hence, the financial assets of a constrained firm grow at a rate of at least \( r \). Note that if we assume that idiosyncratic productivity is bounded above, then the optimal scale of the firm is bounded above. Therefore, since financial assets for a constrained firm are growing at rate \( r \), it must be that at some point in time, the amount of collateral available to the firm, \( \lambda a \), exceeds the maximum optimal scale and the firm becomes unconstrained. Thus, in a stationary equilibrium, all public firms must be unconstrained.

Given that public firms are unconstrained, the asset holdings of public firms in a stationary equilibrium are not determinate. Since these firms always operate at their optimal scale, indeterminacy of asset leads to an indeterminacy of available funds for the firm. We summarize this discussion in the following proposition:

**Proposition 1**  Suppose that in a stationary equilibrium, the interest rate is positive and productivity is bounded above. Then the collateral constraint never binds for any public firm. In addition, the amount of external funds used by public firms to finance investment is indeterminate.

The Effects of Collateral Shocks on Unconstrained Firms—Here, we describe how a shock to the collateral constraint may impact unconstrained, public firms. We argue that general equilibrium effects lead to shocks to the collateral constraints to spillover to these unconstrained firms and can dampen and even overturn their incentives to increase production in response to a tightening of the constraints. To do so, in Appendix C.3, we analyze a static, partial equilibrium version of our model. We develop sufficient conditions for the equilibrium output of every firm to be decreasing in the tightness of the collateral constraints. The key parameters that determine the strength of these general equilibrium spillovers are the elasticity of household labor supply, \( \varepsilon \), the elasticity of substitution across goods, \( \rho \), the labor share parameter, \( \alpha \), and the intermediate input share parameter, \( \eta \). We now state our main result from Appendix C.3.
Proposition 2 Suppose that $\beta = 0$ and that there exists a positive measure of constrained firms. If $1 + \varepsilon \geq \eta \rho (1 - \alpha)$, then output of all firms is increasing in the collateral constraint parameter, $\lambda$.

The intuition for this result follows our argument in the introduction. A tightening of the collateral constraint causes constrained firms to reduce their demand for capital. Since capital and labor are complements in production, constrained firms also decrease their demand for labor. The decrease in the demand for labor causes the wage rate to fall. The decline in the wage leads to a decline in demand for the final good. Given our assumed production function, the elasticity of output of an unconstrained firm with respect to the wage rate is simply $\eta \rho (1 - \alpha)$ (in our static version, we hold the interest rate fixed). If household preferences have the GHH form, the elasticity of aggregate demand with respect to the wage is just $1 + \varepsilon$. The condition, $1 + \varepsilon > \eta \rho (1 - \alpha)$, simply ensures that the demand and intermediate input effects dominate the reduced marginal cost arising from the reduction in the wage.

This result is useful for understanding our results in the dynamic economy below. If we assume a labor supply elasticity of 2.6, labor share of 0.66, input share of 0.5 and an elasticity of substitution of 4, then we should expect shocks to the collateral constraint to generate an aggregate recession where all firms decrease output (at least steady state to steady state).

4 Calibration and Quantitative results

In this section, we calibrate the model and undertake exercises intended to illustrate the contribution of changes in financial frictions to business cycle frequency fluctuations. We calibrate the model parameters using simulated method of moments by matching steady state moments of the model to their counterparts in the data. We then perform impulse response analysis and analyze the macroeconomic impact of financial shocks. In addition, we compare the effects of financial shocks to those arising from fluctuations in aggregate productivity. Lastly, we examine the sensitivity of our findings to changes in model parameters.

4.1 Calibration

To calibrate the model, we partition our model parameters into those that we fix in advance following estimates typically found in the macroeconomic literature and those that we estimate by matching model implied moments with the data, i.e., simulated method of moments.
Our calibration is novel in the sense that we use evidence on the use of external financing in the cross-section presented in section 2. We view the data on external financing as a useful source of information to discipline the importance of the role financial markets play in reallocating funds from cash-rich, low productivity firms to cash-poor, high productivity firms. We argue that the responsiveness of output to financial market shocks depends on the amount of external funds firms use. In Appendix C.4, we use a stylized version of our dynamic model to show that a change in parameter values that increases external financing, also increases the responsiveness of the model to a financial shock. In this sense, external financing is a key moment in our calibration.

The top panel of Table 4 lists the set of calibrated parameters. The key parameters of the steady state calibration are the parameters governing the tightness of the collateral constraint, $\lambda$, and the process for idiosyncratic firm level productivity. We assume that firm level productivity follows an AR(1) process so that

$$\log z_{it} = \rho z_{i,t-1} + \epsilon_{it}, \epsilon_{it} \sim N(0, \sigma_z^2).$$

We jointly calibrate the parameters $\lambda, \rho_z$, and $\sigma_z^2$ to target three moments in the data. Specifically, we target the average aggregate debt to total assets in the U.S. economy since 1986, which is 0.49; the use of external funds for investment by private firms, which is 0.82 (for U.K. firms); as well as the time series average of the standard deviation of size-weighted net debt-to-total asset ratio of private firms, which is 0.54 (for U.K. private firms).

Additionally, we jointly calibrate the remaining parameters: $s$, the measure of private firms; $\eta$, the parameter governing the intermediate input share in firm production; and $\psi$, capturing household’s disutility of labor to match the following moments. First, private firms in the U.S. account for 40% of gross output\(^{11}\); second, input’s share of gross output is 43% (Jones (2011)); and third, aggregate hours account for 30% of households’ time. We summarize our model targets in the data and the parameter values under which our model attains these targets in the following table.

The bottom panel in Table 4 contains the set of fixed parameters. We fix the discount rate to 0.96, targeting an annual real interest rate of 4%. We set the annual depreciation rate, $\delta$, to be 0.07 and we assume that the exit rate of private firms, $\zeta$, is 10%, which implies a ten year survival rate of 34%, which is consistent with estimates from Dunne\(^{11}\).

\(^{11}\)We arrive at this number by aggregating domestic (U.S.) gross output by public firms in Compustat and comparing this aggregate measure of public firm production to aggregate non-financial corporate gross output. The time series average value of this moment is 60%. Hence, we estimate that private firms account for roughly 40% of gross output.
We set \( \alpha \), the parameter governing the capital share in the firm production function to 0.3 and choose the elasticity of substitution across firms or goods to be \( \rho = 4 \), in line with estimates from micro data evidence (see Burstein and Hellwig (2008)).

We assume that household preferences are given by

\[
U(C, L) = \log \left( C - \frac{\psi}{1 + \frac{\varepsilon}{\varepsilon}} L^{1+\frac{1}{\varepsilon}} \right)
\]

We choose an elasticity of labor supply, \( \varepsilon \), to be 2.6. As in Greenwood et al. (1988), we assume that there are no wealth effects on the labor-leisure tradeoff to highlight the role of the complementarity between public and private firms in our model. However, it is important to keep in mind that choosing a more standard utility function as used in the business cycle literature will reduce the sensitivity of output to changes in the severity of financial frictions because declines in output by constrained firms will generate increases in output by unconstrained firms. Our choice of \( \varepsilon \) is in the range of typical macro estimates (see Chetty et al. (2011)).

Consider the calibrated parameter values which align the model moments with the data. First, observe that the fraction of private firms is only marginally larger than 40%, our target for the fraction of gross output that private firms produce. This suggests that in the steady state, the collateral constraint does not bind severely for a large fraction of private firms. Indeed, only 28% of firms face a binding collateral constraint in the steady

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collateral Constraint (( \lambda ))</td>
<td>6.98</td>
<td>External Financing</td>
<td>.82</td>
</tr>
<tr>
<td>Persistence of Idio. TFP (( \rho_z ))</td>
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<td>Debt-to-Total Assets</td>
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</tr>
<tr>
<td>Std. of Idio. TFP (( \sigma_z ))</td>
<td>0.33</td>
<td>Dispersion in Net Debt-to-Assets</td>
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</tr>
<tr>
<td>Disutility of labor (( \psi ))</td>
<td>0.41</td>
<td>Aggregate Hours</td>
<td>0.3</td>
</tr>
<tr>
<td>Share of private firms (( s ))</td>
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<td>Private Firms Share of Gross Output</td>
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</tr>
<tr>
<td>Share of Intermediate Inputs (( \eta ))</td>
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<td>Intermediate Input Share</td>
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</tr>
<tr>
<td><strong>Fixed Parameters</strong></td>
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<tr>
<td>Discount Rate (( \beta ))</td>
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<td>Exit Risk of Private Firms (( \zeta ))</td>
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</table>

Table 4: Parameters Values
state of our model. The fact that under 30% of firms face a binding collateral constraint is perhaps not surprising given our calibration. The collateral constraint parameter is large in the sense that private firms are able to rent capital up to 7 times their net worth in any period. Put differently, this means that the amount of net debt outstanding for a given firm is at most equal to 86% of their capital stock. In our sample of private firms, we find that size-weighted net debt-to-fixed assets is on average 35%. However, we also find that in any given year, roughly 25% of firms have a net debt to fixed asset ratio greater than 85%. In this sense, we view this value of the collateral constraint not inconsistent with the fact that many firms appear able to borrow heavily against their physical assets. Combined with the large collateral constraint parameter, we also find our calibration selects parameter values with high levels of persistence in idiosyncratic productivity. These numbers are larger than those in Midrigan and Xu (2014) but also in line with those in Bachmann and Bayer (2014), who estimate the persistence of firm level TFP to be 0.9675.

To evaluate our measure of persistence and the volatility of shocks in the idiosyncratic productivity process, we test the model by evaluating the volatility in firm level employment growth rates. We do so because the literature on whether financial frictions can successfully explain cross-country differences in aggregate productivity has emphasized that it is critical to match estimates of firm level volatility as opposed to the persistence of productivity alone in order to ensure our model economy is not artificially subjecting firms to financial constraints too frequently (see Moll (2014) and Midrigan and Xu (2014)). To address this issue, we compute the size-weighted standard deviation of employment growth for private firms in our model as in Davis et al. (2007). In particular, let \( g_{it} = (l_{it} - l_{it-1})/\sqrt{5(l_{it} + l_{it-1})} \) be employment growth for firm \( i \) from period \( t-1 \) to \( t \) and let \( \omega_{it} = (l_{it} + l_{it-1})/\sum_i (l_{it} + l_{it-1}) \) be the share of employment in the two periods of firm \( i \). Then we compute the \( \omega_{it} \)-weighted standard deviation of \( g_{it} \). In our model, this value is 0.47 while in the data from Davis et al. (2007), this value is 0.42. We conclude that employment growth volatility in our model is roughly comparable to that found in the data.

4.2 Analyzing the Impact of Shocks

Here, we describe the response of the model economy to an aggregate financial shock as modeled by a tightening of the collateral constraint. To assess the magnitude of the response of the economy, we compare the response to that of a TFP shock.

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12 Recall that there is no direct analog of this statistic in the data, which is why we use the financial flows data to calibrate the model instead of targeting this feature of the model.
The Effects of Financial Shocks—We model a financial shock as an aggregate shock to \( \lambda \). We assume that the shock to \( \lambda \) declines upon impact and this decline decays over time. We fix the decay of the impulse so that the shock has a half-life of one year. We calibrate the initial impulse so that on impact, the shock generates a one standard deviation decline in aggregate debt-to-total assets ratio. In the U.S., since 1986, the standard deviation of the aggregate debt-to-total assets ratio for non-farm, non-financial corporate businesses is 0.015 which, given that the average value of debt-to-total assets is 0.49 implies such a shock should induce a 3% decline in this ratio.\(^{13}\) We then compute the impulse response path of the economy as it transitions back to the steady state (under perfect foresight). Figure 3 displays impulse responses of output (net of intermediate inputs), measured productivity, the wage and interest rate, consumption, employment and investment, and sales by public and private firms all as percentage deviations from the steady state of the model.

We draw four important observations from this impulse response exercise. The first observation is that the impulse to the economy generates a roughly 0.4% decline in output on impact. Given that the aggregate capital stock is fixed, it is perhaps surprising that output and labor fall upon the impact of the shock. The collateral shock generates a recession by inducing a greater degree of misallocation in the model relative to the steady state outcomes. Specifically, the decline in output is driven by a decline in aggregate measured productivity. This decline in productivity is initially driven because constrained firms in the model are now more constrained and must reduce their capital demand (and therefore labor demand and production). Constrained firms are those firms that experience a positive shock to idiosyncratic productivity, either in the same period of, or in those leading up to, the aggregate collateral shock. These high productivity firms then grow more slowly along the impulse path than they would have in the steady state.

Additionally, a general equilibrium response amplifies the direct impact of the collateral shock. The reduction in capital demand by constrained firms induces a downward movement in the interest rate. This downward movement induces a response of unconstrained firms to increase their demand for capital. Unconstrained firms are those firms that experience a negative shock to idiosyncratic productivity, either in the same period of, or in those periods leading up to, the collateral shock and are thus, on average, low productivity firms. These low productivity firms then shrink more slowly along the impulse path than they would have in the steady state. Both the direct and indirect general equilibrium effects induce a decline in aggregate productivity and generate a mild recess-

\(^{13}\)Figure 15 displays the residuals of the aggregate debt-to-assets ratio. In our calibrated model, we require a 30% decline in \( \lambda \) as depicted in Figure 3 to induce this change.
Figure 3: Effects of a Financial Shock on Macroeconomic Outcomes.

In all, the misallocation of resources caused by the financial shock causes measured aggregate productivity to decline by 0.2%.\footnote{We define measured aggregate productivity in any period as $\text{TFP}_t = (Q_t - I_t)/K_t^{\alpha}L_t^{1-\alpha}$.}

The second observation is that financial shocks generate a persistent effect on output. Recall that our impulse to the collateral constraint recovers by half each year. However, output recovers by half after roughly two years so that the endogenous response of the economy is twice as persistent as the shock. This persistence is also driven by the misallocation of capital. Because private firms must accumulate wealth by saving out of their profits, it takes more time for high productivity firms to reach their optimal scale along the impulse path. Consequently, even as the collateral shock recovers, output recovers more slowly.

Third, we observe that the financial shock causes consumption, employment, and
investment all to move in the same direction of output. The decline in employment is driven by the reduction in the wage rate given the fall in aggregate productivity (and by our assumption that household labor supply decisions are independent of their wealth). Because the financial shock in the model generates a decline in aggregate productivity on impact, the initial decline in output coincides with a decline in investment and consumption. Notice that this is in contrast to models in which the financial constraint only constrains new investment. In those types of models, a financial shock has no immediate impact on output and therefore a decline in investment coincides with an increase in consumption (see Kiyotaki and Moore (2008) for an example).

Our fourth observation is that while sales of private firms fall by roughly 1.4% on impact and remain below steady state for roughly 20 periods, sales by public firms actually rise by roughly 0.25% on impact of the shock. The different initial response of sales by public and private firms is caused by the increased use of capital by public firms – none of which face binding collateral constraints – and the fact that in response to the 0.1% decline in the wage rate, aggregate labor does not decline dramatically. Within two periods, however, as the supply of capital grows more slowly than in steady state following the tightening of the collateral constraints, sales of both types of firms fall below the initial steady state. Sales of public firms in the model reach a minimum of roughly 0.1% below steady state. This pattern for output is a direct result of the complementarity in production across firms that we discuss theoretically above.

![Graph of Public Firm's Share of Gross Output](image1)

![Graph of Deviations from Linear Trend](image2)

**Figure 4:** Response of Share of Gross Output Accounted for by Public Firms in Model (Panel A) and in Data (Panel B: Deviations from a linear trend of Compustat’s share of Gross Output in U.S. Data).

Non-financial linkages in our model play a role in generating comovement of sales among public and private firms. However, independent of the non-financial linkages, our model predicts that the share of output among public firms should rise following a financial shock. We contrast this finding with evidence from Compustat.
Figure 4 plots the share of output by public firms in our model following a financial shock as well as share of output accounted for by public firms in Compustat (plotted as deviations from a linear trend) in Figure 4. Observe that in the data, this share varies by roughly 6% but not only around business cycle dates. For example, although the share of output by Compustat firms rises from 2000 to 2001 and between 2007 and 2008, there are also significant movements (both increases and decreases) in the middle of business cycle recoveries. Interestingly, the data do suggest a large increase in the share of output produced by public firms in the midst of the financial crisis in late 2007 and during the 2000-2001 recession. This evidence, while tentative, suggests that the prediction of our model, that the share of output accounted for by public firms should rise following a financial shock, is not largely in contradiction with the observations from Compustat.

**The Effects of Productivity Shocks**—We now compare the effects of financial shocks to the effects of aggregate productivity shocks in our model. This exercise allows us to provide context for the magnitude of the effects of financial shocks and demonstrate differences in the predictions of the model following different types of impulses.

Here, we consider the transition dynamics that result from a purely unanticipated decline in aggregate productivity which slowly returns to steady state. Again, we fix the half-life of the impulse to one year. We choose the size of the productivity shock so that measured productivity in the model falls on impact by one standard deviation of the measured Solow residual in the United States. This corresponds to roughly a 1% decline in measured productivity (at an annual rate). We also compare the effects of productivity shocks in a version of our model without collateral constraints. Specifically, we analyze our model economy in the case where there are no private firms, but the process for idiosyncratic risk for public firms is the same as in our calibrated model above.

Figure 5 displays the impulse path for measured productivity following the shock to aggregate productivity and gross value added (and, as in all of the figures that follow, the dashed line represents the corresponding path from the response of our model economy to the calibrated financial shock). On impact, gross value added falls by 0.9%. We find then that a financial shock which induces a one standard deviation decline in the aggregate debt-to-asset ratio yields a recession roughly half as large as that caused by a one standard deviation decline in aggregate productivity. In this sense, we view financial shocks as having a sizable impact on the economy.

In Figure 18 in the Appendix, we plot gross output of all non-financial firms in Compustat in the U.S. and gross output of all non-financial firms in the U.S. (data from the BEA) as percentage deviations from a linear trend. This figure shows that output of public firms in Compustat is highly correlated with that of all non-financial firms, but not perfectly so.
Also note that in our model, the effects of productivity shocks are dampened. In other words, a 1% decline in measured productivity causes gross value added to fall by only roughly 0.9%. This is due primarily to the monopoly distortions, which become less severe in response to a decline in aggregate productivity. We also find that our model, with and without collateral constraints, generates roughly the same effect on output. In this sense, in our economy, a fixed collateral constraint does not significantly amplify the effects of productivity shocks as found in earlier work by Kocherlakota (2000) and more recently Khan and Thomas (2013).

Figure 5: Impulse Response to a TFP Shock in the Constrained and Unconstrained model

Finally, Figure 6 depicts the response of aggregate debt-to-total assets ratio as well as the use of external funds in the model (with collateral constraints). Notice that debt-to-assets do not move at all in response to a productivity shock and the use of external funds as defined in equation (2) rises in response to the shock. In response to a decline in productivity for all firms, demand for debt by firms falls since the level of capital they optimally want to install is lower. However, their demand for capital has also fallen by roughly an equivalent amount, which causes the debt-to-asset ratio to remain roughly stable. The response of the use of external funds, on the other hand, is in the opposite direction of the response following a financial shock. The reason for this is that when aggregate productivity declines, the collateral constraints for constrained firms are relaxed since the unconstrained level of investment they want to undertake is lower. As a result, investment by constrained firms falls by less than the aggregate, leading the use of external funds to rise. In this sense, although productivity shocks and financial shocks have similar size effects on output, they affect financial flows in very different ways. It is not surprising that time series variation in measures from aggregated financial flows may be useful in distinguishing between productivity and financial shocks. Given longer time series data on financial flows for private firms, these differences could be useful in undertaking such an analysis.

The Effects of a Large Financial Shock—Here, motivated by the great recession, we an-
Figure 6: Impulse response of external financing and leverage to a TFP shock

analyze the response of the model to a large shock to collateral constraint. In particular, we consider a drop in $\lambda$ calibrated to match the peak-to-trough movement in debt-to-asset ratio in the United States during the great recession. As shown in Figure 16, aggregate debt-to-assets in the United States experienced a peak-to-trough decline of around 5% (or about two standard deviations) during the Great Recession.

Figure 7: Response of the Model to a Large Financial Shock

Figure 7 shows the response of the economy to a two standard deviation shock to debt-to-asset ratio. We find that the response of gross value added is about twice that of the benchmark financial shock. That is, the model behaves roughly linearly in response to a shock to $\lambda$. This is not very surprising as the model is similar to the model in Aiyagari (1994) and Krusell and Smith (1998). While our model is not designed to generate the increase in leverage in the run-up to the Great Recession, we conclude that a financial shock alone cannot generate the type of decline in economic activity experienced in the United States during the Great Recession.

An alternative exercise, which is closer in spirit to the analysis of Khan and Thomas (2013), is to impose a large enough shock to the collateral constraint to generate a 30% decline in the stock of debt. This exercise is intended to capture the decline in the amount of outstanding commercial and industrial loans which occurred in the United States from 2008 to 2011. To induce such a decline in the stock of debt, we require roughly a 20 percentage point drop in the value of firms’ collateral (in steady state, they can collateralize roughly 86 percent of their capital stock and on impact in this crisis simulation they
can only collateralize roughly 65 percent of their capital stock).\textsuperscript{16} When we impose this shock, which is roughly twice as large as our above crisis simulation exercise, we find that aggregate GDP falls by 2%, suggesting that with larger shocks, our model is capable of explaining a sizable percentage of the decline in GDP observed in 2007.

4.3 Sensitivity Analysis

In this subsection, we study the sensitivity of our quantitative exercise to changes in various parameters. We illustrate how our model’s implications for the magnitude and composition of the macroeconomic effects of financial shocks are sensitive to observed moments in the data as well as some of the parameters we did not estimate. We focus on examining how much gross value added responds to financial shocks in our model under different assumptions on either the probability of exit faced by private firms or the elasticity of substitution across goods. We also demonstrate how our findings are sensitive to our observations on the use of external funds found in the data and our ability to match external financing patterns of both public and private firms.

**Sensitivity with Respect to Exit Risk**—Uninsurable exit risk faced by private firms in our model, captured by the parameter $\zeta$, plays an important role in determining the strength of the precautionary motives of the owners of private firms. Here, we show that a reduction in $\zeta$, holding the remaining model parameters fixed, reduces the response of GDP in our model to a financial shock. However, this change also causes our model to yield counterfactual predictions for financial flows. When we recalibrate our model given a lower value of $\zeta$, we obtain similar results as in our benchmark analysis.

Consider modifying our benchmark calibration by only lowering the parameter, $\zeta$, from 0.1 to 0.05 and then imposing the same 30% negative shock to the collateral constraint parameter that we analyzed above. Figure 19 shows the collateral constraint shock, and the responses of the aggregate debt-to-total asset ratio and gross value added in the benchmark economy and in this alternative parameterization. This impulse yields only a 0.1% decline in gross value added.

The magnitude of the impact is only one quarter of that in our benchmark calibration. In this sense, our findings are sensitive to our choice of the exit risk faced by private firms. However, in steady state, this model yields counterfactual implications for both the use of external funds by private firms and their aggregate indebtedness. Table 6 reports steady state moments of our model under this alternative parameter specification. Reducing the

\textsuperscript{16}Khan and Thomas (2013) require an 88 percentage point decline in the value of firms’ collateral to generate this decline, which suggests that debt is much more sensitive in our model. This sensitivity is most likely due to the assumption that all capital is reversible and can be sold without a discount.
exit risk of private firms makes the owners of these firms effectively more patient and thus strengthens the precautionary motive. We see this effect in the model implied moments as the amount of debt used by private firms falls from 0.49 to 0.14. Interestingly, the use of external funds for investment actually rises in steady state as private firms have more relaxed collateral constraints. These constraints are more relaxed in equilibrium because these firms endogenously accumulate more net worth.

We then recalibrate our model under the assumption that $\zeta = 0.05$. Table 7 reports our benchmark calibration when $\zeta = 0.1$ as well as this new calibration with $\zeta = 0.05$. We then subject the economy to an aggregate tightening of the collateral constraint which induces a one standard deviation decline in the debt-to-asset ratio. Figure 20 illustrates the response of the economy.

To induce the same decline in the aggregate debt-to-total assets ratio under the alternative calibration, we must impose a larger financial shock – roughly a 40% decline in $\lambda$ versus 30% in our benchmark calibration. We see that the shock induces over a 0.2% decline in gross value added, which is smaller than our benchmark calibration though over twice as large than when we only adjust the exit risk of private firms. We conclude that matching the use of external funds, independent of the amount of exit risk imposed in the model, plays a key role in determining the responsiveness of the economy to financial shocks. One interpretation of this finding is that in an economy in which financial shocks are forecastable, the likelihood of financial shocks would directly impact the precautionary motives of entrepreneurs. However, such a change in the model also impacts the use of external funds by private firms; we suspect if such a model matched patterns of external financing in the data, the model would obtain effects of financial shocks with similar magnitudes to our benchmark findings.

**Sensitivity with respect to Non-Financial Trade Linkages**—Trade linkages in our model arising from the imperfect substitutability of goods in the economy play a critical role in the steady state. In the absence of these trade linkages, private firms would not produce output because their binding collateral constraint makes them effectively less productive. Here, we compare our results to those we obtain using an elasticity of substitution across goods of $\rho = 10$ in line with estimates typically used in the macro literature (see Basu and Fernald (1997) and Golosov and Lucas (2007) for examples). We find that changing this elasticity has little impact on the aggregate consequences of financials shocks but has important consequences for the responses of private and public firms.

Holding the remaining model parameters fixed, an increase in the elasticity of substitution across goods reduces profit margins of firms as markups from monopolistic com-
petition are much smaller (10% with $\rho = 10$ as opposed to roughly 30%). Consequently, firms use much less debt and less external financing as their unconstrained optimal scale is much lower. We report the model implied moments in Table 9.

Figure 21 demonstrates the impact of the same financial shock we imposed in our benchmark model under this alternative parameterization. We find that the impact on gross value added is much larger – roughly -0.8% on impact – under this parameterization the financial shock is essentially larger because it induces a larger change in debt-to-assets. Figure 21 shows that the direct effect on sales of private firms is larger and the indirect general equilibrium effect which tends to increase output of public firms is also larger. Notice also that output of public firms largely remains above trend following a financial shock. Lastly, we find that the effect of the shock is more persistent. This last finding is driven by the fact that with smaller profit margins, it takes private firm owners longer to accumulate wealth lost (relative to the steady state) following the financial shock. In the Appendix, we also report results from recalibrating the model given a choice of $\rho = 10$ and find the aggregate response of the economy is almost identical to our benchmark findings, but public firms’ sales remain above trend essentially the entire time following the financial shock (see table 8 and figure 22).

We conclude that trade linkages are essential for ensuring that both types of firms produce in the steady state and that unconstrained public firms reduce their sales following a financial shock (at least in the medium run when $\rho = 4$); however, trade linkages are less critical for understanding the aggregate response of the economy to a financial shock.

Sensitivity with Respect to External Financing—Here, we examine the sensitivity of our quantitative findings to the evidence on firms’ use of external funds. To illustrate this sensitivity, we attempt to recalibrate our model to match a counterfactual data sample in which private firms use external funds to finance only 20% of their investment. The calibrated parameter values and model moments from this calibration are displayed in Table 10.

Our model is relatively successful at yielding moments similar to the counterfactual data, however it cannot quite generate the correct amount of debt or sufficient dispersion in debt-to-assets across firms while also matching the low use of external funds. Figure 23 shows the impact of a financial shock in a model economy calibrated as in Table 10.

The impact of the financial shock leads to roughly 0.3% decline in gross value added as opposed to 0.4% decline under our benchmark calibration. Hence, if private firms used external funds to the same degree as public firms but were otherwise as they are in the data, we would find that the impact of financial shocks would be only 75% of what we
observe in our benchmark model.

**Sensitivity with Respect to the Share of Private Firms**—Lastly, we examine the importance of matching patterns of external financing for both public and private firms for our quantitative findings. Specifically, we maintain the calibration from our benchmark economy but we set the measure of private firms in our model to 1. This change in parameterization has essentially no impact on the model’s predictions for data moments implied by the steady state of our economy. When we subject this economy to a financial shock which induces a 3% decline in the aggregate debt-to-assets ratio, we find that aggregate GDP falls by roughly 4.5%, capturing essentially 80% of the decline in aggregate GDP observed during the Great Recession in the United States. In the absence of a large group of firms that are less constrained and fully diversified, our model is capable of generating a substantial decline in aggregate economic activity following a financial shock. This finding points towards the importance of disciplining the model with data on external financing for all firms in the economy. Once external financing patterns in the cross-section for both public and private firms are taken into account, our model of financial shocks faces challenges in generating particularly large recessions.

## 5 Conclusion

We have, theoretically and empirically, analyzed the extent to which firms use external financing and its implication for how financial shocks affect the economy. We used data on firm-level financial flows to argue that private firms use external funds much more extensively than public firms. In the context of our model, we demonstrated that replicating patterns of external financing are important for understanding the macroeconomic response to financial shocks.

Through the lens of a canonical model of reallocation, we showed that financial shocks do induce sizable fluctuations in output, but do not account for particularly severe recession episodes. Of course, incorporating additional nominal or real rigidities such as sticky wages or adjustment costs may increase the relative importance of financial shocks. An interesting question for future research is whether a model that incorporates such additional rigidities and is consistent with cross-sectional patterns of external financing can generate severe declines in economic activity.
References


A Data Appendix – NOT FOR PUBLICATION

A.1 Aggregate Data Sources

A.1.1 U.S. Flow of Funds

We use quarterly data from the flow of funds seasonally adjusted annual rates table F.102 covering Non-Financial Corporate Business. We define Available Funds as the sum of “Total Internal Funds + IVA” (FA106000105.Q) and “Net Dividends” (FA106121075.Q). We define Investment as “Capital Expenditures” (FA105050005.Q). We normalize Available Funds and Investment by “Gross Value Added of Non-Financial Corporate Business” which we obtain from the Bureau of Economic Analysis, Table 1.14 (“Gross Value Added of Domestic Corporate Business in Current Dollars and Gross Value Added of Non-Financial Domestic Corporate Business in Current and Chained Dollars”) Line 17.

To construct aggregate Debt-to-Total Assets, we use annual data from the flow of funds seasonally adjusted annual rates table B.102 covering Non-Financial Corporate Business. We define Debt as “Liabilities” (FL104190005.Q) and Total Assets as “Assets” (FL102000005.Q).

A.1.2 U.K. Flow of Funds

We use quarterly data from the UK Economic Accounts, tables 1.1.1 (“National Accounts Aggregates”), 3.3.3 (“Income and Capital Accounts: Private non-financial corporations: Allocation of Primary Income Account”), 3.3.4 (“Income and Capital Accounts: Private non-financial corporations: Secondary Distribution of Income Account”) and 3.3.7 (“Income and Capital Accounts: Private non-financial corporations: Accumulation Accounts”). We define Available Funds as the sum of “Gross Disposable Income” (RPKZ, Table 3.3.4) and “Dividend Payments” (RVFT, Table 3.3.3). We define Investment as the sum of “Gross Fixed Capital Formation” (ROAW, Table 3.3.7), “Changes in Inventories” (DLQY, Table 3.3.7) and “Acquisitions less disposals of non-produced non-financial assets” (RQBW, Table 3.3.7). We normalize Available Funds and Investment by “Gross National Income at market prices” (ABMZ, Table 1.1.1).

A.2 Cross-Sectional Data Source

For all cross-sectional data sources, we exclude firms in the following industries according to 4-digit SIC classifications: Postal Services (4300-4399), Utilities (4900-4999), Finance, Insurance and Real Estate (6000-6999), and Other Government (≥9000).
A.2.1 U.S. Compustat

We obtain data on U.S. publicly traded firms from the Compustat Monthly Updates – Fundamentals Annual File, North America from WRDS. We restrict attention to firms in Compustat which report Consolidated statements and standardized data, are in the domestic population source, and are active. We use the Historical Standard Industrial Classification to classify firms into industries.

For each firm, we define Available Funds as “Operating Activities – Net Cash Flow” (annual data item 308) if the firm reports its statement of cash flows using format code 7 (annual data item 318). Otherwise, we define Available Funds as “Funds from Operations – Total” (annual data item 110). We define Investment as the sum of “Capital Expenditures (Statement of Cash Flows)” (annual data item 128) and “Acquisitions (Statement of Cash Flows)” (annual data item 129) less “Sale of Property, Plant and Equipment (Statement of Cash Flows)” (annual data item 107).

We define net debt as “Liabilities – Total” (annual data item 181) minus “Cash and Short-Term Investments” (annual data item 1) and “Receivables – Total” (annual data item 2). We use “Assets – Total” (annual data item 6) for total assets. We then define a firm’s return on assets in period $t$ as Available Funds in period $t$ divided by Total Assets in period $t - 1$. Similarly, we define investment to total assets in period $t$ as investment in period $t$ divided by total assets in period $t$ and net debt to total assets as net debt in period $t$ divided by total assets in period $t$.

We drop firms who do not report available funds (data items 308 or 110) or capital expenditures (data item 128). We code missing values for acquisitions, sale of property, plant and equipment, cash and short-term investments, and receivables as 0 unless the firm reports a missing value or combined data item for these objects in which case we drop the entire firm-year observation. Finally, we restrict attention to firms with positive assets, liabilities, and sales.

A.2.2 U.K. Compustat

We obtain data on U.K. publicly traded firms from the Compustat Monthly Updates – Fundamentals Annual File, Global from WRDS. We restrict attention to firms in Compustat which report Consolidated statements and standardized data, report their location as Great Britain, report their financial statements in British Pounds, and are active. We use the Historical Standard Industrial Classification to classify firms into industries.

For each firm, we define Available Funds as “Operating Activities – Net Cash” (data item G692). We define Investment as the sum of “Capital Expenditures” (data item G676)
and “Acquisitions” (data item G681) or “Acquisitions and Disposals - Net Cash Flow” (mnemonic ACQDISN) less “Proceeds from Sale of Fixed Assets” (mnemonic PSFIX) or “Sale of Tangible Fixed Assets” (mnemonic STFIXA).

We define net debt as “Liabilities – Total” (data item G171) minus “Cash and Cash Equivalents - Increase (Decrease)” (data item G684) and “Accounts Receivables/Debtors – Total” (data item G629). We use “Assets – Total” (data item G107) for total assets.

As with public firms in the U.S., we then define a firm’s return on assets in period t as Available Funds in period t divided by Total Assets in period t − 1. Similarly, we define investment to total assets in period t as investment in period t divided by total assets in period t and net debt to total assets as net debt in period t divided by total assets in period t. Additionally, we exclude firm-year observations which do not report available funds (data item G692) or capital expenditures (data item G676). We code missing values for acquisitions, sale of property, plant and equipment, cash and short-term investments, and receivables as 0 unless the firm reports a missing value or combined data item for these objects in which case we drop the entire firm-year observation. Finally, we restrict attention to firms with positive assets, liabilities, and sales.

A.2.3 U.K. Amadeus

We obtain data on U.K. privately held firmst from the Amadeus database created by the Bureau van Dijk and maintained by WRDS. We obtain data from all firm size datasets (Very Large, Large, Medium, and Small). We restrict attention to firms located in Great Britain with stated company type as "Private Limited Company" and which are not publicly quoted (variable QUOTED is "No"). Further, we restrict attention to those firms reporting positive total assets (mnemonic TOAS), positive fixed assets (mnemonic FIAS), positive total liabilities where total liabilities is defined as the sum of Current (mnemonic CULI) and Non-Current (mnemonic NCLI) Liabilities. We require firm’s balance sheet to add up correctly (TOAS is equal to total shareholders’ funds and liabilities, TSHF).

For a given firm-year observation, we construct available funds as the sum of “Profit (loss) for Period)” (mnemonic PL) and “Depreciation” (mnemonic DEPR). We construct investment in year t as tangible fixed assets (mnemonic TFAS) at end of year (in year t) minus lagged tangible fixed assets from year t − 1 plus depreciate declared in year t. As with public firms in the U.S. and the U.K, we then define a firm’s return on assets in period t as Available Funds in period t divided by Total Assets in period t − 1. Similarly, we define investment to total assets in period t as investment in period t divided by total assets in period t and net debt to total assets as net debt in period t divided by total assets in period t.
Lastly, we exclude firms for which lagged total assets, available funds, or investment (or any component of investment) are not available.
Figure 8: U.S. Available Funds, Internal Funds, and Investment normalized by Non-Financial Corporate Business GDP.

Figure 9: U.K. Available Funds, Internal Funds, and Investment, normalized by Gross National Income, 1987-2011.
Figure 10: U.K. Available Funds, Internal Funds, and Investment, normalized by Gross National Income, 1997-2013.
Figure 11: Investment Share by Industry and Listing Status

Figure 12: External Financing by Industry and Listing Status
Figure 13: Investment Shares by Asset Quartile and Listing Status

Figure 14: External Financing by Asset Quartile and Listing Status
Figure 15: U.S. Aggregate Debt-to-Total Assets of Non-Farm Non-Financial Corporate Businesses, 1951-2011.

Figure 16: U.S. Percentage Deviations from HP Trend of Aggregate Debt-to-Total Assets of Non-farm Non-Financial Corporate Businesses, 1985-2011.
Figure 17: Share of Gross Output Accounted for by All Non-Financial Firms in Compustat.

Figure 18: Share of Gross Output in Compustat: Deviations from a Linear Trend
Figure 19: Response to a Financial Shock in Benchmark Calibration and Alternative Parameterization with $\zeta = .05$. 
Figure 20: Response to a Financial Shock in Benchmark and Alternative Calibration with $\zeta = 0.05$. 
Figure 21: Response to a financial shock in benchmark calibration and alternative parameterization with $\rho = 10$. 
Figure 22: Response to a financial shock in the benchmark and alternative calibration with $\rho = 10$. 

Collateral Constraint Shock, $\lambda$

Debt-to-Total Assets

Gross Value Added

Gross Output of Public and Private Firms

Interest Rate
Figure 23: Response to a Financial Shock in Benchmark Calibration and Alternative Calibration with Exfin = 0.2.
<table>
<thead>
<tr>
<th>Quartile</th>
<th>Private</th>
<th>Public</th>
<th>Private</th>
<th>Public</th>
<th>Private</th>
<th>Public</th>
<th>Private</th>
<th>Public</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>631,028</td>
<td>2,519</td>
<td>1.11</td>
<td>9.19</td>
<td>2.00</td>
<td>12.23</td>
<td>0.04</td>
<td>0.28</td>
</tr>
<tr>
<td>Q2</td>
<td>45,596</td>
<td>2,506</td>
<td>24.53</td>
<td>43.04</td>
<td>38.08</td>
<td>55.84</td>
<td>0.94</td>
<td>1.87</td>
</tr>
<tr>
<td>Q3</td>
<td>20,709</td>
<td>2,507</td>
<td>115.46</td>
<td>181.21</td>
<td>143.59</td>
<td>237.52</td>
<td>4.49</td>
<td>9.15</td>
</tr>
<tr>
<td>Q4</td>
<td>6,135</td>
<td>2,508</td>
<td>1,512.20</td>
<td>4,095.51</td>
<td>1,103.15</td>
<td>3,378.74</td>
<td>46.00</td>
<td>190.12</td>
</tr>
</tbody>
</table>

Table 5: Sample Statistics for Public and Private firms by Public Firm Asset Quartiles in the United Kingdom. N: Number of Firm-Year Observations. Total Assets, Sales, Investment reported in Millions of Pounds.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>$\zeta = 0.1$</th>
<th>$\zeta = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Financing</td>
<td>.82</td>
<td>.82</td>
<td>1.13</td>
</tr>
<tr>
<td>Debt-to-Total Assets</td>
<td>.49</td>
<td>.49</td>
<td>.14</td>
</tr>
<tr>
<td>Dispersion in Net Debt-to-Assets</td>
<td>.54</td>
<td>.54</td>
<td>.60</td>
</tr>
<tr>
<td>Aggregate Hours</td>
<td>.3</td>
<td>.3</td>
<td>.31</td>
</tr>
<tr>
<td>Private Firms Share of Gross Output</td>
<td>.4</td>
<td>.4</td>
<td>.40</td>
</tr>
<tr>
<td>Intermediate Input Share</td>
<td>.43</td>
<td>.43</td>
<td>.43</td>
</tr>
</tbody>
</table>

Table 6: Target Moments from the model for a lower value of $\zeta$

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Benchmark Calibration</th>
<th>Alternative Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit Risk ($\zeta$)</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Collateral Constraint ($\lambda$)</td>
<td>6.98</td>
<td>11.20</td>
</tr>
<tr>
<td>Persistence of Idio. TFP ($\rho_z$)</td>
<td>0.95</td>
<td>0.89</td>
</tr>
<tr>
<td>Standard Deviation of Idio. TFP ($\sigma_z$)</td>
<td>0.33</td>
<td>0.39</td>
</tr>
<tr>
<td>Disutility of labor ($\psi$)</td>
<td>0.41</td>
<td>0.1</td>
</tr>
<tr>
<td>Share of private firms ($s$)</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>Share of Intermediate Inputs ($\eta$)</td>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 7: Alternative calibration with lower exit risk

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Benchmark Calibration</th>
<th>Alternative Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit Risk ($\zeta$)</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Collateral Constraint ($\lambda$)</td>
<td>6.98</td>
<td>3.13</td>
</tr>
<tr>
<td>Persistence of Idio. TFP ($\rho_z$)</td>
<td>0.95</td>
<td>0.91</td>
</tr>
<tr>
<td>Standard Deviation of Idio. TFP ($\sigma_z$)</td>
<td>0.33</td>
<td>0.9</td>
</tr>
<tr>
<td>Disutility of labor ($\psi$)</td>
<td>0.41</td>
<td>0.33</td>
</tr>
<tr>
<td>Share of private firms ($s$)</td>
<td>0.41</td>
<td>0.56</td>
</tr>
<tr>
<td>Share of Intermediate Inputs ($\eta$)</td>
<td>0.43</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 8: Alternative Calibration with $\rho = 10$
<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>$\rho = 4$</th>
<th>$\rho = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Financing</td>
<td>.82</td>
<td>.82</td>
<td>.74</td>
</tr>
<tr>
<td>Debt-to-Total Assets</td>
<td>.49</td>
<td>.49</td>
<td>.36</td>
</tr>
<tr>
<td>Dispersion in Net Debt-to-Assets</td>
<td>.54</td>
<td>.54</td>
<td>.3</td>
</tr>
<tr>
<td>Aggregate Hours</td>
<td>.3</td>
<td>.3</td>
<td>.06</td>
</tr>
<tr>
<td>Private Firms Share of Gross Output</td>
<td>.4</td>
<td>.4</td>
<td>.37</td>
</tr>
<tr>
<td>Intermediate Input Share</td>
<td>.43</td>
<td>.43</td>
<td>.52</td>
</tr>
</tbody>
</table>

Table 9: Target Moments from the model with a higher value of $\rho$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collateral Constraint</td>
<td>$\lambda$</td>
<td>Aggregate Debt-to-Assets</td>
<td>.47</td>
<td>.49</td>
</tr>
<tr>
<td>Persistence of Idio. TFP</td>
<td>$\rho_z$</td>
<td>Use of External Funds</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Standard Deviation of Idio. TFP</td>
<td>$\sigma_z$</td>
<td>Dispersion in Debt-to-Assets</td>
<td>0.50</td>
<td>0.54</td>
</tr>
<tr>
<td>Disutility of labor</td>
<td>$\psi$</td>
<td>Aggregate Hours</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Share of private firms</td>
<td>$s$</td>
<td>Private Share of Gross Output</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Share of Intermediate Inputs</td>
<td>$\eta$</td>
<td>Share of Inputs in Gross Output</td>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 10: Alternative targets for External financing and calibrated parameters
B.1 Size Analysis by Asset Decile

Here, we replicate our findings from Section 2.3 on the use of external funds by firm’s asset size using deciles instead of quartiles.

Figure 24: Investment Shares by Asset Decile and Listing Status

Figure 25: External Financing by Asset Decile and Listing Status
<table>
<thead>
<tr>
<th>Company Type</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>Q7</th>
<th>Q8</th>
<th>Q9</th>
<th>Q10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Firm-Year Observations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>560,416</td>
<td>53,995</td>
<td>29,366</td>
<td>19,509</td>
<td>13,338</td>
<td>9,484</td>
<td>8,499</td>
<td>4,704</td>
<td>3,065</td>
<td>1,092</td>
</tr>
<tr>
<td>Public</td>
<td>1,010</td>
<td>1,005</td>
<td>1,005</td>
<td>1,004</td>
<td>1,001</td>
<td>1,004</td>
<td>1,007</td>
<td>999</td>
<td>1,003</td>
<td>1,002</td>
</tr>
<tr>
<td><strong>Total Assets (Millions of pounds)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>0.43</td>
<td>5.60</td>
<td>11.29</td>
<td>21.79</td>
<td>39.30</td>
<td>67.70</td>
<td>130.32</td>
<td>284.43</td>
<td>768.98</td>
<td>5,699.51</td>
</tr>
<tr>
<td>Public</td>
<td>3.55</td>
<td>10.74</td>
<td>20.39</td>
<td>35.64</td>
<td>60.30</td>
<td>103.39</td>
<td>189.10</td>
<td>403.23</td>
<td>1,059.26</td>
<td>8,948.46</td>
</tr>
<tr>
<td><strong>Sales (Millions of pounds)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>0.86</td>
<td>9.52</td>
<td>18.81</td>
<td>35.00</td>
<td>57.49</td>
<td>91.42</td>
<td>163.49</td>
<td>315.85</td>
<td>665.00</td>
<td>3,627.30</td>
</tr>
<tr>
<td>Public</td>
<td>5.54</td>
<td>14.16</td>
<td>25.25</td>
<td>46.63</td>
<td>78.65</td>
<td>137.03</td>
<td>257.65</td>
<td>517.53</td>
<td>1,325.85</td>
<td>6,811.84</td>
</tr>
<tr>
<td><strong>Investment (Millions of pounds)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>0.02</td>
<td>0.21</td>
<td>0.43</td>
<td>0.83</td>
<td>1.54</td>
<td>3.08</td>
<td>4.65</td>
<td>11.70</td>
<td>30.56</td>
<td>144.40</td>
</tr>
<tr>
<td>Public</td>
<td>0.10</td>
<td>0.33</td>
<td>0.77</td>
<td>1.74</td>
<td>2.45</td>
<td>4.89</td>
<td>9.11</td>
<td>19.09</td>
<td>43.38</td>
<td>422.25</td>
</tr>
</tbody>
</table>

Table 11: Sample Statistics for Public and Private Firms by Public Firm Asset Deciles in the United Kingdom.
<table>
<thead>
<tr>
<th>Company Type</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>Q7</th>
<th>Q8</th>
<th>Q9</th>
<th>Q10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Use of External Funds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>3.77%</td>
<td>2.97%</td>
<td>3.09%</td>
<td>3.79%</td>
<td>4.32%</td>
<td>4.85%</td>
<td>8.19%</td>
<td>10.28%</td>
<td>15.69%</td>
<td>24.68%</td>
</tr>
<tr>
<td>Public</td>
<td>0.15%</td>
<td>0.18%</td>
<td>0.25%</td>
<td>0.39%</td>
<td>0.40%</td>
<td>0.50%</td>
<td>1.03%</td>
<td>1.29%</td>
<td>2.73%</td>
<td>11.20%</td>
</tr>
<tr>
<td><strong>Investment Share</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>2.02%</td>
<td>2.59%</td>
<td>2.84%</td>
<td>3.72%</td>
<td>4.68%</td>
<td>6.70%</td>
<td>9.29%</td>
<td>12.64%</td>
<td>20.58%</td>
<td>39.94%</td>
</tr>
<tr>
<td>Public</td>
<td>0.03%</td>
<td>0.09%</td>
<td>0.23%</td>
<td>0.50%</td>
<td>0.59%</td>
<td>1.35%</td>
<td>2.11%</td>
<td>3.61%</td>
<td>7.24%</td>
<td>84.24%</td>
</tr>
<tr>
<td><strong>Investment Share of Firms Using External Funds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>1.57%</td>
<td>1.96%</td>
<td>2.15%</td>
<td>2.75%</td>
<td>3.47%</td>
<td>4.16%</td>
<td>7.66%</td>
<td>9.88%</td>
<td>17.29%</td>
<td>28.71%</td>
</tr>
<tr>
<td>Public</td>
<td>0.03%</td>
<td>0.06%</td>
<td>0.17%</td>
<td>0.27%</td>
<td>0.38%</td>
<td>0.71%</td>
<td>1.09%</td>
<td>2.01%</td>
<td>4.17%</td>
<td>27.24%</td>
</tr>
</tbody>
</table>

Table 12: Time Series Average of Use of External Funds, Investment Share, and Investment Share of Firms using External Funds for Public and Private firms by Public Firm Asset Deciles in the United Kingdom.
C Proof of Theoretical Results – NOT FOR PUBLICATION

C.1 Production and Saving Decisions of Privately Held Firms

In this section, we provide a characterization of the privately held firm’s optimal capital, labor, and savings decisions. The problem of a surviving privately held firm can be written recursively as

\[
V_u(a, z) = \max_{c, a', k, l} u(c) + \beta \zeta E[V_u(a', z') | z]
\]

s.t.

\[
c + a' \leq p(a, z) z^{\frac{1}{p-1}} \left( k^\alpha l^{1-\alpha} \right) \eta l^{1-\eta} - wl - (r + \delta)k - I + (1 + r)a
\]

\[
p(a, z) = Q^{\frac{1}{\rho}} \left( z^{\frac{1}{p-1}} \left( k^\alpha l^{1-\alpha} \right) \eta l^{1-\eta} \right)^{-\frac{1}{\rho}}
\]

\[
k \leq \lambda a
\]

\[
a_0 \text{ given}
\]

The decision of how much capital to install and how much labor to hire is a static one. We therefore can use the results from our static model, namely Lemma (4) to define profits of a privately held firm, which we denote \(\Pi_u(a, z; w, r, Q)\). Then the problem of a private firm can be simplified to a consumption and savings problem written as

\[
V^u(a, z) = \max_{c, a'} u(c) + \beta \zeta [V^u(a', z') | z]
\]

s.t.

\[
c + a' \leq \Pi_u(a, z; w, r, Q) + (1 + r)a
\]

As in Aiyagari (1994), the only intertemporal effect of the borrowing constraint comes from distorting the savings decisions. The nature of the borrowing constraint ensures that this happens in a smooth way. Thus, the optimal savings decision can be solved as in Aiyagari (1994), except for the extra term that captures the effect of savings on the next period’s profit function. We then have the following lemma, in which we suppress the dependence of \(\Pi_u\) on equilibrium parameters \(w, r,\) and \(Q\) and write it only as \(\Pi_u(a, z)\).

**Lemma 3** For a private firm, the optimal asset position policy is given by a function \(a'(a, z)\) that satisfies

\[
u'(\Pi(a, z) + (1 + r)a - a'(a, z)) = \beta \zeta E \left\{ \left[ \Pi_u(a'(a, z), z') + (1 + r) \right] u'(\Pi(a'(a, z), z') + (1 + r)a'(a, z) - a'(a'(a, z), z')) | z \right\}
\]

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C.2 Equilibrium Results Concerning Publicly Firms

Proof of Lemma Proposition 1.
Publicly held firms solve

\[
\max E_0 \sum \beta_t d_t \\
d_t + a_{t+1} \leq f(z_t, k_t) + (1 + r)a_t \\
k_t \leq \lambda_t a_t \\
d_t \geq 0
\]

or

\[
\max E_0 \sum \beta_t \left[ (f_t(z_t, k_t) + (1 + r)a_t - a_{t+1})(1 + \eta(z_t^t)) + \mu(z_t^t) (\lambda a_t - k_t) \right]
\]

FOCs

\[
f'(z_t, k_t) = \mu(z_t^t) \\
(1 + \eta(z_t^t)) = \beta E_t \left[ (1 + r) (1 + \eta(z_{t+1}, z_t^t)) + \mu(z_{t+1}, z_t^t) \lambda \right]
\]

where the last can be re-written using \( \beta (1 + r) = 1 \):

\[
\eta(z_t^t) = \beta E_t \left[ (1 + r)\eta(z_{t+1}, z_t^t) + \mu(z_{t+1}, z_t^t) \lambda \right]
\]

Thus, if \( \eta(z_{t+1}, z_t^t) \) or \( \mu(z_{t+1}, z_t^t) \) are positive for any \( z_{t+1} \) with strictly positive probability following \( z_t^t \) then \( \eta(z_t^t) = 0 \) and

\[
a_{t+1}(z_t^t) = f(z_t, k_t) + (1 + r)a_t
\]

Since \( f \geq 0 \) and \( 1 + r > 1 \) we have that assets grow as long as constraints ever bind in the future.

Now, optimal unconstrained capital scale satisfies

\[
f'(z_t, k_t) = 0.
\]

Let the optimal scale be defined as

\[
k^*(z) = (f')^{-1}(0, z).
\]

We can easily show that \( k^*(z) \) is increasing in \( z \). Suppose \( z \in [z, \bar{z}] \). Then, if \( a_t \geq \lambda k^*(\bar{z}) \),
the constraint does not bind currently and choosing any $a_{t+1} \geq a_t$ will ensure that the constraint never binds again in the future. Suppose at time $s$ the firm’s assets satisfy $a_s \geq \lambda k^*(z)$. In this case, the firm solves

$$\max E_s \sum_{t \geq s} \beta^{t-s} \left[ (f_t(z_t, k_t) + (1 + r)a_t - a_{t+1}) \right]$$

and since $\beta(1 + r) = 1$ this simplifies to

$$E_s \sum_{t \geq s} \beta^{t-s} [(f_t(z_t, k_t)] + a_t(1 + r)$$

Note then, in steady state that no publicly held firms will be constrained (assets are increasing until the constraint never binds and then are indeterminate above $\bar{a}$). Then, investment for any firm is simply

$$k^*(z_{t+1}) - (1 - \delta)k^*(z_t)$$

Thus, investment is bounded above by

$$k^*(z) - (1 - \delta)k^*(z).$$

However, available funds, in steady state, are not pinned down since they are equal to

$$f(z_t, k^*(z_t)) - r(k^*(z_t) - a_t)$$

Since the firm is indifferent between any $a_t \geq \bar{a}$, there are a continuum of equilibria with different steady state asset holdings of publicly held firms each corresponding to different amounts of external financing.

C.3 Analysis of Non-financial Linkages in a Static Version of our Benchmark Model

In this section, we develop a static version of our model in order to establish the main economic mechanism of the model. We use the static model to illustrate how shocks to the collateral constraint affects both firms for whom the constraint binds and also those for whom the constraint does not bind. We derive a sufficient condition on the model parameters under which a tightening of the collateral constraint leads all firms to reduce output in equilibrium.
Consider a static economy populated by a continuum of intermediate good firms, a representative final good producer, and a representative household.

**Intermediate Good Producers.** Each intermediate good firm \( i \in [0, 1] \) has an asset level \( a_i \) and productivity \( z_i \). Moreover, \((a_i, z_i)\) is distributed according to \( F(a, z) \). An intermediate good firm with productivity \( z_i \) and assets \( a_i \), rents labor, \( L \), and capital, \( K \), rents out its assets \( a_i \) and purchases an amount of the final good, \( I \), to be used as an input to production according to the production function

\[
q_i = z_i^{\frac{1}{\rho-1}} (k^{\alpha}l^{1-\alpha})^{\eta} I^{1-\eta}
\]

Each firm may rent capital up to a multiple of the value of the firm’s assets. Specifically, we impose a *collateral constraint* so that the amount of capital rented by a firm with asset level \( a_i \) is bounded by \( \lambda a_i \) where \( \lambda \geq 1 \). One can rationalize this type of constraint by a model of moral hazard or limited enforcement. In line with the rest of the literature, we impose this constraint and do not provide a formal micro foundation for it.

**Final Good Producer.** The final good producer uses a bundle of inputs purchased from intermediate good firms and takes their prices as given. Given a bundle \( \{q_i\}_{i \in [0, 1]} \), the final good producer uses the following Dixit-Stiglitz production function:

\[
Q = \left[ \int_0^1 q_i^\rho \frac{\rho-1}{\rho} dF(i) \right]^\frac{1}{\rho-1}
\]

with \( \rho > 1 \).

**Households.** We assume that there is a representative households who buys the final good and provides labor to intermediate good producers. Following Greenwood et al. (1988), we assume that household preferences are given by

\[
U \left( c - \psi \frac{1+\frac{1}{\xi}}{1+\frac{1}{\xi}} \right)
\]

**Markets.** We assume that the labor market is competitive at wage level \( w \). As for capital market, we assume that the economy is small and open. That is, there exist suppliers of capital that have deep pockets and inelastically supply capital at a given interest rate \( r \). This is an assumption that simplifies the analysis; under this assumption, when we perform comparative statics, we do not need to consider general equilibrium effects that arise from changes in the interest rate for capital. Similar analysis can be done in a closed economy.
Moreover, we assume that there is monopolistic competition across intermediate good firms and prices are given by $p_i$. The final good producer takes these prices as given and intermediate good producers take the demand function for intermediate output as given. We normalize the price of final good to 1.

Given the above market structure, the final good producer’s maximization problem is given by

$$\max_{q_i} \left[ \int_0^1 \frac{p-1}{q_i} \frac{dF(i)}{p-1} - \int_0^1 p_i q_i dF(i) \right].$$

The resulting demand for intermediate good $i$ is given by

$$q_i = \frac{1}{p_i} Q \frac{1}{\rho} = p_i.$$

Given this demand function, each intermediate good firm maximizes its profit subject to its collateral constraint:

$$\pi_i = \max_{k,l,L,p_i} p_i z_i^{1-\frac{1}{p}} \left( k^{\alpha L^{1-\alpha}} \right)^{\eta} L^{1-\eta} - wL - rk - I + ra_i \tag{15}$$

subject to

$$p_i = Q \frac{1}{\rho} \left( z_i^{1-\frac{1}{p}} \left( k^{\alpha L^{1-\alpha}} \right)^{\eta} L^{1-\eta} \right)^{-\frac{1}{p}}$$

$$k \leq \lambda a_i$$

We say that a firm is financially constrained if the collateral constraint is binding for a firm in equilibrium and is financially unconstrained otherwise.

To complete the definition of competitive equilibrium, we need to specify the household’s optimization problem as well as market clearing conditions. The representative household’s optimization problem is given by

$$\max_{c,L} U \left( c - \psi L^{1+\frac{1}{\varepsilon}} \right) \frac{1}{1+\frac{1}{\varepsilon}}$$

subject to

$$c \leq wL + \int_0^1 \pi_i dF(i).$$
Labor market and product market clearing are given by

\[ \int_0^1 l_i dF(i) = L \]

\[ c + \int_i I_i dF(i) = Q. \]

Hence a competitive equilibrium of this economy is given by \( \{ k_i, l_i, I_i, p_i \}_{i \in [0,1]} \), \( c, L, Q, w \) that satisfies the above conditions.

Because of monopolistic competition, the revenue function of the firm exhibits decreasing returns to scale. As a result, for every \( z \), there is an unconstrained optimal scale, which is increasing in \( z \). Not surprisingly, then, every \( z \), there is a threshold in assets, say \( a^*(z) \) such that firms with assets and productivity \( (a, z) \) with \( a \geq a^*(z) \) are financially unconstrained and if \( a < a^*(z) \) the firm is financially constrained. We state this result along with optimal capital, labor, and intermediate input decisions for firms in the following lemma (the proof is omitted).

**Lemma 4** For every \( z \), there exists \( a^*(z) \) such that for \( a \geq a^*(z) \), \( k(a, z) \leq \lambda a \) and for \( a < a^*(z) \), \( k = \lambda a \). Furthermore, \( a^*(z) \) satisfies

\[ a^*(z) = \frac{1}{\lambda} [\nu(1-\eta)]^{(1-\eta)\nu} \left( \frac{\alpha \eta \nu}{\bar{\nu}} \right)^{1+\frac{\alpha \eta \nu}{1-\nu}} \left( \frac{\nu(1-\alpha)\eta}{w} \right)^{\frac{1-\alpha}{1-\nu}} Qz. \]

If \( a \geq a^*(z) \) then

\[ k(a, z) = [\nu(1-\eta)]^{(1-\eta)\nu} \left( \frac{\alpha \eta \nu}{\bar{\nu}} \right)^{1+\frac{\alpha \eta \nu}{1-\nu}} \left( \frac{\nu(1-\alpha)\eta}{w} \right)^{\frac{1-\alpha}{1-\nu}} Qz \]

if \( a < a^*(z) \) then \( k(a, z) = \lambda a \). Finally,

\[ l = \left( \frac{\nu(1-\alpha)\eta}{w} \right)^{\frac{1-(1-\eta)\nu}{1-(1-\alpha)\nu}} (\nu(1-\eta))^{\frac{(1-\eta)\nu}{1-(1-\alpha)\nu}} (Qz)^{\frac{1-\nu}{1-(1-\alpha)\nu}} k^{\frac{\alpha \eta \nu}{1-(1-\alpha)\nu}} \]

\[ I = \left[ \nu(1-\eta)(Qz)^{-\nu} \left( k^\alpha l^{1-\alpha} \right)^{\eta \nu} \right]^{\frac{1}{1-(1-\eta)\nu}} \]

where \( \nu = 1 - \frac{1}{\rho} \).

Given the decisions of firms along with optimal labor supply of households, we can characterize equilibrium output, \( Q \) and the wage rate, \( w \) using the production function
of the final good producer and the labor market clearing conditions. Given these equilibrium values, we can show that the equilibrium wage rate is decreasing in the collateral constraint parameter \( \lambda \). This final result follows because a relaxing of the constraint must increase aggregate capital demand and therefore labor demand causing the wage to rise. We have the following lemma.

**Lemma 5** Any competitive equilibrium must satisfy the following:

1. \( Q = \frac{1}{(1-\alpha)\eta(1-\frac{\nu}{2})} \psi^{-\varepsilon} w^{1+\varepsilon} \),

2. The equilibrium wage, \( w \), is decreasing in \( \lambda \).

**Proof.** Given capital, labor and intermediate input demand, we can construct aggregate excess output and labor as functions of prices \( r, w \), and output \( Q \). Specifically, let \( A^* = \{(a, z) : a \geq a^*(z)\} \)

\[
Q^\nu = \int q^\nu G(da, dz)
\]

\[
= \left( \frac{\nu(1-\alpha)\eta}{w} \right)^{\frac{\nu(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left( \frac{\alpha \eta \nu}{f} \right)^{\frac{\alpha \eta \nu}{1-\nu}} Q^\nu \int_{(a,z) \in A^*} zG(da, dz)
\]

\[
+ \left( \frac{\nu(1-\alpha)\eta}{w} \right)^{\frac{\nu(1-\alpha)}{1-\alpha \eta \nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\alpha \eta \nu}} Q^{\frac{\nu(1-\eta)(1-\alpha \eta \nu)}{1-(1-\alpha \eta \nu)\nu}} \lambda^{\frac{\alpha \eta \nu}{1-(1-\alpha \eta \nu)\nu}} \times
\]

\[
\int_{(a,z) \in A^*} z^{\frac{\nu}{1-(1-\alpha \eta \nu)}} a^{\frac{\alpha \eta \nu}{1-(1-\alpha \eta \nu)\nu}} G(da, dz)
\]

Labor Demand is just a function of output. We have

\[
l = \left( \frac{(1-\alpha)\eta \nu}{w} \right) Q^{1-\nu} q^\nu.
\]

Thus

\[
\int lG(da, dz) = \left( \frac{(1-\alpha)\eta \nu}{w} \right) Q^{1-\nu} \int q^\nu G(da, dz)
\]

Household labor supply given our assumed form for household preferences (GHH) is just \( \psi^{-\varepsilon} w^\varepsilon \). Thus labor market clearing is simply

\[
\psi^{-\varepsilon} w^\varepsilon = \left( \frac{(1-\alpha)\eta \nu}{w} \right) Q
\]

and, therefore, aggregate output in equilibrium satisfies

\[
Q = \psi^{-\varepsilon} (1-\alpha)\eta \nu^{-1} w^{1+\varepsilon}.
\]
Re-write $Q$ from (16) as

$$
1 = \left( \frac{\nu(1-\alpha)\eta}{\nu} \right) \frac{\eta}{\nu} \left( \frac{\nu(1-\eta)}{1-\nu} \right) \frac{\alpha\eta\nu}{\tilde{f}} \int_{(a,z)\in A^*} zG(da, dz) \tag{18}
$$

$$
+ \left( \frac{\nu(1-\alpha)\eta}{\nu} \right) \frac{\eta}{\nu} \left( \frac{\nu(1-\eta)}{1-\nu} \right) \frac{\alpha\eta\nu}{\tilde{f}} \int_{(a,z)\notin A^*} zG(da, dz) \tag{19}
$$

Analyzing the derivative with respect to $\lambda$, we have

$$
0 = \left( \frac{\nu(1-\alpha)\eta}{\nu} \right) \frac{\eta}{\nu} \left( \frac{\nu(1-\eta)}{1-\nu} \right) \frac{\alpha\eta\nu}{\tilde{f}} \int_{(a,z)\in A^*} zG(da, dz)
$$

$$
\times \left[ -\frac{\nu\eta(1-\alpha)}{1-\nu} \frac{1}{w} \frac{dw}{d\lambda} \right]
$$

$$
+ \left( \frac{\nu(1-\alpha)\eta}{\nu} \right) \frac{\eta}{\nu} \left( \frac{\nu(1-\eta)}{1-\nu} \right) \frac{\alpha\eta\nu}{\tilde{f}} \int_{(a,z)\notin A^*} zG(da, dz)
$$

$$
\times \left[ -\frac{\nu\eta(1-\alpha)}{1-\nu} \frac{1}{w} \frac{dw}{d\lambda} - \frac{\alpha\eta\nu}{1-\nu} \frac{1}{Q} \frac{dQ}{d\lambda} + \frac{\alpha\eta\nu}{1-\nu} \frac{1}{\lambda} \right]
$$

Suppose $\frac{dw}{d\lambda} \leq 0$. Then $\frac{dQ}{d\lambda} \leq 0$. Then we must have

$$
-\frac{\nu\eta(1-\alpha)}{1-\nu} \frac{1}{w} \frac{dw}{d\lambda} - \frac{\alpha\eta\nu}{1-\nu} \frac{1}{Q} \frac{dQ}{d\lambda} + \frac{\alpha\eta\nu}{1-\nu} \frac{1}{\lambda} \leq 0
$$

and since

$$
\frac{\alpha\eta\nu}{1-\nu} \frac{1}{\lambda} > 0
$$

we have

$$
\frac{\nu\eta(1-\alpha)}{1-\nu} \frac{1}{w} \frac{dw}{d\lambda} + \frac{\alpha\eta\nu}{1-\nu} \frac{1}{Q} \frac{dQ}{d\lambda} > 0
$$

but the coefficients are all positive so this is a contradiction. As a result, the wage must be increasing in $\lambda$. ■

We are now ready to state our necessary and sufficient condition for a tightening of the collateral constraint to cause both financially constrained and financially unconstrained firms to decrease output. Using the optimal production decisions of firms, we can show
that a financially unconstrained firm’s output satisfies
\[ q_i = \left( \frac{\nu (1 - \alpha)}{w} \right)^{\frac{\eta(1-\alpha)}{1-\nu}} \left( \nu (1 - \eta) \right)^{\frac{1-\eta}{1-\nu}} \left( \nu \eta \right)^{\frac{\nu}{1-\nu}} \frac{\alpha \eta \nu}{f} \left( \nu \right)^{\frac{\nu}{1-\nu}} Qz^{\frac{1}{\nu}} \]

A one percent increase in \( w \) causes \( Q \) to increase by \( 1 + \varepsilon \) percent and \( w^{-\frac{\eta(1-\alpha)}{1-\nu}} \) to decrease by \( \eta \left( \frac{1-\alpha}{1-\nu} \right) \). Hence, output of financially unconstrained firms is increasing in \( w \) or \( \lambda \) if and only if \( 1 + \varepsilon \geq \eta \rho (1 - \alpha) \). We then have the following proposition.

**Proposition 6** Suppose there exists a positive measure set of constrained firms. If \( 1 + \varepsilon \geq \eta \rho (1 - \alpha) \), then output of all firms is increasing in the collateral constraint parameter, \( \lambda \).

### C.4 Analysis of the Importance of the Use of External Funds in a Stylized Version of our Benchmark Model

In this section, we analyze a version of our model with perfect competition, perfect substitutes, and an i.i.d. process for firm level productivity. Specifically, we assume that in every period, each firm has a probability \( \pi \) of having productivity equal to 1 and probability \( 1 - \pi \) of having probability equal to 0. We solve analytically for the equilibrium and the amount of external financing used by firms in the model. We then compare the effect of changes in the collateral constraint parameter, \( \lambda \), across economies with different probabilities of high productivity, \( \pi \).

In particular, for each \( \pi \)-economy, we choose the collateral constraint parameter, \( \lambda(\pi) \) so that the aggregate debt-to-assets ratio in the model is the same across all \( \pi \)-economies. We show that even though the debt-to-asset ratio is held constant, the amount of external financing is decreasing the probability of receiving a high productivity shock. Then, for each \( \pi \)-economy, we compare steady state wealth in the \( \lambda(\pi) \) economy to that in the economy when \( \lambda = 1 \), in other words, the autarkic version of that economy. We show that the difference in steady state wealth between the \( \lambda(\pi) \), high debt economy, and the \( \lambda = 1 \), no debt economy monotonically decreasing in the probability of receiving a high productivity shock.

**Model and Solution**

In this simplified version of our model, firms are identical, produce a homogeneous
final output good, and the process for firm level productivity is given by

\[ z_t = \begin{cases} 
1 & \text{with prob } \pi \\
0 & \text{with prob } 1 - \pi 
\end{cases} \]

where the shocks are independent and identically drawn across firms and time. We assume a small open economy with a fixed interest rate that satisfies \( 0 \leq r \leq \frac{1}{\beta} - 1 \).

The problem of a firm in any period can be written recursively as

\[ V(a, z) = \max \ln(c) + \beta EV(a', z') \]

subject to

\[ c + a' \leq (1 + r)a + \max_{k \leq \lambda a, l} zk^\alpha l^{1-\alpha} - wl - (r + \delta)k \]

Clearly a firm with \( z = 0 \) chooses \( k = 0, l = 0 \). It is straightforward to show that profits for a firm with \( z = 1 \) are given by

\[ \left[ \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}} - (r + \delta) \right] \lambda a. \]

Using this result, we may write the recursive problem of the firm as

\[ V(a, z) = \max \ln(c) + \beta EV(a', z') \]

s.t.

\[ c + a' \leq \{ z\lambda \left[ \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}} - (r + \delta) \right] + (1 + r) \} a. \]

Given our assumed form of preferences along with i.i.d. shock process for productivity, we immediately have that the savings functions are linear in asset holdings and given by

\begin{align*}
a'(a, 1) &= \beta \left\{ \lambda \left[ \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}} - (r + \delta) \right] + (1 + r) \right\} a \quad (20) \\
a'(a, 0) &= \beta (1 + r)a.
\end{align*}

The law of motion for assets in a steady state equilibrium yields the equilibrium wage rate which must satisfy

\[ 1 = \beta (1 + r) + \beta \pi \lambda \left[ \alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}} - (r + \delta) \right]. \]
Labor market clearing (with aggregate labor normalized to 1), then, defines steady state wealth:

$$\bar{A}(\lambda, \pi) = \left[ \frac{\alpha \beta (\pi \lambda)^{\alpha}}{1 - \beta (1 + \tau) + \beta (r + \delta) \pi \lambda} \right]^{\frac{1}{1 - \alpha}}.$$  

We now turn to analyzing the amount of external financing firms rely on as well as the amount of aggregate debt and assets.

**External Financing**

First, as in our quantitative model, we define available funds and investment by re-writing the budget

$$c_t + a_{t+1} = z_t k_t^{\alpha t} 1^{-\alpha} - w t_l - (r + \delta) k_t + (1 + r) a_t$$

of a firm with an explicit definition of debt \(b_t = k_t - a_t\). We then have

$$c_t + k_{t+1} - (1 - \delta) k_t = z_t^{\alpha t} k_t^{\alpha t} 1^{-\alpha} - w t_l - r b_t + b_{t+1} - b_t.$$  

We then define available funds, debt, and investment as

$$AF_t = k_t^{\alpha t} 1^{-\alpha} - w t_l - r b_t$$

$$b_t = k_t - a_t$$

$$X_t = k_{t+1} - (1 - \delta) k_t$$

Available funds for specific firms depends on their asset holdings and their productivity in any period. All of the derivations are included below. Available funds for firms with assets \(a_t\) and productivity \(z_t\) satisfy

$$AF_t(a_t, z_t) = \begin{cases} 
    a_t \left[ \frac{1 - \beta (1 + \tau)}{\beta \pi} + \lambda \delta + r \right] & \text{if } z_t = \bar{z} \\
    a_t r & \text{if } z_t = 0
\end{cases}$$

Investment, of course, depends on productivity and assets in period \(t + 1\) since these factors determine the amount of capital a firm uses in period \(t + 1\). Since assets in period \(t + 1\) are functions of assets and productivity in period \(t\) we may define investment as

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functions only of $a_t, z_t$ and $z_{t+1}$. We have

$$X_t(a_t, z_t, z_{t+1}) = \begin{cases} 
    a_t \lambda \left[ \delta + \frac{1-\pi}{\pi} (1 - \beta (1 + r)) \right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \\
    -a_t (1 - \delta) \lambda & \text{if } z_t = \bar{z}, z_{t+1} = 0 \\
    a_t \lambda \beta (1 + r) & \text{if } z_t = 0, z_{t+1} = \bar{z} \\
    0 & \text{if } z_t = 0, z_{t+1} = 0
\end{cases}.$$ 

To aid us in defining external financing, it is useful to define the amount of excess available funds a firm has for investment. Define $AF - X$ for each firm:

$$AF - X = \begin{cases} 
    a_t \left[ r + (1 - \beta (1 + r)) \frac{1 - \beta \lambda (1 - \pi)}{\beta \pi} \right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \\
    a_t \left[ \frac{1 - \beta (1 + r)}{\beta \pi} + \lambda + r \right] & \text{if } z_t = \bar{z}, z_{t+1} = 0 \\
    a_t \left[ r (1 - \lambda \beta) - \lambda \beta \right] & \text{if } z_t = 0, z_{t+1} = \bar{z} \\
    a_t r & \text{if } z_t = 0, z_{t+1} = 0
\end{cases}.$$ 

We use this expression to get a sense of which firms are likely to rely on external financing. Clearly unproductive firms in period $t+1$ will not typically rely on outside funds since both firms have 0 or negative investment. Typically, the firm that switches from unproductive in period $t$ to productive in period $t+1$ ($z_t = 0, z_{t+1} = \bar{z}$) will rely on outside funds since that firm has low available funds in period $t$ but a high amount of investment (when $\lambda$ is sufficiently large). Finally the firm that is productive in two consecutive periods ($z_t = \bar{z}, z_{t+1} = \bar{z}$) will typically not rely on external funds since that firm’s available funds are large in period $t$, however this is sensitive to the choice of $\lambda$ since, as $\lambda$ becomes large, even though the firm has high available funds, the amount the firm invests grows as well.

Before turning to the effects of changes in $\lambda$, we point out that in the aggregate, independent of the collateral constraint and the probability of being productive, in the aggregate firms can self finance all of their investment. To see this, notice that in the aggregate, investment is simply

$$\delta \pi \lambda \bar{A}$$

as productive firms are maintaining the capital stock, and available funds are given by

$$\bar{A} \left[ r + \frac{1 - \beta (1 + r)}{\beta} + \pi \lambda \delta \right]$$

Thus, in the aggregate, firms can self-finance all of their investment as the aggregate ex-
cess is given by

\[
\bar{A} \left[ r + \frac{1 - \beta (1 + r)}{\beta} \right]
\]

Finally, we have the aggregate debt-to-asset ratio:

\[
\frac{\pi (\lambda - 1)}{\pi \lambda + 1 - \pi}.
\] (21)

To see this final result, note that debt is just \( k - a \) for firms with \( k \geq a \). The only firms with \( k \geq a \) are those with \( z = 1 \). Hence, aggregate debt is simply \( \pi \bar{A} (\lambda - 1) \). Total assets, however, is not simply wealth, or \( \bar{A} \). Total assets are capital installed by firms with \( z = 1 \) and assets of firms with \( z = 0 \) since these firms, in effect, have claims to financial assets. Hence, total assets are given by \( \pi \lambda \bar{A} + (1 - \pi) \bar{A} \). Note that the aggregate debt-to-asset ratio for any \( \pi \) varies from 0 to 1 as \( \lambda \) varies from 1 to \( \infty \).

Relating External Financing to the Importance of Financial Markets

Consider the following exercise. For any \( \pi \), choose \( \lambda \) so that the debt-to-asset ratio is constant (same amount of aggregate debt relative to assets in every \( \pi \) economy). Then, consider the difference in steady state wealth when \( \lambda = \lambda (\pi) \) and when \( \lambda = 1 \) (or when debt-to-assets falls from the constant level to 0). I do this because the metric is easier to analyze (with respect to \( \pi \)) than is the derivative of steady state wealth. We have the following proposition.

**Proposition 7** Suppose \( 0 < r < \frac{1}{\beta} - 1 \). Let \( \pi \in [\underline{\pi}, \bar{\pi}] \) and define \( \lambda (\pi) \) such that the debt-to-asset ratio in the \( \pi \)-economy with parameter \( \lambda (\pi) \) is equal to \( \bar{B} \). If for all \( \pi, \frac{1}{\beta} < \lambda (\pi) < \frac{1}{\beta (1 - \pi)} \) then external financing is decreasing in \( \pi \) and \( \log(\bar{A} (\lambda (\pi), \pi) - \log(\bar{A} (1), \pi)) \) is decreasing in \( \pi \). (The result is the same for output).

**Proof.** The assumptions of the proposition ensure that the only firm relying on external funds for investment is the firm switching from unproductive to productive. Formally, these assumptions place bounds on \( \pi \) for a given \( \bar{B} \). To see this, using the definition of debt-to-assets in equation (21), we have that

\[
\lambda (\pi) = \frac{\bar{B}}{\pi (1 - \bar{B})} + 1.
\] (22)
Since \( \lambda(\pi) \) is decreasing in \( \pi \), we can replace the assumption on \( \lambda(\pi) \) by ensuring that

\[
\frac{1}{\beta} \leq \frac{\bar{B}}{\pi(1 - \bar{B})} + 1
\]

and

\[
\frac{\bar{B}}{\pi(1 - \bar{B})} + 1 \leq \frac{1}{\beta(1 - \bar{B})}
\]

It can be shown that these conditions are consistent with \( \bar{\pi} > \pi \).

Recall the definitions of external financing for the \((z_t = 0, z_{t+1} = \bar{z})\) and the \((z_t = \bar{z}, z_{t+1} = \bar{z})\) firms:

\[
a_t \left[ r(1 - \lambda \beta) - \lambda \beta \right] \text{ if } z_t = 0, z_{t+1} = \bar{z}
\]

\[
a_t \left[ r + (1 - \beta(1 + r)) \left[ \frac{1 - \beta \lambda(1 - \pi)}{\pi \delta \lambda} \right] \right] \text{ if } z_t = \bar{z}, z_{t+1} = \bar{z}
\]

Since \( 0 \geq 1 - \lambda \beta \), it must be that firm switching from unproductive to productive \((z_t = 0, z_{t+1} = \bar{z})\) uses external funds for investment and since \( 1 - \beta \lambda(\pi)(1 - \pi) \geq 0 \) the firm that is productive for two consecutive periods does not use external funds for investment. Therefore, our statistic on the amount of external funds used for investment satisfies

\[
\frac{\pi(1 - \pi)}{\delta} \left[ \beta(1 + r) - \frac{r}{\bar{B}} \right]
\]

Using the definition of \( \lambda \) in (22), we have

\[
\frac{1}{\lambda} = \frac{\pi(1 - \bar{B})}{\bar{B} + \pi(1 - \bar{B})}
\]

so that our external financing statistic satisfies

\[
\frac{(1 - \pi)}{\delta} \left[ \beta(1 + r) - \frac{r\pi(1 - \bar{B})}{\bar{B} + \pi(1 - \bar{B})} \right]
\]

\[
= \frac{(1 - \pi)}{\delta} \left[ \beta(1 + r) - \frac{r}{\pi(1 - \bar{B}) + 1} \right]
\]

As a result, we immediately see that our statistic is decreasing in \( \pi \). Consider now the
definition of steady state wealth. We have

\[ \bar{A}(\lambda, \pi) = \left[ \frac{\alpha \beta (\pi \lambda) \alpha}{1 - \beta (1 + r) + \beta (r + \delta) \pi \lambda} \right]^{\frac{1}{1 - \alpha}} \]

and

\[ \bar{A}(1, \pi) = \left[ \frac{\alpha \beta \pi \alpha}{1 - \beta (1 + r) + \beta (r + \delta) \pi} \right]^{\frac{1}{1 - \alpha}} \]

Then (recalling that \( \pi \lambda = \frac{\bar{B}}{1 - \bar{B}} + \pi \))

\[
(1 - \alpha) \left[ \log(\bar{A}(\lambda(\pi), \pi)) - \log(\bar{A}(1, \pi)) \right] = \alpha \log \left( 1 + \frac{\bar{B}}{1 - \bar{B}} \frac{1}{\pi} \right) - \log \left( 1 + \frac{\bar{B}}{1 - \bar{B}} \frac{1}{\frac{1 - \beta (1 + r)}{\beta (r + \delta)} + \pi} \right)
\]

Analyzing this equation, let \( c = \frac{\bar{B}}{1 - \bar{B}} \), \( d = \frac{1 - \beta (1 + r)}{\beta (r + \delta)} \). Then I claim that

\[ f(\pi) = \log \left( 1 + c \pi^{-1} \right) - \log(1 + c(d + \pi)^{-1}) \]

is decreasing in \( \pi \). To see this, we have

\[ f'(\pi) = \frac{-c \pi^{-2}}{1 + c \pi^{-1}} - \frac{-c(d + \pi)^{-2}}{1 + c(d + \pi)^{-1}} = c \left[ (d + \pi)^{-2}(1 + c(d + \pi)^{-1})^{-1} - \pi^{-2}(1 + c \pi^{-1})^{-1} \right] \]

And we must have

\[ (d + \pi)^{-2}(1 + c(d + \pi)^{-1})^{-1} \leq \pi^{-2}(1 + c \pi^{-1})^{-1} \]

since

\[ \pi^2 + c \pi \leq (d + \pi)^2 + c(d + \pi) \]

\[ 0 \leq d^2 + 2d\pi + cd \]

And we have that \( \beta (1 + r) \leq 1 \) so that \( d \geq 0 \) and \( c \geq 0 \). ■

Output is given by
\[ Y(\pi, \lambda) = \left( \frac{1 - \alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \pi \lambda \bar{L}(\pi, \lambda) \]

\[ = (\pi \lambda \bar{L}(\pi, \lambda))^{\alpha} \]

\[ = \left[ \frac{\alpha \beta \pi \lambda}{1 - \beta(1 + r) + \beta(r + \delta) \pi \lambda} \right]^{\alpha \frac{1}{1-\alpha}} \]

\[ = \left[ \frac{\alpha \beta}{(1 - \beta(1 + r)) (\pi \lambda)^{-1} + \beta(r + \delta)} \right]^{\alpha \frac{1}{1-\alpha}} \]

Then

\[ \frac{1 - \alpha}{\alpha} \left[ \log Y(\pi, \lambda(\pi)) - \log Y(\pi, 1) \right] \]

\[ = \log \left( \frac{1}{\pi} (1 - \beta(1 + r) + \beta(r + \delta)) \right) - \log \left( \frac{1}{\pi \lambda}(1 - \beta(1 + r)) + \beta(r + \delta) \right) \]

\[ = \log \left( \frac{1}{\pi} c_1 + c_2 \right) - \log \left( \frac{1}{\bar{c} + \pi} c_1 + c_2 \right) \]

Differentiating with respect to \( \pi \) we have

\[ \frac{-c_1 \pi^{-2}}{\pi^{-1} c_1 + c_2} - \frac{-c_1 (\bar{c} + \pi)^{-2}}{(\bar{c} + \pi)^{-1} c_1 + c_2} \]

\[ = c_1 \left[ \frac{1}{(\bar{c} + \pi) c_1 + (\bar{c} + \pi)^2 c_2} - \frac{1}{\pi c_1 + \pi^2 c_2} \right] \]

which must be negative.

**Derivations for External Financing**

- Available Funds:
- If $z_t = 1$ then

$$AF_t = \alpha \left( \frac{1-\alpha}{w^\alpha} \right)^{\frac{1-\alpha}{\alpha}} k_t - r(k_t - a_t)$$

$$= a_t \left[ \lambda \alpha \left( \frac{1-\alpha}{w^\alpha} \right)^{\frac{1-\alpha}{\alpha}} - r(\lambda - 1) \right]$$

$$= a_t \left[ \frac{1 - \beta(1 + r)}{\beta\pi} + \lambda(r + \delta) - r\lambda + r \right]$$

$$= a_t \left[ \frac{1 - \beta(1 + r)}{\beta\pi} + \lambda\delta + r \right]$$

If $z_t = 0$ then

$$AF_t = -rb_t = -r(-a_t) = ra_t$$

- Investment

  - If $z_t = z_{t+1} = \bar{z}$,

$$X_t = \lambda a_{t+1} - (1 - \delta)\lambda a_t$$

$$= \lambda \beta \left\{ \lambda \left[ \alpha \left( \frac{1-\alpha}{w^\alpha} \right)^{\frac{1-\alpha}{\alpha}} - (r + \delta) \right] + (1 + r) \right\} a_t - (1 - \delta)\lambda a_t$$

$$= \lambda a_t \left[ \beta \lambda \alpha \left( \frac{1-\alpha}{w^\alpha} \right)^{\frac{1-\alpha}{\alpha}} - \beta\lambda(r + \delta) + \beta(1 + r) - (1 - \delta) \right]$$

$$= \lambda a_t \left[ \frac{1 - \beta(1 + r)}{\pi} + \beta\lambda(r + \delta) - \beta\lambda(r + \delta) + \beta(1 + r) - (1 - \delta) \right]$$

$$= \lambda a_t \left[ \delta + \frac{1 - \pi}{\pi} (1 - \beta(1 + r)) \right]$$

  - If $z_t = \bar{z}, z_{t+1} = 0$,

$$X_t = -(1 - \delta)\lambda a_t$$

  - If $z_t = 0, z_{t+1} = \bar{z}$,

$$X_t = \lambda a_{t+1} = \lambda\beta(1 + r)a_t$$

  - If $z_t = z_{t+1} = 0, X_t = 0$.

- $AF - X$

  - If $z_t = z_{t+1} = 0$,

$$AF_t - X_t = ra_t$$

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- If $z_t = 0, z_{t+1} = \bar{z}$,

\[ AF_t - X_t = ra_t - \lambda \beta (1 + r) a_t \]
\[ = a_t [r - \lambda \beta (1 + r)] \]

- If $z_t = \bar{z}, z_{t+1} = 0$,

\[ AF_t - X_t = a_t \left[ \frac{1 - \beta (1 + r)}{\beta \pi} + \lambda \delta + r + (1 - \delta) \lambda \right] \]
\[ = a_t \left[ \frac{1 - \beta (1 + r)}{\beta \pi} + \lambda + r \right] \]

- If $z_t = z_{t+1} = \bar{z}$,

\[ AF_t - X_t = a_t \left[ \frac{1 - \beta (1 + r)}{\beta \pi} + \lambda \delta + r - \lambda \left[ \frac{1 - \pi (1 - \beta (1 + r))}{\pi} \right] \right] \]
\[ = a_t \left[ \frac{1 - \beta (1 + r)}{\beta \pi} + \lambda \delta + r - \lambda \left[ \frac{1 - \pi (1 - \beta (1 + r))}{\pi} \right] \right] \]
\[ = a_t \left[ r + (1 - \beta (1 + r)) \left[ \frac{1 - \beta \lambda (1 - \pi)}{\beta \pi} \right] \right] \]