Reputation and Persistence of Adverse Selection in Secondary Loan Markets*

V.V. Chari† Ali Shourideh‡ Ariel Zetlin-Jones§
chari002@umn.edu shouride@wharton.upenn.edu azj@andrew.cmu.edu

November 3, 2013

Abstract

The volume of new issuances in secondary loan markets fluctuates over time and falls when collateral values fall. We develop a model with adverse selection and reputation that is consistent with such fluctuations. Adverse selection ensures that the volume of trade falls when collateral values fall. Without reputation, the equilibrium has separation, adverse selection is quickly resolved and trade volume is independent of collateral value. With reputation, the equilibrium has pooling and adverse selection persists over time. The equilibrium is efficient unless collateral values are low and originators reputational levels are low. We describe policies that can implement efficient outcomes.

*We are grateful to the editor, four anonymous referees and Mark Aguiar, Hugo Hopenhayn, Roozbeh Hosseini, Larry Jones, Patrick Kehoe, Guido Lorenzoni, Chris Phelan as well as seminar participants at ASU, Kellogg, Yale, the 2009 SED Meeting, New York and Minneapolis Fed, the Conference on Money and Banking at the University of Wisconsin, and XII International Workshop in International Economics and Finance in Rio for helpful comments. Chari and Shourideh are grateful to the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. This paper is a revised version of our earlier paper titled 'Adverse Selection, Reputation, and Sudden Collapses in Secondary Loan Markets'.

†University of Minnesota, FRB Minneapolis
‡Wharton School, University of Pennsylvania
§Tepper School, Carnegie Mellon University
1 Introduction

Secondary loan markets allow loan originators to sell all or part of their loan portfolios to other financial institutions. These markets include the market for syndicated loans as well as the market for publicly traded securitized loans. This market is economically important. In 2007, for example, 1.3 trillion dollars of new loans were syndicated and from 1986 to 2012, approximately 500 billion dollars of new loans were syndicated each year\(^1\). The volume of new issuances in secondary loan markets arising from originators’ sales decisions fluctuates a great deal and sometimes collapses, typically when the value of the assets underlying the original loans falls. Ivashina and Scharfstein (2010), for example, present data showing that the volume of loans to large corporations, almost all of which are syndicated, declined by 37% in August and September 2008 relative to the previous year. In Chari et al. (2010), we document that in the late 1920s, the volume of new issuances in the securitized loan market fell sharply.

In the wake of the recent financial crisis, policy makers were concerned about the effect of the collapse on broader economic activity and initiated a variety of asset purchase policies intended to restore the volume of new issuances in secondary loan markets. Evaluating the effects of these policies requires models in which trade volume falls when the underlying asset values fall.

In this paper, we develop a dynamic model of the volume of new issuances of secondary loans, show that reductions in underlying asset values can generate a fall in this volume, and use the model to evaluate policies. Adverse Selection and Reputation play key roles in our model. We assume that sellers are better informed than buyers about the quality of their loan portfolios so that our model features adverse selection. Adverse selection ensures that the volume of trade falls when the value of the underlying asset falls. Incorporating reputational concerns is necessary in the sense that in the absence of these concerns adverse selection is quickly resolved. When such concerns are sufficiently strong, our model has pooling outcomes in the sense that banks of different quality levels make the same decisions. This pooling implies that buyers cannot use past actions to infer perfectly the quality levels of individual banks so that adverse selection persists.

The persistence of adverse selection implies that our model can generate fluctuations in trade volume associated with fluctuations in underlying asset values even in the long run. In terms of policy, we show that our equilibrium outcomes are efficient unless underlying asset values are low and secondary loan issuers have poor reputations. Thus, intervention is

\(^1\)We obtained data from Thompson Reuters LPC. This data set includes loans made to a subset of non-financial corporations.
needed only when underlying asset values are low and is best directed at issuers with poor reputations.

In our model, financial institutions which originate primary loans, called banks for convenience, sell all or part of these loans to other institutions. Motivated by an empirical literature discussed below, we assume that banks are better informed about the likelihood of default of these loans than buyers. This asymmetry of information creates an adverse selection problem because buyers understand that banks have an incentive to sell loans with a high likelihood of default. In the static version of our model, buyers use the quantity sold as a screening device to induce high quality banks to separate themselves from low quality banks. In particular, buyers offer nonlinear pricing schedules in which the price per unit of loans falls with the fraction of a bank’s loan portfolio that is purchased. Banks with high-quality loans are attracted to contracts that offer a high per unit price with a small fraction sold because holding loans is not very costly. Banks with low-quality loans are attracted to contracts that offer a low per unit price and allow a large fraction of their loans to be sold. The equilibrium is necessarily separating. Pooling outcomes in which banks with different qualities of loan portfolios choose the same amount of loans to be sold cannot be equilibria because buyers have strong incentives to offer cream-skimming contracts that are relatively attractive only to high quality banks. Because the equilibrium is separating, the quality level of individual banks becomes known to future buyers.\textsuperscript{2}

Adverse selection is particularly acute when the value of the loans in default, called collateral value, is low. In the static version of our model, we show that when the collateral value is low, the fraction of loans sold by high quality banks is low. Thus, declines in collateral values lead to declines in volume. This feature of our model also appears in other models in economics and finance that study the endogenous determination of trade volume with adverse selection. (See for example Glosten and Milgrom (1985), Kurlat (2013), Guerrieri and Shimer (2011), Garleanu and Pedersen (2004), Fishman and Parker (2012), among many others.)

While adverse selection is promising in accounting for volume fluctuations, the feature that the equilibrium is separating creates a challenge in accounting for fluctuations in volume in periods other than the first period in our dynamic model. The challenge arises because with separating equilibria, the quality levels of banks are revealed to buyers in future periods. There is then no adverse selection in future periods, if the same banks participate regularly in secondary markets and if quality levels are persistent. The empirical literature on sec-

\textsuperscript{2}A familiar issue with adverse selection models is that a separating equilibrium in pure strategies sometimes does not exist. We follow Dasgupta and Maskin (1986) in studying equilibria in mixed strategies. See Guerrieri et al. (2010) for an alternative approach to resolving non-existence of equilibria in pure strategies.
ondary loan markets discussed below documents that issuers of secondary loans are typically well-established institutions which regularly participate in these markets and are therefore not anonymous traders. This literature also suggests that quality and reputations are persistent. These considerations lead us to develop a dynamic model in which reputational considerations play a central role.

Our main contributions are to show that reputational concerns are necessary and sufficient for fluctuations in collateral values to lead to fluctuations in trade volume. In our dynamic model, we show that when reputational concerns are absent, the equilibrium has no asymmetry of information in any period other than the first one. We go on to show that when reputational concerns are sufficiently strong, asymmetry of information must persist in the sense that there is no equilibrium with complete revelation of information about quality levels of banks.

We focus on equilibria which maximize trade. When reputational concerns are sufficiently strong, low quality banks have strong incentives to mimic the behavior of high quality banks. We show that these mimicking incentives are so strong that they overcome cream-skimming attempts by buyers. Specifically when the reputational level of the bank is relatively high, the mimicking incentives are so strong that the equilibrium outcome has complete pooling with full trade of loan portfolios by banks of all qualities. Since the equilibrium has complete pooling no information is revealed about the quality of bank portfolios. When a bank’s reputational level is relatively low, the mimicking incentives are still very strong. The equilibrium has partial pooling in the sense that high quality banks sell a relatively small portion of their loan portfolios and low quality banks randomize between selling all their loans and pooling with high quality banks. This randomization implies that the contract choice reveals some but not complete information about the quality levels of the bank’s portfolio. The equilibrium has partial pooling because banks with high quality loans have strong incentives to retain their entire loan portfolios rather than accepting a low complete pooling price.3

The potential for our model to be consistent with substantial fluctuations in the volume of trade is best exemplified in the infinite horizon version of our model with aggregate shocks to collateral values. We show that the aggregate volume of trade is relatively low when collateral values are low and high when collateral values are high. We show that the model can generate discontinuous fluctuations in the volume of trade at critical levels of collateral values. We present an illustrative simulation in which the volume of trade rises gradually over time and then abruptly falls.

We address policymakers’ concerns about fluctuations in the volume of trade in secondary

3Pooling equilibria also emerge in the literature on the so-called Ratchet effect. See, for example, Freixas et al. (1985), Laffont and Tirole (1988).
loan markets by analyzing the efficiency properties of our model. We show that contracts which induce greater separation in allocations between banks of different qualities reduce welfare by distorting current allocations but raise future welfare by revealing more information about the quality level of banks. We show that this trade-off is always resolved by reducing the extent of separation in allocations and the amount of information revelation to its lowest feasible value. Indeed, when a bank’s reputation level is high, efficient allocations have no separation and full trade. These allocations therefore coincide with maximal trade equilibrium outcomes. Efficient allocations and equilibrium allocations also coincide when reputation is low as long as adverse selection is not too severe. The two differ from each other only when a bank’s reputation level is low and adverse selection is severe. In this case, competitive forces induce more separation and more information revelation than do efficient allocations. We show that efficient outcomes can be implemented by policies which limit private trade.

Our model also generates inefficient fluctuations in the volume of trade through another channel. Like many signaling models, our model has multiple equilibria, and fluctuations can occur if market participants switch from coordinating on one equilibrium to coordinating on another equilibrium. We analyze policies intended to eliminate inefficient equilibria and ensure a unique equilibrium. In some models such as those analyzing deposit insurance policies (see Diamond and Dybvig (1983) for example), inefficient equilibria can be eliminated by a commitment from the government to pay off depositors using the proceeds from bank assets. This policy does not need to be activated in equilibrium and therefore does not use external resources. In our model, we argue that conventional asset purchase policies in which the government offers to purchase loans at an actuarially fair price are ineffective in the sense that these policies do not eliminate inefficient equilibria. We show that more unconventional policies which limit private trade can be effective in ensuring uniqueness of equilibrium.

1.1 Evidence and Related Literature

Here, we present evidence that adverse selection and reputation play important roles in secondary loan markets as well as evidence supporting implications of our model. We also discuss related literature.

Ivashina (2009), in a study of the syndicated loan market, finds that when the share of the loan retained by the originator rises, other participants accept a lower per unit return on the loan. This feature of the data is consistent with adverse selection models and is hard

---

4See Spence (1973) for the classic example of this phenomenon and Vickers (1986) for an application in the Barro and Gordon (1983) model.
to reconcile with other models. Downing et al. (2009) find that loans that banks held on their balance sheets yielded higher returns on average than did similar loans which they securitized and sold. Drucker and Mayer (2008) argue that underwriters of prime mortgage-backed securities are better informed than buyers. Specifically, the tranches on which such underwriters bid perform better than the tranches on which they do not bid. See also Dewatripont and Tirole (1994), Ashcraft and Schuermann (2008), and Arora et al. (2009)) for arguments and evidence of adverse selection in secondary loan markets.

Ivashina (2009) shows that the syndicated loan market is dominated by the same banks over time so that originators are not anonymous traders. Ross (2010) and Fang (2005) present evidence on the importance of reputation in secondary loan markets. Ross (2010) finds that when borrowers obtain loans from high reputation banks the stock price response of borrowers is favorable relative to borrowers who obtain primary loans from lower reputation banks. He also finds that borrowers who obtain loans from high reputation banks receive lower interest rates than borrowers who obtain loans from lower reputation banks. He interprets these findings as suggesting that banks are heterogeneous in their ability to screen and monitor primary loans. Since this heterogeneity persists in the data, Ross’s evidence provides strong support for our assumption that the quality of loans originated by banks is persistent over time.

Fang (2005) studies the role of bond underwriting in the investment banking industry. She finds that reputable investment banks charge higher fees but obtain larger issues for the borrowers whose loans they underwrite. She argues that these findings are evidence both of heterogeneity in the quality of underwriting services and of the importance of reputational considerations which induce reputable investment banks to provide underwriting services conscientiously.

Our model implies that when adverse selection problems become more severe, the average quality of loans that are sold falls compared to the average quality of loans that are retained. Elul (2011) presents evidence on the quality of loans that were sold relative to those that were held and finds that in 2006 there was not much difference in these quality levels. Starting in 2006, the quality of loans that were sold worsened relative to the quality of loans that were held. Our model is consistent with Elul (2011)’s finding given that underlying asset values seem to show signs of weakening in 2006. Mian and Sufi (2009) present evidence that securitized loans were more likely to default than non-securitized loans. This evidence is also consistent with our model.

We build on an extensive literature on adverse selection in asset markets, including the work of Myers and Majluf (1984), Glosten and Milgrom (1985), Kyle (1985), and Garleanu and Pedersen (2004) as well as to the related securitization literature, specifically, the work
of DeMarzo and Duffie (1999) and DeMarzo (2005). Our work is particularly related to that of Fishman and Parker (2012) and Guerrieri and Shimer (2011) who study trade volume fluctuations in models with adverse selection. As in Fishman and Parker (2012) and in Dang et al. (2012), the equilibrium in our model sometimes features inefficiently high levels of information revelation. Our work is also related to Eisfeldt (2004), and Kurlat (2013) who study dynamic environments with persistent adverse selection. In these papers, either the type of an agent is not persistent over time or agents are anonymous. In our paper, in contrast, we have argued that the natural assumptions are that types are persistent and agents are not anonymous. With these assumptions, reputational concerns necessarily play a central role.\footnote{Camargo and Lester (2011) and Daley and Green (2012) study adverse selection in dynamic models with persistent types. Sellers are endowed with an asset or a commodity at the beginning of the game and must decide when to sell. In our model, in contrast, current decisions affect the prices at which banks can sell their assets in the future.}

We also build on an extensive literature on reputation. See for example Kreps and Wilson (1982), Milgrom and Roberts (1982), Diamond (1989), Mailath and Samuelson (2001), Ely and Välimäki (2003), Ely et al. (2008), and Ordoñez (2013). Much of that literature develops reputational models in which certain types of players play fixed strategies rather than maximizing payoffs. In those models, players develop reputations by deciding whether or not to mimic their non-strategic counterparts. We have no non-strategic players. In our model, banks of all quality types must decide whether or not to mimic each other.

Our analysis of policy is closely related to recent work by Philippon and Skreta (2012) as well as Tirole (2012) who analyze policies in models with adverse selection. The main difference between our work and theirs is that we focus on the incentives induced by reputation while these authors analyze static models.

\section{Static Model of Adverse Selection in Secondary Loan Markets}

In this section, we introduce and analyze a static model of a secondary loan market that features adverse selection. The static model sets the stage for the dynamic models with reputation. In the model, the primary economic role of the secondary loan market is to allow loans to be reallocated from originators to buyers who have a comparative advantage in holding and managing loans. Adverse selection arises because originators are better informed about the quality of their loans. We show that such a model has a unique separating equilibrium and we show that fluctuations in collateral values induce fluctuations in aggregate
trading volume.

2.1 Model

Consider an economy with a large number of loan originators referred to as banks and a large number of buyers. Both banks and buyers are risk neutral. Each bank is endowed with a loan portfolio. We normalize the size of the loan portfolio to be 1. The loans in a portfolio can also be thought of more generally as investment opportunities such as projects, mortgages, or asset-backed securities.

Loans are risky because of the possibility of a default. If no default occurs, they yield a return of $\bar{v}$ and if a default occurs, they yield a return of $v$. We refer to $\bar{v}$ as the collateral value of the loan and to $v = \bar{v} - v$ as the spread in returns. Note for future reference that a fall in the collateral value, $\bar{v}$, induces an increase in the spread, $v$. The probability of no default is denoted by $\pi$. For simplicity, we assume that the probability of default is the same for all loans in a given bank’s portfolio. Banks are heterogeneous in the sense that the probability of default differs across different banks. We assume that there are two types of banks so that the probability of no default can take on one of two values $\pi \in \{\bar{\pi}, \pi\}$ with $\bar{\pi} < \pi$. We refer to a bank that has a loan portfolio of type $\bar{\pi}$ as a high-quality bank and one with a loan portfolio of type $\pi$ as a low-quality bank.

We assume that the buyers have a comparative advantage in managing loans. We model this comparative advantage by assuming that the cost for the bank of holding and managing a loan is $c > 0$ and we normalize the holding and management cost for buyers to be 0. The holding and management costs represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank’s portfolio, among other potential factors.

The bank chooses how much of its loan portfolio to sell in the secondary market. Let $x$ denote the fraction of the loan portfolio that the bank sells. Let $t$ denote the payment the bank receives from buyers for selling $x$ loans so that the ratio $t/x$ is the price per loan. The payoff for a bank of type $\pi$ which sells a fraction $x$ of its loan portfolio at payment $t$ can be written, subject to a normalization, as

$$t + (1 - x) (\pi v - c)$$

and the profit of a buyer who purchases a fraction of loans $x$ for a payment $t$ from a bank of type $\pi$ is given by $x\pi v - t$. (In the Appendix A, we provide details of this normalization.) If buyers are perfectly informed about the types of banks, it is clearly efficient to have banks sell their entire loan portfolios.
We introduce adverse selection by assuming that the bank knows the type of loans in its portfolio and that potential buyers do not. Buyers believe that the bank is high-quality with probability \( \mu \) and low-quality with probability \( 1 - \mu \). We refer to \( \mu \) as the reputation of the bank. Banks differ in their reputation levels and the distribution of banks by reputation levels is given exogenously. In the dynamic models that follow, this reputation evolves endogenously. We model buyers as engaging in Bertrand-style price competition, so it suffices to restrict the number of potential buyers to two.

We begin by analyzing the secondary loan market for a given bank with reputation \( \mu \) and then extend the analysis to aggregate outcomes. In the secondary loan market, buyers simultaneously offer contracts to the bank. A contract specifies the fraction of loans the buyer will purchase and the payment for such a purchase. Since we have two types of banks, a contract, \( z \), consists of a four-tuple \((x_h, t_h, x_l, t_l)\). Here a pair \((x_i, t_i)\) is an offer that is intended for a bank of type \( i = l, h \). After buyers offer contracts, the bank chooses which buyer’s contract and offer to accept. Since the bank can choose which offer to accept, we can restrict attention to contracts that are incentive compatible in the sense that they satisfy

\[
\begin{align*}
& t_h + (1 - x_h) (\bar{\pi} v - c) \geq t_i + (1 - x_i) (\bar{\pi} v - c) \quad (2) \\
& t_l + (1 - x_l) (\bar{\pi} v - c) \geq t_h + (1 - x_h) (\bar{\pi} v - c) \quad (3)
\end{align*}
\]

The incentive constraints (2) and (3) ensure that facing a contract \( z \), a bank of quality type \( i, i = l, h \), prefers the offer \((x_i, t_i)\) intended for it to the offer intended for the bank of the other type. Let \( Z \) denote the set of incentive compatible contracts.

A well known problem with adverse selection models is that equilibria in pure strategies sometimes do not exist (see for example Rothschild and Stiglitz (1976). Dasgupta and Maskin (1986) show that in many such environments, mixed strategy equilibria do exist. We follow Rosenthal and Weiss (1984) in allowing for mixed strategies on the part of buyers and pure strategies by banks.

A strategy for buyer \( j = 1, 2 \) is a distribution function \( F_j(z) \) over the set \( Z \). A strategy for the bank consists of an action \( \delta_j(z_j, z_{-j}; \bar{\pi}) \in [0, 1] \) for \( j = 1, 2 \) where \( \delta_j \) denotes the probability that contract \( j \) is accepted. Obviously, the bank may not accept both contracts. But we do allow the bank to reject both contracts. Given the mixed strategy by the other buyer, \( F_{-j} \), and the strategy of the bank, \( \delta \), the profits earned by a buyer offering contract \( z \) are given by

\[
\int [\mu \delta_j(z, z_{-j}; \bar{\pi}) (x_h \bar{\pi} v - t_h) + (1 - \mu) \delta_j(z, z_{-j}; \bar{\pi}) (x_l \bar{\pi} v - t_l)] dF_{-j}(z_{-j}) \quad (4)
\]

An equilibrium consists of strategies for buyers and a strategy for the bank such that (i) for all \( z_j \) in the support of \( F_j \) there is no alternative contract \( \hat{z}_j \) which earns strictly
higher profits and (ii) the bank’s strategy specifies that its choice is maximizes its payoff. We require that our equilibria be monotone in the sense that a low-quality bank prefers a contract $\hat{z}$ to a contract $z$ if and only if a high-quality bank also prefers the contract $\hat{z}$ to $z$. In any monotone equilibrium, $\delta_j (z, z_{-j}; \bar{\pi}) = 1$ if and only if $\delta_j (z, z_{-j}; \bar{\pi}) = 1$. We restrict ourselves to tie-breaking rules in which if contracts offered by both buyers give the same payoff to a bank of given quality, the bank accepts either offer with probability $1/2$. This assumption is purely for convenience. Since buyers can offer a contract $(0, 0, 0, 0)$ we can restrict ourselves to equilibria in which the bank chooses some offer. An equilibrium is symmetric if $F_1 = F_2 = F$.

We say that an equilibrium is separating if the offers accepted by low- and high-quality banks are different. In such a situation, after trades have occurred, the type of the bank is known. Separating equilibria are of interest in dynamic versions of our model because future buyers could exploit knowledge of choices by the bank in previous periods and thus affect the behavior of the bank in the current period.

We now show that our model has a unique separating equilibrium. We begin by using arguments similar to those in Dasgupta and Maskin (1986) to show that any monotone equilibrium outcome must have four key properties. (See the Appendix for the proof.) At each contract in the support of $F$, the low-quality bank sells its entire loan portfolio ($x_l = 1$), the incentive constraint for the low type, (3), holds with equality, buyers make zero profits in the sense that for each point in the support of $F$, the expression in brackets in (4) is zero, and the intended offers to the low-quality bank do not yield positive profits so that $t_l \geq \bar{\pi}v$. Since $x_l = 1$, the zero profit condition at each point in the support of $F$ can be written as

$$\mu (x_h \bar{\pi} v - t_h) + (1 - \mu) (\bar{\pi} v - t_l) = 0.$$  

(5)

Since any contract must also satisfy the incentive constraint of the low-quality bank with equality and $x_l = 1$, it follows that given any payment to the low-quality bank $t_l$, the equations (3) with equality and (5) can be solved to uniquely obtain the the other elements of the contract, $x_h, t_h$. Thus, any distribution $F$ can be represented by an associated distribution over payments to the low-quality bank $t_l$. With some abuse of notation, we use $F$ to denote the distribution over $t_l$.

Next we characterize the pure strategy equilibrium and the range of reputation levels for which such an equilibrium exists. Following Rothschild and Stiglitz (1976), it is straightforward to show that any pure strategy equilibrium must have buyers breaking even on each type of bank so that $t_l = \bar{\pi}v$ and $t_h = x_h \bar{\pi}v$. Since the incentive constraint for the low-quality bank (3) must hold with equality, we can substitute for $t_l$ and $t_h$ into (3), to obtain that the
fraction of loans sold by the high-quality bank is given by
\[ x_h = \frac{1}{1 + \frac{(\bar{\pi} - \pi)v}{c}} = \frac{1}{1 + d} \]  \hspace{1cm} (6)
where \( d = (\bar{\pi} - \pi)v/c \). We refer to \( d \) as the Adverse Selection Discount. This discount captures the extent to which adverse selection reduces trade compared to the full information volume of trade. Note that as the dispersion in quality \( \bar{\pi} - \pi \) or the loan spread \( v \) increase, the adverse selection discount increases. Recall that \( v = \bar{v} - \bar{\pi} \). Thus the adverse selection discount increases as the collateral value \( v \) falls.

We say that the contract \( (x_h, x_h \bar{\pi}v, 1, \pi v) \) where \( x_h \) is given by (6) is the least-cost separating outcome. This outcome is that associated with the pure strategy equilibrium. In the proof of the proposition below, we show that when the bank’s reputation is below a threshold, \( \tilde{\mu} \), the least-cost separating outcome is an equilibrium outcome. To determine the threshold \( \tilde{\mu} \) compare the payoffs of the high-quality bank in the least-cost separating outcome given by \( x_h \bar{\pi}v + (1 - x_h)(\bar{\pi}v - c) \) with the payoffs it would receive from accepting a zero profit full trade pooling contract. Such a contract specifies \( x_h = x_l = 1 \) with \( t_h = t_l = \hat{p}(\mu) \) where \( \hat{p}(\mu) \) is given from the zero profit condition for buyers as
\[ \hat{p}(\mu) = \mu \bar{\pi}v + (1 - \mu)\pi v. \]  \hspace{1cm} (7)
Using (6) and (7), straightforward algebra shows that the high-quality bank is indifferent between the least-cost separating outcome and the zero profit pooling contract at a reputation level
\[ \tilde{\mu} = \frac{d}{1 + d}. \]  \hspace{1cm} (8)

Above \( \tilde{\mu} \), the high-quality bank strictly prefers the zero profit pooling contract to the least-cost separating outcome. Thus, when the bank’s reputation is relatively high, no pure strategy equilibrium exists because a deviation by one of the buyers to an allocation near the pooling outcome is profitable. With appropriately chosen mixed strategies, deviations are not profitable. The idea is to construct the mixed strategies so that any deviation attracts low-quality banks with disproportionate probability. In the Appendix, we show that our model has a mixed strategy equilibrium in which the distribution \( F \) is given by
\[ F(t_l) = \left( \frac{t_l - \pi v}{\pi v} \right)^{\tilde{\mu} - 1} \]  \hspace{1cm} (9)
with support given by \([\pi v, \hat{p}(\mu)]\) where the pooling price \( \hat{p}(\mu) \) is given by (7). Notice that since the payments to the low-quality bank are higher than \( \pi v \) with positive probability, the equilibrium features cross-subsidization in the sense that profits from high-quality banks are used by buyers to subsidize losses from low-quality banks. We summarize this discussion in
the following Proposition which is proved in the Appendix.

**Proposition 1** The static model has a separating equilibrium. If \( \mu \leq \bar{\mu} \) the equilibrium outcome is the least cost separating outcome. If \( \mu \geq \bar{\mu} \), then the equilibrium has mixed strategies by buyers and the distribution over contracts is given by (9). Furthermore, the separating equilibrium is unique in the class of monotone equilibria.

This static equilibrium induces payoffs for the high and low quality banks as a function of their initial reputation levels, \( \mu \), given by

\[
V(\mu; \pi) = \begin{cases} 
\pi v + \frac{\max\{\mu - \bar{\mu}, 0\}}{1 - \bar{\mu}} (\bar{\pi} - \pi) v & \pi = \bar{\pi}, \\
\hat{p}(\bar{\mu}) + \frac{\max\{\mu - \bar{\mu}, 0\}^2}{\mu(1 - \bar{\mu})} (\bar{\pi} - \pi) v & \pi = \bar{\pi}.
\end{cases}
\] (10)

Note that in the least cost separating allocation, the payoff of the high quality bank is equal to its payoff in a pooling outcome with \( \mu = \bar{\mu} \) given by \( \hat{p}(\bar{\mu}) \). This value function plays a central role in our dynamic model with reputational concerns.

Note that we have described an equilibrium in which buyers possibly mix and banks do not. Using standard cream-skimming arguments, it is straightforward to show that there is no equilibrium in which banks use mixed strategies. When we turn to the dynamic model, we will show that standard cream-skimming arguments fail so that there are equilibria in which banks do mix.

### 2.1.1 Trade Volume and the Spread in the Static Model

Next, we turn to the promise of adverse selection models in generating fluctuations in the volume of trade associated with changes in collateral values. Specifically, we will show that an increase in the adverse selection discount results in a fall in trade volume for all levels of reputation. To show this result, consider the expected volume of trade for a given bank with reputation level \( \mu \). For low levels of reputation, \( \mu \leq \bar{\mu} \), from Proposition 1, the equilibrium outcome is the least cost separating outcome so that the expected volume of trade, \( T_L(\mu) \), is given by

\[
T_L(\mu) = \mu \frac{1}{d + 1} + (1 - \mu). \] (11)

For high levels of reputation, \( \mu > \bar{\mu} \), from Proposition 1, the equilibrium outcome has mixed strategies. Straightforward computation using the form of \( F \) in (9) yields that the expected volume of trade in our model \( T_H(\mu) \) is given by

\[
T_H(\mu) = \mu \left[ 1 - \frac{1 - \mu}{\mu} \left( \frac{1}{d} + \frac{1}{d^2} \right)^{-1} \right] + (1 - \mu). \] (12)
Consider now an increase in the adverse selection discount, \(d\). From (8) we see that the threshold \(\tilde{\mu}\) increases as \(d\) increases. Therefore, a bank with reputation level \(\mu\) in the least-cost separating equilibrium stays in that equilibrium and from (11) we see that the expected volume of trade falls for such a bank. A bank with reputation level \(\mu\) sufficiently greater than \(\tilde{\mu}\) stays in the mixed strategy equilibrium and from (12) it is easy to show that the expected volume of trade falls for such a bank. A bank with reputation level near \(\tilde{\mu}\) switches from the mixed strategy equilibrium to the least-cost separating equilibrium. Straightforward computation shows that \(T_L(\mu) \leq T_H(\mu)\). Thus, for such banks, the volume of trade falls as well.

We have shown that if the adverse selection discount increases, either due to a decrease in the collateral value or to an increase in the dispersion in bank quality, the expected volume of trade for a bank of a given reputation, \(\mu\), falls. Since the expected volume of trade for banks of each reputation level falls, it follows that for any distribution of banks by reputation levels, the aggregate volume of trade falls as well. We summarize this discussion in the following proposition.

**Proposition 2 (The Promise of Adverse Selection Models in Generating Trade Volume Fluctuations)** Decreases in collateral values and increases in the dispersion of bank quality reduce expected trade volumes for banks of all reputational levels and decrease aggregate trade volume.

While the static adverse selection model is promising in generating a fall in the expected volume of trade when collateral values fall, note that all of the decline is due to a decline in volume of trade of high-quality banks. Low-quality banks always sell their entire loan portfolios. Declines in the volume of trade in secondary markets seem very widespread, suggesting that the volume of trade across banks of varying qualities is highly correlated. Since the static adverse selection model cannot generate a decline in volume of trade by all quality-types of banks, it faces a correlation challenge when confronted with the data. Next we turn to dynamic models primarily to show that adverse selection can persist and secondarily to show that such models can address the correlation challenge.

## 3 Reputational Concerns in a Two Period Model

We begin our analysis of reputation in financial markets by analyzing a two period model. We show that when reputational concerns are weak, adverse selection cannot persist because the quality of the bank can be perfectly inferred from its past actions. When reputational concerns are strong, we show that adverse selection persists in the sense that the quality of
the bank cannot be perfectly inferred from its past actions. We show that any equilibrium necessarily features partial pooling for low values of bank reputation in the sense that future buyers are able to obtain some but not complete information about the bank’s quality from its past actions. For high values of reputation, equilibria with the highest volume of trade have complete pooling in the sense that future buyers are able to obtain no information about the bank’s quality from its past actions. The incompleteness of information revelation implies that fluctuations in collateral values can induce fluctuations in the aggregate volume of trade in both periods as well as in the volume of trade by banks of different qualities in the first period.

The two period model is a straightforward extension of the static model. As in the static model, in each period of the dynamic model, banks originate a loan portfolio of size normalized to be 1, and then buyers offer contracts intended for high- and low-quality banks. Banks then choose the contract that gives them the highest payoffs. Banks discount future payoffs at rate $\beta$. In order for reputation to play a role, we assume that buyers observe the contract chosen by an individual bank in the previous period. Buyers use this observation to update their beliefs about the quality level of banks.

We make a variety of simplifications which are intended to allow us to focus on the role of reputation and to suppress other links between buyers and banks over time. We assume that the quality types of banks is completely persistent in the sense that it is the same in both periods. (In our infinite horizon model we analyze equilibria with partial persistence.) We assume that loans originated in any period can only be sold in that period and that each bank interacts with a new set of buyers in each period. We also assume that buyers do not observe the returns on loans in previous periods. Our simplifications imply that the only variable that links behavior over time is the reputation level of the bank which is endogenously determined by the bank’s past choices.

A Markov equilibrium for this economy consists of (possibly mixed) strategies by both banks and buyers, updating rules for buyers which satisfy Bayes rule and is defined in the usual fashion.

3.1 The Model without Reputational Concerns

We begin by analyzing a dynamic adverse selection model without reputational concerns. Formally, we suppress reputational concerns by setting banks’ discount factor $\beta$ equal to 0. Note that buyers still update their beliefs about the quality level of an individual bank from past contract choices.

Since the discount factor $\beta$ is zero and since each buyer interacts with an individual
bank only in one period, all decisions in the first period are unaffected by future payoffs. It follows that the equilibrium in the first period coincides with the equilibrium in the static model. Since this equilibrium features complete separation of banks by quality type, it also features complete learning by buyers. In the second period, buyers believe that the bank is either of high- or low-quality depending on its first period contract choice. Hence, the equilibrium in the second period has the bank selling its entire loan portfolio at a price of $\bar{\pi}v$ if buyers believe the bank is of high-quality and at a price of $\pi v$ if buyers believe the bank is of low-quality. The volume of trade in the second period is independent of the collateral value and the dispersion in bank quality. We summarize this discussion in a proposition.

**Proposition 3** Suppose that the discount factor of banks, $\beta$, equals 0. The equilibrium of the dynamic model features full separation and complete learning in the first period. The volume of trade in the second period is independent of the collateral value and the dispersion in bank quality.

By continuity, it follows that for sufficiently small values of the discount factor $\beta$, the equilibrium has full separation. This proposition can be extended trivially to a model with many periods. It illustrates starkly that when reputational concerns are weak, adverse selection cannot persist and raises challenges for theories based on adverse selection in accounting for fluctuations in the volume of trade.

### 3.2 The Model with Reputational Concerns

Here we show that if banks are sufficiently patient, complete separation in the first period cannot be an equilibrium outcome. The proof of this result is by contradiction. Suppose that an equilibrium exists with complete separation. With such separation, buyers in the last period know the quality level of the bank so their beliefs are either 1 or 0. The equilibrium in the last period coincides with the static equilibrium with beliefs of 1 or 0 and the associated payoffs of banks can be calculated from (10).

Incentive compatibility of buyers’ contracts in the first period implies

\[ t_h + (1 - x_h)(\bar{\pi}v - c) + \beta V(1; \bar{\pi}) \geq t_l + (1 - x_l)(\bar{\pi}v - c) + \beta V(0; \bar{\pi}), \]  
\[ t_l + (1 - x_l)(\bar{\pi}v - c) + \beta V(0; \bar{\pi}) \geq t_h + (1 - x_h)(\bar{\pi}v - c) + \beta V(1; \bar{\pi}), \]  

where the continuation value $V(\mu, \pi)$ is given in (10). Adding the constraints (13) and (14) and substituting for the continuation values from (10) gives

\[ x_l - x_h \geq \beta \frac{d}{1 + d}. \]
Clearly, if the right side of (15) is greater than 1, since \( x_l \leq 1 \) and \( x_h \geq 0 \), no incentive compatible contract can achieve full separation in the first period. When the right side of (15) is bigger than 1, we say that banks are sufficiently patient. We have proved the following proposition.

**Proposition 4 (Patience and Persistence of Adverse Selection)** Suppose that banks are sufficiently patient. Then no equilibrium in the two period economy has complete separation of high- and low-quality banks in the first period.

This proposition shows that if banks are sufficiently patient, any equilibrium must have partial or complete pooling by banks. Since any equilibrium must have partial or complete pooling, banks of both quality types must either use mixed strategies or choose the same contract. In what follows, we focus on equilibria in which in the first period buyers use pure strategies. As in the static model, buyers simultaneously offer contracts \( z = (x_h, t_h, x_l, t_l) \) and banks choose which offer if any to accept. If they choose to accept one of the offers, high-quality banks accept the offer \((x_h, t_h)\) with probability \( \alpha_h \) and the offer \((x_l, t_l)\) with complementary probability and low-quality quality banks choose \((x_l, t_l)\) with probability \( \alpha_l \) and \((x_h, t_h)\) with the complementary probability. We will say that \( z \) has complete pooling if \((x_h, t_h) = (x_l, t_l)\) and has partial pooling if \((x_h, t_h) \neq (x_l, t_l)\). Incentive compatibility implies that

\[
\begin{align*}
t_h + (1 - x_h)(\bar{\pi} v - c) + \beta V(\mu'_h; \bar{\pi}) &\geq t_l + (1 - x_l)(\bar{\pi} v - c) + \beta V(\mu'_l; \bar{\pi}), \\
t_l + (1 - x_l)(\bar{\pi} v - c) + \beta V(\mu'_l; \bar{\pi}) &\geq t_h + (1 - x_h)(\bar{\pi} v - c) + \beta V(\mu'_h; \bar{\pi})
\end{align*}
\]

(16) (17)

where \( \mu'_h \) (\( \mu'_l \)) is the belief of future buyers that the bank is of high quality upon observing the accepted offer \((x_h, t_h)\) \((x_l, t_l)\). In addition, when \( \alpha_i < 1, \ i = h, l \), the relevant incentive constraint holds with equality. If the contract has complete pooling, Bayes’ Rule implies that beliefs of future buyers are the same as initial beliefs so that \( \mu'_h = \mu'_l = \mu \). If the contract has partial pooling, Bayes’ Rule implies that

\[
\begin{align*}
\mu'_h &= \frac{\mu \alpha_h}{\mu \alpha_h + (1 - \mu) (1 - \alpha_l)}, \\
\mu'_l &= \frac{\mu (1 - \alpha_h)}{\mu (1 - \alpha_h) + (1 - \mu) \alpha_l}.
\end{align*}
\]

(18)

An *equilibrium* consists of an incentive compatible contract \( z \), mixing probabilities \( \alpha_h \) and \( \alpha_l \), and a belief function \( \mu' (x, t) \) such that (i) banks’ choices about which offer to accept, if any, maximize their payoffs, (ii) buyers maximize expected profits given by

\[
\begin{align*}
\mu \alpha_h \left(x_h \bar{\pi} v - t_h\right) + \mu \left(1 - \alpha_h\right) \left(x_l \bar{\pi} v - t_l\right) \\
+ (1 - \mu) \alpha_l \left(x_l \bar{\pi} v - t_l\right) + (1 - \mu) \left(1 - \alpha_l\right) \left(x_h \bar{\pi} v - t_h\right)
\end{align*}
\]

(19)

and (iii) the belief function satisfies Bayes’ rule when applicable. We focus on equilibria in which buyers make zero profits and hence all the surplus is captured by banks.
If the bank chooses to reject both offers, its payoff is given by \( \pi v - c + \beta V(\mu'; \pi) \). Since the continuation value \( V \) is non-decreasing in \( \mu \), it follows that a necessary condition for an equilibrium is that it satisfy a participation constraint for the high-quality bank given by

\[
t_h + (1 - x_h) (\pi v - c) + \beta V(\mu'_h; \hat{\pi}) \geq \pi v - c + \beta V(0; \bar{\pi}),
\]

and an analogous participation constraint for the low-quality bank. Note that any equilibrium in which banks hold their loans can be represented as an equilibrium in which buyers offer to buy none of the loans at a zero payment. Therefore, we can restrict attention to equilibria in which the bank accepts one of the offers and require that any equilibrium satisfy (20) and its analog for the low-quality bank.

In much of what follows, we focus on equilibria with the highest volume of trade referred to as Maximal Trade Equilibria. These equilibria are attractive for three reasons: First, in our analysis of optimal policy below we show that such equilibria Pareto dominate any other equilibrium. Second, the maximal trade outcomes are closest to the maximal trade outcomes under full information. Third, as we show below, such equilibria also maximize first period ex ante expected payoffs of banks.

Given our focus on maximal trade outcomes, we ask when complete pooling with full trade, \( x_h = x_l = 1 \), and associated payments \( t_h = t_l = \hat{p}(\mu) \) where \( \hat{p}(\mu) \) is given by (7) is an equilibrium outcome. We show that full trade is an equilibrium outcome if \( \mu \geq \mu^* \) where

\[
\mu^* = 1 - \frac{c}{(\bar{\pi} - \bar{\pi}) v} = 1 - \frac{1}{d}.
\]

The threshold \( \mu^* \) is set so that the high-quality bank is just indifferent between accepting the full trade contract and holding its loans, so that (20) is satisfied with equality. To determine this threshold, note first that with complete pooling, \( \mu' = \mu \). Since \( \mu^* < \hat{\mu} \), the continuation values are equal so that \( V(\mu'; \hat{\pi}) = V(0; \bar{\pi}) \). Evaluating (20) with equality at the full trade contract using (7) gives us (21). If \( \mu \geq \mu^* \), the high-quality bank strictly prefers to sell all of its loans at the pooling price to holding and and if \( \mu < \mu^* \), it prefers to hold rather than to sell. Thus, full trade is potentially an equilibrium outcome for \( \mu \geq \mu^* \) and cannot be an equilibrium for \( \mu < \mu^* \). In what follows, we assume that \( \mu^* \geq 0 \) so that \( d \geq 1 \).

Consider supporting full trade as an equilibrium by the following out of equilibrium belief function

\[
\mu'(x', t') = \begin{cases} 
0 & \text{if } t' + (1 - x') (\pi v - c) \leq \hat{p}(\mu) \\
1 & \text{otherwise}
\end{cases}.
\]

Note that this belief function says that future buyers believe that the bank is low-quality
if and only if it accepts an offer which is statically less favorable for the high-quality bank than the pooling contract. In the Appendix, we show that these strategies and beliefs are indeed equilibria. For some intuition for why full trade is an equilibrium, note that as in the static model, buyers would like to offer cream-skimming contracts that are attractive to high quality banks and unattractive to low quality bank. With our beliefs, attracting high quality banks requires offering them a price higher than \( \hat{p}(\mu) \) in return for some retention, \( x_h < 1 \). The key step in our proof is to show that if the right side of (15) is greater than 1, low quality banks are also attracted to such a deviation. Therefore, buyers cannot make positive profits with this deviation. We summarize this result in the following lemma:

**Lemma 1** Suppose that \( \mu \geq \mu^* \) and that banks are sufficiently patient. Then full trade by banks of both quality types is an equilibrium outcome.

In the Appendix, we prove a more general version of lemma 1 in which we characterize the entire set of complete pooling equilibria with \( x_l = x_h \), possibly less than 1.

Recall that under full information it is efficient to have banks of all qualities to sell their entire portfolios, so that, for high values of reputation, these outcomes coincide in the first period with full information outcomes.

Consider, next the case when banks are sufficiently patient and \( \mu < \mu^* \). We have already shown that full trade cannot be an equilibrium outcome. Since banks are sufficiently patient, complete separation cannot be an equilibrium outcome. Next, we argue that banks of both quality types cannot mix in any equilibrium. If they both did, then any contract together with mixing probabilities must satisfy (16)–(17) with equality as well as the zero profit condition. Straightforward but extensive algebraic calculations available on request show that any such contract violates (20)\(^6\). Thus the equilibrium must have partial pooling with \( 0 < \alpha_l < 1 \).

Consider then the problem of maximizing trade volume given by

\[
T = [\mu + (1 - \mu) (1 - \alpha_l)] x_h + (1 - \mu) \alpha_l x_l
\]

subject to the requirement that the contract be an equilibrium. We have shown that incentive compatibility (16)-(17), zero profits, namely that (19) is zero and (20) are necessary conditions that any equilibrium must satisfy. It turns out that Bertrand competition between the buyers requires us to impose an additional necessary condition that

\[
\frac{1}{2} \mu (x_h \bar{\pi} v - t_h) + (1 - \mu) (\bar{\pi} v - t_l - (1 - x_l) (\bar{\pi} v - c)) \leq 0.
\]

\(^6\)It is immediate that if banks are sufficiently patient, no equilibrium can have only the high quality bank mixing. The argument is identical to that of proposition 4.
To see why this condition is necessary, consider a deviation by one of the buyers to a contract which has the same offer for high quality banks and an offer to the low quality bank which makes that bank slightly better off and delivers the highest profits to the buyer. Specifically, let the deviation offer be given by \( z' = (x_h, t_h, 1, t'_l) \) where \( t'_l = t_l + (1 - x_l)(\bar{\pi}v - c) + \varepsilon \) with a small enough value of \( \varepsilon \). In such a deviation, the low quality bank chooses \((1, t'_l)\) with probability 1, independent of the beliefs assigned to \((1, t'_l)\). To see why this choice is optimal for the low quality bank, note first that the low quality bank is statically better off. Since the value function is weakly increasing and \( \mu'_l = 0 \), the continuation values following \((1, t'_l)\) cannot be lower than \( V(0; \bar{\pi}) \) so that the low quality bank is weakly better off dynamically. The high quality bank is indifferent between the buyers’ contracts and chooses the two buyers with equal probability. Hence, the profits for the buyer from such deviation converge to the left side of (23) as \( \varepsilon \) converges to 0. The inequality in (23) guarantees that such a deviation is not profitable.

A candidate for the maximal trade equilibrium then is a contract that maximizes (22) subject to (16)-(17), the expression in (19) is non-negative, (20) and (23). In the Appendix, we show that this candidate outcome is indeed an equilibrium by showing that (16), (17), zero profits, (20), and (23) are not just necessary but also sufficient conditions that any equilibrium must satisfy. Hence, we have the following lemma:

**Lemma 2** Suppose that \( \mu < \mu^* \) and that banks are sufficiently patient. Then the maximal trade outcome maximizes (22) subject to incentive compatibility (16)-(17), zero profits, (23) and (20).

Notice that this characterization of equilibrium allocations does not depend on any specification of beliefs off the equilibrium path.

Next we show that the maximal trade equilibrium maximizes first period ex-ante payoffs given by

\[
\mu[t_h + (1 - x_h)(\bar{\pi}v - c)] + (1 - \mu)[\alpha_l(t_l + (1 - x_l)(\bar{\pi}v - c) + (1 - \alpha_l)(t_l + (1 - x_h)(\bar{\pi}v - c))].
\]  

Substituting for \( t_h \) and \( t_l \) from the zero profits constraint, (24) reduces to \( \hat{p}(\mu) - c(1 - T) \). Clearly maximizing \( T \) is equivalent to maximizing (24). By the same type of reasoning, if \( \mu \geq \mu^* \), the maximal trade equilibrium maximizes first-period ex-ante expected payoffs.

Let \( \bar{z} = (\bar{x}_h, \bar{t}_h, \bar{x}_l, \bar{t}_l) \) and \( \bar{\alpha}_l \) denote the maximal trade allocation. Here we provide a characterization of this allocation. Clearly, it must have \( \bar{x}_l = 1 \) because it is feasible to increase volume by increasing \( x_l \) by \( \varepsilon \) and \( t_l \) by \( (\bar{\pi}v - c)\varepsilon \). Next, we show that the constraints to the problem of maximizing trade volume can be reduced to two inequalities. To obtain these inequalities, substitute for \( t_h \) and \( t_l \) from the incentive constraint (17) and the zero
profit condition into (20) and (23), to obtain
\[
x_h \leq \bar{x}(\mu'_h, v) = \frac{\beta \left[ \Delta(\mu'_h; \bar{\pi}) - \Delta(\mu'_h; \bar{\pi}) \left(1 - \frac{\mu}{\mu'_h}\right)\right] + \left(1 - \frac{\mu}{\mu'_h}\right)c}{\bar{\pi}v - c - \hat{p}(\mu) + \left(1 - \frac{\mu}{\mu'_h}\right)c}
\] (25)
where \(\Delta(\mu; \pi) = V(\mu; \pi) - V(0; \pi)\). (The algebra underlying these substitutions can be found in the Appendix). The maximal trade outcome maximizes (22) subject to (25) and (26). Since the volume of trade is increasing in \(x_h\) and since (25) imposes an upper bound on \(x_h\), it follows that in the maximal trade equilibrium (25) must be binding. Furthermore, in the proof of the following lemma in the Appendix, we show that if (26) is not binding, then \(\mu'_h = \bar{\mu}\). We summarize this characterization in the following lemma:

**Lemma 3** Suppose that \(\mu \leq \mu^*\) and that banks are sufficiently patient. Then, in the maximal trade equilibrium, \(x_1 = 1\), (25) is binding. In addition, \(\mu'_h = \bar{\mu}\) if (26) is not binding and \(\mu'_h \geq \bar{\mu}\) otherwise.

In summary, lemmas 1, 2, and 3 show that the maximal trade equilibrium features pooling in the sense that buyers in the second period remain uncertain about the quality levels of individual banks. Thus, adverse selection persists in our model.

### 3.2.1 Volume of Trade and the Spread

Here, we show that in the maximal trade equilibrium a temporary unexpected aggregate increase in the spread results in a reduction in the aggregate trading volume. It is straightforward to adapt the proposition for the two period model to allow for the adverse selection discount to be different in the two periods.

Consider the effect on volume associated with such an increase for a bank with a high reputation level so that \(\mu \geq \mu^*\). From (21), it follows that \(\mu^*\) increases. If the increase in \(v\) leaves the bank in the full trade equilibrium, the volume of trade is unaffected. If the increase raises the threshold \(\mu^*\) above the bank’s reputation level, the equilibrium switches from full trade to partial pooling so that the volume of trade falls. Thus, all banks with reputation levels in a neighborhood above \(\mu^*\), experience a decline in the volume of trade.

Consider, next, the effect on volume associated with an increase in the spread for a bank with reputation level \(\mu \leq \mu^*\). Since \(\mu^*\) increases, such a bank continues to remain in the partial pooling equilibrium. Recall that in the maximal trade equilibrium, \(\bar{x}_1 = 1\) and (25) is binding. Moreover, because the shock to the spread is temporary, it has no effect on
continuation values. Therefore, an increase in \( v \) reduces the right side of (25) for given \( \mu'_h \). We have shown that when (26) is not binding, \( \mu'_h = \bar{\mu} \) so that in this case, the volume of trade decreases. In the Appendix, we show that when (26) is also binding, the volume of trade decreases. We have established the following proposition for the maximal trade equilibrium:

**Proposition 5** (Fluctuations in Trade Volume with Reputational Concerns) If banks are sufficiently patient, a temporary decrease in collateral values weakly reduces expected trade volumes for banks of all reputational levels. Furthermore, if the distribution of reputation has mass below \( \mu^* \) or in a neighborhood above \( \mu^* \), aggregate trade volume strictly falls in the first period.

Recall from proposition 2, that an unanticipated shock to collateral values in the last period reduces volume in that period. These propositions taken together illustrate that if reputational concerns are sufficiently strong, adverse selection can persist and temporary fluctuations in collateral values can induce fluctuations in the volume of trade in both periods. Note also that the volume of trade declines for both high- and low-quality banks so that reputation can help address the correlation challenge.

Notice that the changes in aggregate volume associated with changes in the spread depend on the nature of the initial distribution of reputation. In our infinite horizon model, we endogenize this distribution and show that aggregate volume falls in response to increases in the spread. In that model, we also analyze the role of anticipations of future shocks to loan spreads on the current volume of trade.

3.2.2 Multiplicity of Equilibria

While the main focus of our analysis is maximal trade equilibria, for the sake of completeness we describe other equilibria. Since our model has multiple equilibria, the volume of trade may fluctuate even when fundamentals are unchanged.

Consider the set of equilibria for values of \( \mu \leq \mu^* \). Recall that incentive compatibility (16)-(17), zero profits, (23) and (20) are not just necessary but also sufficient conditions for an equilibrium. Thus, any contract that satisfies these conditions can be supported as an equilibrium. Clearly, a range of values of \( x_l \) and \( x_h \) can be supported as equilibria. These other equilibria have lower volumes of trade.

Consider next equilibria for \( \mu \geq \mu^* \). Recall that our model has a range of complete pooling equilibria with lower volumes of trade than the maximal trade equilibrium. In our proof of lemma 1, we show that the range of complete pooling equilibria is characterize by a lower bound \( \underline{x}(\mu) \) and that any value of \( x \in [\underline{x}(\mu), 1] \) can be sustained as a complete pooling
equilibrium. In that proof, we also show that $x(\mu)$ equals 0 for sufficiently high values of $\mu$. This result illustrates that the model is consistent with complete collapses in the volume of trade that are independent of fundamentals, at least for high values of reputation.

Clearly, sunspot like fluctuations can induce a switch from one equilibrium to another and thereby induce fluctuations in the volume of trade that are unrelated to fundamentals. In Chari et al. (2010), we use global game techniques to refine the multiplicity of equilibria in a similar environment. In the resulting unique equilibrium, there is a critical threshold in the collateral value above which aggregate trading volume is high and below which aggregate trading volume is low.

Next we ask whether policy interventions can ensure uniqueness of equilibrium and also whether policy interventions are needed to address possible inefficiency of equilibria.

4 Implications for Policy

In the wake of the 2007 collapse of secondary loan markets, policy makers proposed a variety of programs intended to restore the volume of trade and to remedy perceived inefficiencies in the market for secondary loans. Under some of these policies, labeled conventional asset purchase policies, the government planned to purchase asset-backed securities at prices roughly equal to its perception of the value.

One possible motivation for intervention is that policy makers perceived equilibrium outcomes in the secondary loan market as inefficient. We address this motivation by analyzing the efficiency of equilibria in our model. We show that when the adverse selection discount is low, the maximal trade equilibrium is efficient for all reputation levels of banks and that the equilibrium outcomes are inefficient only when the adverse selection discount is high and reputation levels are low. We show that when equilibrium outcomes are inefficient, unconventional policies that limit private trade can support efficient outcomes.

A second possible motivation for such policies is that policy makers desired to restore the volume of trade because they perceived the collapse as arising from a switch by private agents from one equilibrium to another. Given that our model has multiplicity of equilibria, we ask whether conventional asset purchase policies are effective. We show that policies in which the government attempts to purchase securities at prices that prevailed before the market collapsed do not by themselves eliminate multiplicity and therefore do not induce increased volume of trade. In this sense, conventional asset purchase policies are ineffective when reputational concerns are strong. We show that unconventional policies can eliminate multiplicity.
4.1 Optimal Policy

Here, we analyze the efficiency properties of our equilibria. Consider the problem of a planner who is able to control only allocations in the first period and takes as given the continuation payoff functions as well as the way future buyers infer reputation levels. We assume that the planner maximizes ex-ante expected payoffs of banks (including the discounted continuation values). To derive the objective of the planner, let \( \bar{W}(z, \alpha, \bar{\pi}) \) denote the left side of (16), \( W(z, \alpha, \bar{\pi}) \) denote the right side of (16), \( \bar{W}(z, \alpha, \bar{\pi}) \) the left side of (17) and \( W(z, \alpha, \bar{\pi}) \) the right side of (17) where future beliefs are given from the mixing probabilities \( \alpha = (\alpha_h, \alpha_l) \) from (18). The planner’s objective is to maximize

\[
\mu[\alpha_h \bar{W}(z, \alpha, \bar{\pi}) + (1 - \alpha_h)W(z, \alpha, \bar{\pi})] + (1 - \mu)[\alpha_l \bar{W}(z, \alpha, \bar{\pi}) + (1 - \alpha_l)W(z, \alpha, \bar{\pi})] 
\]

(27)

by choosing the first period contract, \( z \), and mixing probabilities \( (\alpha_h, \alpha_l) \). The idea that the planner can only control first period allocations respects the restriction in our model that buyers can only offer one period contracts. A more general analysis which we leave for future work would allow for limited commitment by the planner.

Formally, a first period allocation \( z = (t_h, x_h, t_l, x_l) \) and mixing probabilities \( (\alpha_h, \alpha_l) \) is ex-ante efficient if it maximizes (27) subject to (i), incentive compatibility, (16) and (17), the Bayes’ rule requirement (18), (ii) the participation constraint for the high quality bank in (20) and its analogue for low quality banks, and (iii) the break-even constraint for buyers, or feasibility, which implies that the expression in (19) be non-negative.

We now characterize the set of ex-ante efficient allocations. As we have shown above, the maximal trade equilibrium maximizes ex-ante first period payoffs. Ex-ante efficiency includes continuation payoffs as well. Thus, ex-ante efficiency yields different outcomes only if the static loss from deviating from the maximal trade equilibrium is outweighed by a dynamic gain.

Consider \( \mu \geq \mu^* \). Here, we prove that the maximal trade equilibrium allocation is ex-ante efficient and provide some intuition for the result. We show that the static loss of a small deviation from this allocation is of an order of magnitude greater than the dynamic gain from doing so. Consider a small perturbation of the maximal trade outcome in which the mixing probabilities are given by \( \alpha_h = 1/2 + \varepsilon \) and \( \alpha_l = 1/2 \) together with a perturbed contract \( (x_h', t_h', 1, t_l') \) which is incentive compatible and yields non-negative profits. From Bayes’ rule (18), this perturbation raises \( \mu_h' \) and lowers \( \mu_l' \) by approximately \( 2\mu (1 - \mu) \varepsilon \) for

\( ^7 \) The restriction to allocations which consists of offers to two types of agents in an environment with limited commitment is without loss of generality. See Bester and Strausz (2001).

\( ^8 \) Analogously to the analysis of equilibrium, it is possible to prove that in any ex-ante efficient allocation, it is optimal to set \( \alpha_h = 1 \) so that high-quality banks do not mix.
small values of $\varepsilon > 0$. Consider the increase in the continuation value for the high-quality bank given by

$$\alpha_h V (\mu_h'; \bar{\pi}) + (1 - \alpha_h) V (\mu_l'; \bar{\pi}) - V (\mu; \bar{\pi}) = \Delta \varepsilon$$

Taking a Taylor series expansion of this expression shows that $\Delta \varepsilon$ is of order of $\varepsilon^2$. A similar argument applies for the low quality banks. Notice that this perturbation induces a spread in continuation values, $V (\mu_h'; \pi) - V (\mu_l'; \pi)$. To ensure that the incentive constraints (17) and (16) are satisfied, this increase in spread in continuation values implies that $x_h$ must fall by an amount proportional to $\varepsilon$. Therefore first period ex-ante expected payoffs also falls by an amount proportional to $\varepsilon$. Since continuation payoffs rise by a term of the order of $\varepsilon^2$ and first period payoffs fall by a term in the order of $\varepsilon$, it follows that such a perturbation reduces ex-ante payoffs. In the Appendix, we show that this argument generalizes to any perturbation of the maximal trade equilibrium. We then have the following proposition.

**Proposition 6** Suppose that $\mu \geq \mu^*$. The maximal trade equilibrium is ex-ante efficient.

Next, consider ex-ante efficient allocations when $\mu \leq \mu^*$. Consider a partial pooling allocation in which $\alpha_h = 1$ and $\alpha_l$ is such that $\mu_h' = \bar{\mu}$. This allocation has the feature that the spread in continuation values $\Delta (\mu_h'; \pi) = V (\mu_h'; \pi) - V (0; \pi)$ is zero. As the discussion of the proceeding proposition shows, increasing the spread in continuation values reduces the volume of trade in the first period. In the Appendix, we show that using the strong submodularity property of the value function $V (\mu; \pi)$, the static losses from reducing volume in the first period outweigh the dynamic gains. Hence, any ex-ante efficient partial pooling allocation must have no spread in continuation values, so that $\mu_h' = \bar{\mu}$.

Hence, maximal trade equilibria are efficient if they have the feature that $\mu_h' = \bar{\mu}$ and inefficient otherwise. Recall from lemma 3 that if the (26) is not binding, $\mu_h' = \bar{\mu}$. In the Appendix, we show that if $d \leq 2$, the efficient allocation satisfies (23) and it violates (23) only if $d > 2$ and $\mu$ is sufficiently below $\mu^*$.

In order to get some intuition for this result, notice that holding the contract fixed, an increase in the spread $v$, raises the right side of (20) by more than the left side. Thus, an increase in the spread creates a force that makes a reduction in $x_h$ necessary. A fall in $x_h$ reduces profits generated from the high quality bank and therefore reduces the subsidy available to low quality banks at the offer $(1, t_l)$. This reduction in the subsidy is particularly large when $\mu$ is low. These considerations suggests that when $v$ is large and $\mu$ is low, the payment $t_l$ tends to be low. This low payment creates strong incentives for one of the buyers to deviate to a contract which induces the low quality bank to accept an offer close to $(1, t_l)$ with probability 1 and thereby not to accept the offer $(x_h, t_h)$. When these incentives are
sufficiently strong, that is when (23) is violated, the ex-ante efficient allocation cannot be supported as an equilibrium.

Next, we compare volume of trade in the efficient allocations and the equilibrium outcomes. It is possible to show that an increase in $\mu'_{h}$, reduces the right side of (25). Since this constraint is binding in the maximal trade equilibrium, such a reduction reduces $x_{h}$ and therefore, trade volume. Thus, when the equilibrium allocation is inefficient so that $\mu'_{h} > \tilde{\mu}$, the volume of trade is lower than in the efficient allocation. Note, also, that when $\mu'_{h} > \tilde{\mu}$, the equilibrium reveals inefficiently too much information. We summarize these findings in the following proposition:

**Proposition 7** Suppose that $\mu \leq \mu^{*}$ and that banks are sufficiently patient. If $d \leq 2$, the maximal trade equilibrium is ex-ante efficient. If $d > 2$, there exists some $\hat{\mu} < \mu^{*}$ such that for $\mu \geq \hat{\mu}$ the maximal trade equilibrium is efficient and for $\mu < \hat{\mu}$ the maximal trade equilibrium has inefficiently low levels of volume.

This proposition shows intervention is desirable only when the adverse selection discount is high and that this intervention should be targeted to banks with low levels of reputation.

### 4.1.1 Efficient Interventions

Here, we discuss policies that can implement ex-ante efficient allocations. We show how ex-ante efficient allocations can be weakly implemented in the sense that one equilibrium of the resulting game is ex-ante efficient.

Propositions 6 and 7 show that we need only focus on cases where $\mu \leq \hat{\mu}$ and $d \geq 2$. We have argued that in this case, the efficient allocation violates the Bertrand competition constraint. In particular, if one of the buyers, say buyer 2, offers the efficient contract, buyer 1 would find it optimal to deviate it to a contract $(x_{h}, t_{h}, x_{l}, t_{l} + \varepsilon)$. Such a contract attracts the low-quality bank to the offer $(x_{l}, t_{l} + \varepsilon)$ with probability 1 and attracts high-quality banks with probability $1/2$. The resulting payoffs to buyer 1 are given by the left side of (23), and, since that constraint is violated in the efficient allocation, this deviation is profitable.

Implementing the efficient allocations requires designing a policy, say a tax policy, which makes such deviations unprofitable. To design such a policy consider the efficient allocation and consider the following set of potentially profitable deviation offers by buyers:

$$A = \{(x, t) : t + (1 - x) (\pi v - c) > t_{l} + (1 - x_{l}) (\pi v - c),$$

$$(1 - \mu) (x \pi v - t) + \frac{1}{2} \mu (x_{h} \pi v - t_{h}) \geq 0\}$$

24
The set $A$ is set of offers which attract the low-quality bank with probability one and make positive profits against the efficient allocation. Since efficient allocation violates (23), $A$ is non-empty. Consider a tax policy with a tax on buyers $\tau_l$ chosen so that

$$(1 - \mu) (x_l \pi v - t_l - \tau_l) + \frac{1}{2} \mu (x_h \pi v - t_h) = 0.$$  

(28)

Apply the tax defined by (28) to any offer in the set $A$. Clearly, if one of the buyers offers the efficient allocation, the other buyer does not have an incentive to make an offer that satisfies the Bertrand competition constraint. We have proved the following proposition.

**Proposition 8** Suppose that $\mu \leq \hat{\mu}$ and $d \geq 2$. Then the ex-ante efficient allocation is an equilibrium outcome given the tax policy defined by (28).

### 4.2 Equilibrium Multiplicity and Asset Purchase Policies

Here, we analyze asset purchase policies which can eliminate multiplicity of equilibria discussed in section 3.2.2. We show that conventional asset purchase policies do not eliminate multiplicity of equilibria in our model but unconventional asset purchase policies can do so.

To set the stage for the discussion of conventional asset purchase policies in our model, we begin by describing an environment in which such policies do eliminate multiplicity of equilibria. Consider a non-strategic version of our static model. In this version, banks and buyers have the same payoffs, but are price takers. In this environment, when $\mu$ is sufficiently high, it is straightforward to show that the model has two equilibria. In one equilibrium, the price is $\hat{p}(\mu)$ and both types of banks sell their entire loan portfolios and in the other the price is $\pi v$ and only the low-quality bank sells its portfolio. This model can clearly generate sudden collapses in trade volume associated with a switch from the high to the low trade equilibrium. Also note that the low trade equilibrium is inefficient in the sense that the high trade equilibrium Pareto dominates the low trade equilibrium. In this environment, if the government commits to a policy of purchasing asset at $\hat{p}(\mu)$, it can eliminate the low price/volume equilibrium and the government does not actually have to buy any loans. This policy resembles that of deposit insurance Diamond and Dybvig (1983) model and is desirable since it is Pareto improving.

We find this way of modeling sudden collapses in trade unattractive for two reasons. First, buyers have strong incentives to offer nonlinear contracts intended to separate high-quality banks from low-quality banks. This nonlinearity is a pervasive feature of models with adverse selection. For example, in this model, a buyer who makes an offer to purchase a small quantity of loans for a price per loan close to $\bar{\pi} v$ will attract only high-quality banks. Second, even if we restricted ourselves to linear contracts, we find the low volume equilibrium
unappealing. The reason is in such an environment, each buyer has a strong incentive to offer to buy the entire loan portfolio at a price per loan slightly less than $\hat{p}(\mu)$ and attract both types of banks.\footnote{It has been suggested to us that models with linear contracts and capacity constraints could possibly generate multiple equilibria. We think that this conjecture is unlikely to be true based on the results of Guerrieri et al. (2010) who develop an adverse selection model with capacity constraints and matching frictions and obtain a unique equilibrium.} Such strategic interaction would eliminate the low volume equilibrium.

Notwithstanding these critiques, we are sympathetic to the idea that asset purchase policies intended to eliminate multiplicity of equilibria could possibly be effective. As we have shown in section 3.2, if $\mu \geq \mu^*$, our model has a range of complete pooling equilibria with varying values of volume. We ask whether asset purchase policies can eliminate multiplicity of equilibria in our two period model. Consider an asset purchase policy in which the government offers a contract $(1, \hat{p}(\mu))$ for any bank type in the first period and does not intervene in the market in the second period. We will show that such a policy does not eliminate pooling equilibria with $x < 1$.

To show this result, consider a low trade pooling equilibrium with $x < 1$. Recall that in the equilibrium without the government, it was feasible for one of the buyers to make the offer $(1, \hat{p}(\mu))$, but such a deviation was not profitable. The reason is that under our equilibrium beliefs, a bank which accepts such an offer ends up with a future reputation of 0. With such future reputation level, high quality banks are unwilling to accept the offer, because the static gains are overweighed by the dynamic losses. In the proof of the following proposition in the Appendix, we show that the offer $(1, \hat{p}(\mu))$ does not attract low quality banks either. We have the following Proposition.

**Proposition 9** Suppose that $\mu \geq \mu^*$ and the government offers to purchase loans at price $\hat{p}(\mu)$ in the first period. Then the two period game has an equilibrium where neither bank type sells to the government and both types sell a fraction of their portfolio, $x$, at price $\hat{p}(\mu)$.

This proposition shows that when reputational concerns are sufficiently strong, conventional asset purchase policies do not restore the volume of trade if equilibrium selection is unaffected by the policy. Clearly, a policy of purchasing all assets at a sufficiently high price will restore the volume of trade. But such a policy necessarily requires using tax revenues from other sources.

Multiplicity of equilibria can be eliminated by more interventionist policies. One example of such a policy prohibits private trade at any contract other than the maximal trade equilibrium. This policy clearly ensures that the maximal trade equilibrium is implemented. Another example is a tax policy similar to that discussed above. In this sense, unconventional policies which limit private trade can eliminate multiplicity of equilibria.
5 The Infinite Horizon Model with Reputational Concerns

We now turn to a stochastic infinite horizon model with reputational concerns. In this model, we show that the main results from our two-period model are robust in the infinite horizon. In addition, the infinite horizon model captures the effect of anticipated stochastic fluctuations in collateral values on aggregate trading volume and allows us to endogenize the initial distribution of reputations. We show that the model is capable of producing a slow build up of volume in secondary loan markets followed by abrupt collapses.

Our stochastic infinite horizon model is a straightforward extension of the two period model studied above. We introduce stochastic fluctuations in collateral values that are perfectly correlated across banks. We assume that the spread $v_t$ is independently distributed over time and that it is drawn from an identical distribution $G(v_t)$ with support $[v_{min}, v_{max}]$ where the lower bound $v_l$ is given by $c/(\bar{\pi} - \pi)$. At this lower bound, there is no adverse selection problem in the sense that for all reputation levels, the high-quality bank is willing to sell its entire loan portfolio at the pooling price $\hat{p}(\mu)$. Let $Ev$ denote the expected value of the spread $v$. For simplicity, we assume that the cost of holding loans, $c$, and the dispersion in bank quality $\bar{\pi} - \pi$ are constant, so that fluctuations in collateral values are the only shocks which induce fluctuations in the adverse selection discount.

We allow the quality of the banks to change according to a Markov process. In particular, at the end of each period, with probability $\lambda$, each bank draws a new quality level. These draws are independent across banks and buyers can observe whether a bank has drawn a new quality level. If the bank draws a new quality level, the bank is of high quality with probability $\mu$ where $\mu$ is distributed according to the distribution function $\Psi(\mu)$ and $\Psi(\mu)$ is continuous with support on $[0, 1]$. Allowing the quality levels of banks to change ensures that the model has a unique invariant distribution of banks’ reputations.

To simplify the analysis, we restrict banks to either sell or hold their entire loan portfolios so that $x = 0$ or $x = 1$. In the Appendix, we show that the equilibrium features that we characterize are robust to relaxing this restriction.

Next, we turn to characterizing a stationary Markov equilibrium in which equilibrium outcomes in any period depend only on the beliefs of buyers about the bank’s quality and the spread $v_t$. In particular, these outcomes do not depend on calendar time. As in the two period model, if banks are sufficiently patient, a separating equilibrium does not exist in the infinite horizon model. We will say that banks are sufficiently patient in the infinite horizon
The following proposition is the analogue of Proposition 4. (The proof is identical to the two period model and is available upon request):

**Proposition 10** (Patience and Persistence of Adverse Selection) If banks are sufficiently patient in the infinite horizon economy, then no equilibrium has complete separation of high- and low-quality banks in any period.

Consider now an equilibrium for the infinite horizon model when banks are sufficiently patient. We show that the infinite horizon model has an equilibrium that is similar to the first period equilibrium of the two-period model. Figure 1 displays the nature of the equilibrium in the infinite horizon model. As in the two period model, when reputation levels are low, the equilibrium has partial pooling in the sense that high-quality banks use pure strategies and low-quality banks use mixed strategies. In the infinite horizon model, partial pooling is an equilibrium if current reputation $\mu_t$ is below a threshold function $\mu^*(v)$ and below a threshold level $\mu_h$. The threshold function $\mu^*(v)$ is chosen so that when $\mu_t = \mu^*(v)$ high-quality banks are statically indifferent between holding holding their loans and receiving a payoff of $\bar{\pi}v - c$ and selling them at the pooling price and receiving a payoff of $\hat{p}(\mu)$. Equating these payoffs

$$\frac{\beta(1 - \lambda)}{1 - \beta(1 - \lambda)} \geq \frac{(\bar{\pi} - \pi)v_{max} + c}{(\pi - \bar{\pi})Ev}.$$  

This inequality ensures that partial pooling is an equilibrium for high-reputation levels.
yields
\[\mu^*(v) = 1 - \frac{c}{(\bar{\pi} - \pi)v}.\]

If \(\mu_t > \mu^*(v)\), banks of both quality types statically strictly prefer to sell their loans rather than hold them. If \(\mu_t < \mu^*(v)\), banks of both quality types statically strictly prefer to hold their loans rather than sell them. Note that the indifference condition used to define \(\mu^*(v)\) is the same as the the indifference condition that defines the threshold \(\mu^*\) in the two period model.

The threshold level \(\mu_h\) is chosen so that the low-quality bank is indifferent between selling their loans for a payment of \(\bar{\pi}v_t\) and receiving a future reputation of 0 and holding their loans and receiving a future reputation of \(\mu_h\). This indifference condition requires that \(\mu_h\) satisfies
\[\bar{\pi}v_t + \beta(1 - \lambda)V(0; \bar{\pi}) + \beta \lambda W = \bar{\pi}v_t - c + \beta(1 - \lambda)V(\mu_h; \bar{\pi}) + \beta \lambda W\]
where \(W\), the continuation value in the event of replacement is given by
\[W = \int_0^1 [\mu V(\mu; \bar{\pi}) + (1 - \mu) V(\mu; \bar{\pi})] d\Psi(\mu).\]

Notice that \(\mu_h\) is independent of the spread \(v_t\). The analogue of \(\mu_h\) in the two period model is an equilibrium in which \(\mu'_h\) is sufficiently greater than \(\bar{\mu}\) and one in which \(x_h = 0\). One difference between the infinite horizon and the two period model is that in the infinite horizon model, the value function \(V(\mu; \pi)\), is increasing in reputation levels \(\mu\) even for relatively low values of \(\mu\) while in the two period model the value function is constant for low values of reputation. The value function is increasing in the infinite horizon model because of the possibility that low future values of the spread \(v\) might make it possible for low- and high-quality banks to sell at a favorable pooling price.

When reputation levels are high, the equilibrium features complete pooling. Depending on the reputation levels, this complete pooling equilibrium has both high- and low-quality banks both selling or both holding. Note that the feature that the equilibrium can have both types of banks holding their loans is unlike that in the two-period model. This feature arises because in our stochastic model, low quality banks have incentives to hold onto their loans in the hope of being able to sell their loans in the future at high prices.

The equilibrium strategy of the high-quality banks is to sell their loans if \(\mu_t > \mu^*(v)\) and to hold their loans if \(\mu_t \leq \mu^*(v)\). The equilibrium strategy of the low-quality banks is, in a sense, to mimic the behavior of high-quality banks. These banks sell their loans if \(\mu_t > \mu^*(v)\), hold their loans if \(\mu_t \leq \mu^*(v)\) and \(\mu_t \geq \mu_h\), and mix between selling their loans and holding them if \(\mu_t \leq \mu^*(v)\) and \(\mu_t > \mu_h\). The mixing probabilities \(\alpha_l(\mu_t, v_t)\) are chosen
so that the reputation level in the next period is \( \mu_h \), so that
\[
\mu_h = \frac{\mu_t}{\mu_t + (1 - \mu_t)(1 - \alpha_l(\mu_t, v_t))}.
\] (31)

These strategies induce continuation payoffs \( V(\mu, \pi) \) in the obvious fashion. Clearly these continuation payoffs are increasing in reputation levels \( \mu \).

Next, consider the beliefs that make these strategies optimal for banks. When \( \mu_t \geq \mu^*(v_t) \), the equilibrium strategies are for banks of both quality types to sell, so that \( \mu_{t+1} = \mu_t \). If a bank deviates to holding its loan, we assume that future buyers assign the same reputation level as under the equilibrium, so that \( \mu_{t+1} = \mu_t \). These beliefs ensure that when banks of both quality types care only about static incentives and therefore prefer to sell their loan portfolios. When \( \mu_t \leq \mu_h \) and \( \mu_t < \mu^*(v_t) \), that is, when the equilibrium falls in the partial pooling region, in equilibrium future buyers assign a reputation of \( \mu_{t+1} = \mu_h \) to a bank which holds its loan portfolio and a reputation of 0 to a bank which sells its loan portfolio. Consider finally reputation levels such that \( \mu_h < \mu_t < \mu^*(v_t) \). For such reputation levels, the equilibrium prescribes that both types of banks hold their entire loan portfolios and future beliefs are given by \( \mu_{t+1} = \mu_t \). We assume that if a bank deviates and sells its loan portfolio, future buyers assign the bank a reputation \( \mu_{t+1} = 0 \). With these out of equilibrium beliefs, high-quality banks statically and dynamically prefer to hold their loans rather than sell them. Therefore low-quality banks can only receive a price of \( \pi v_t \) for selling their loans. Thus, holding loans is preferred to selling them for low-quality banks if
\[
\pi v_t - c + \beta(1 - \lambda)V(\mu, \pi) + \beta\lambda W \geq \pi v_t + \beta(1 - \lambda)V(0; \pi) + \beta\lambda W.
\] (32)

Since continuation payoffs are increasing in \( \mu \), (30) implies (32) for \( \mu \leq \mu_h \).

We have shown that bank strategies are optimal given the equilibrium offers by buyers. In the Appendix, we construct out of equilibrium beliefs in essentially the same manner as in our two-period model to show that there are no profitable deviations by buyers. We summarize this discussion in the following proposition.

**Proposition 11** Suppose that banks are sufficiently patient in the infinite horizon model. The model has a stationary Markov equilibrium with at best partial separation. For a given value of the loan spread \( v_t \), there is a threshold such that when reputation is below this threshold, there is partial learning as the low-quality bank reveals its type with probability \( \alpha_l(\mu_t, v_t) \) satisfying (31). When reputation is above the threshold, the equilibrium has no learning with both quality types selling either all or none of their loan portfolio.

\( ^{10} \)In the Appendix A, we show that this proposition also holds when banks can sell a fraction of their loans. The proof is similar to that of the proof of proof of lemma 2 and uses sub-modularity of the value function.
This proposition shows that when reputational concerns are sufficiently strong, adverse selection persists.

5.1 Volume of Trade and the Spread in the Infinite Horizon

Next we use Proposition 11 to show that the aggregate volume of trade falls in response to a decline in collateral values, and that this decline is discontinuous at a critical value of the spread. To show this result, we begin by characterizing properties of the equilibrium distribution over buyers’ beliefs about bank quality. Let $H_t$ denote the equilibrium distribution over reputation levels $\mu$ in period $t$ for a particular history of realizations of collateral shocks and suppose that $t$ is large enough so that the initial distribution $H_0$ has an insignificant effect on $H_t$. Proposition 11 implies that since $\Psi$ has full support over $[0,1]$, $H_t$ has full support with mass points at 0 and $\mu_h$.

To understand how shocks to collateral value $v_t$ affect volume of trade, consider an increase in the spread $v_t$ from $v^1$ to $v^2$. Consider the effect on the expected volume of trade for a bank with a given reputation $\mu_t$. If the increase in spread does not induce the bank to switch regions, clearly its volume of trade does not change. Consider therefore a bank which switches from selling all its loans to the complete pooling with holding region or to the partial pooling region. If such a bank has reputation $\mu_t > \mu_h$, it switches from selling to complete pooling with holding and the expected volume falls from 1 to 0. If such a bank has reputation $\mu_t \leq \mu_h$, then it switches to the partial pooling region so that the high-quality bank switches from selling to holding and the low-quality bank switches from selling with probability $1 - \alpha_t(\mu_t)$. Since $H_t$ has full support, an increase in the spread leads to a decline in the aggregate volume of trade. Since the distribution $H_t$ has a mass point at $\mu_h$, if the increase in spread induces banks with reputation levels $\mu_t = \mu_h$ to switch regions, the decline in the aggregate volume of trade is discontinuous. A similar argument implies that increases in volume of spread are discontinuous at a critical value of the spread.

We have established the following proposition.

**Proposition 12** If banks are sufficiently patient and if shocks to collateral values are independent over time, then the aggregate volume of trade is declining in the spread $v$. Changes in the volume of trade are discontinuous at critical values of the collateral shock.

As the proof of the proposition makes clear, an increase the spread leads to a decline in the volume of new issuances by banks of both quality types. In this sense, reputational forces can help generate a decline in trading volume by banks of all quality levels. Recall that in
the static model, an increase in the spread leads to a reduction in the volume of trade only by high-quality banks. Thus, reputational forces can help overcome the correlation challenge present in static adverse selection models.

Figure 2 reports results from a numerical example intended to illustrate fluctuations in the trade volume that our model can generate.\textsuperscript{11} We have plotted trade volume for a particular simulation of our model. In this simulation, collateral values gradually rise at a linear rate from period 1 to period 40. In period 41, collateral values fall abruptly and then rise at a linear rate from that period onwards. The figure shows a sizeable increase in volume from period 0 to period 40 and a sharp fall in period 41 after which volume continues to grow.

This illustrative simulation captures the idea that the model can generate slow buildups in the volume of trade followed by abrupt collapses. While exploring whether this model can generate fluctuations of the kind seen in the data is beyond the scope of this paper, the simplicity of our infinite horizon model makes it amenable to future quantitative analyses.

\textsuperscript{11}The parameter values for this simulation are as follows: $\tilde{\pi} = .8, \bar{\pi} = .2, c = 1, \beta = .9, \lambda = .1, v_{\text{min}} = (5/3), v_{\text{max}} = 15, v_t \sim U[v_{\text{min}}, v_{\text{max}}]$ (or $G(v_t) = (v_t - v_{\text{min}})/(v_{\text{max}} - v_{\text{min}}), \Psi(\mu) = \mu$. The value of the loan if no default occurs is set to 16 so that the collateral value is given by $v_t = 16 - v_t$. 


6 Conclusion

We have argued that adverse selection is a promising candidate in accounting for fluctuations in the volume of trade in secondary loan markets. Since issuers of secondary loans are not anonymous and since buyers can observe past transactions, one would expect the market to reveal information about sellers and one would expect that eventually adverse selection would cease to be a factor in generating fluctuations in the volume of trade. We have shown that reputational concerns can impede the revelation of information and indeed in some cases can lead buyers never to learn about the quality of sellers. Thus reputational concerns imply that adverse selection can play a role in generating fluctuations in the volume of trade even in the long run.

The techniques developed here should be widely applicable to the study of other markets which feature both private information and lack of anonymity. Applying these techniques to these other markets can help us to understand price formation, volume of trade, information revelation and the welfare properties of outcomes in these other markets.

References


Appendix

A For Online Publication

A.1 Normalization of Bank and Buyer Payoffs

Suppose buyers offer contracts of the form \((x, \hat{t})\). If a bank accepts such a contract, the banks’ payoffs are given by

\[
\hat{t} + (1 - x) (\pi \bar{v} + (1 - \pi) \nu - c)
\]

Buyers profits from \((x, \hat{t})\) if a bank of type \(\pi\) accepts are given by

\[
x (\pi \bar{v} + (1 - \pi) \nu) - \hat{t}.
\]

Letting \(t = \hat{t} - x \nu\) and \(v = \bar{v} - \nu\) these payoffs can be expressed as

\[
t + (1 - x) (\pi v - c) + \nu
\]

and

\[x \pi v - t.\]

It is then immediate that we may normalize \(\nu\) and analyze the effect of changes in the collateral value, \(\bar{v}\), by analyzing changes in the loan spread, \(v\).

Q.E.D.

A.2 General Properties of Static Equilibrium

Here we show that any symmetric monotone equilibrium of the static model has the properties described in the text.

1. Suppose by way of contradiction that \(x_l < 1\). Consider a deviation to the contract \((x_h, t_h, x_l + \varepsilon, t_l + (\pi v - c) \varepsilon)\). This deviation keeps the low quality bank indifferent while the profit per low type is higher and hence it is profitable.

2. Suppose (3) holds with strict inequality. Then, from (2) and (3), \(x_h < 1\). A deviation to a contract \(z' = (x_h + \varepsilon, t_h' + \varepsilon (\pi v - c), x_l, t_l)\) leaves the utility of the high quality bank unchanged and increases the profits per high quality bank.

3. To prove that buyers makes zero profits on each point in the support of \(z\), first note that in any monotone equilibrium, for each pair of contracts offered by buyers \(j\) and \(-j\), \(\delta_j(z_j, z_{-j}; \bar{\pi}) = \delta_j(z_j, z_{-j}; \pi)\). Thus, in any monotone equilibrium, profits associated with a
A contract \( z \) for buyer \( j \) can be written as

\[
\Pi(z) = \mu(x_h \bar{\pi}v - t_h) + (1 - \mu)(x_l \bar{\pi}v - t_l) \int \delta_j(z, z_{-j}; \bar{\pi}) dF(z_{-j}).
\]

Since the contract \((0, 0, 0, 0)\) guarantees zero profits, it follows that for each \( z \in \text{Supp}(F) \) with \( \int \delta_j(z, z_{-j}; \bar{\pi}) dF(z_{-j}) > 0 \),

\[
\mu(x_h \bar{\pi}v - t_h) + (1 - \mu)(x_l \bar{\pi}v - t_l) \geq 0. \quad (A.1)
\]

Furthermore, in any mixed strategy, at each such \( z \), profits must be identical, or \( \Pi(z) = \bar{\Pi} \).

We now prove that for each \( z \in \text{Supp}(F) \), \( (A.1) \) holds with equality. To do so, consider the lowest payoff of the low-quality bank,

\[
y = \inf_{z \in \text{Supp}(F)} t_h + (1 - x_h)(\bar{\pi}v - c)
\]

Let \( \bar{z} \) denote the contract that attains this infimum. Suppose first that the support of \( F \) has no mass at \( \bar{z} \). Clearly, there is a sequence \( \{z_n = (x_{h,n}, t_{h,n}, x_{l,n}, t_{l,n})\}_{n=1}^{\infty} \subset \text{Supp}(F) \) that converges to \( \bar{z} \). Since banks accept the contract they prefer most, the probability that a contract in this sequence is accepted converges to zero, so that \( \int \delta_j(z_n, z_{-j}; \bar{\pi}) dF(z_{-j}) \rightarrow 0 \). Hence \( \Pi(z_n) \rightarrow 0 \) (since \( \mu(x_{h,n} \bar{\pi}v - t_{h,n}) + (1 - \mu)(x_{l,n} \bar{\pi}v - t_{l,n}) \) is bounded). Now, since \( \Pi(z) = \pi, \forall z \in \text{Supp}(F) \), we must have \( \pi = 0 \).

Suppose next that \( F \) has positive mass \( f_0 \) at \( \bar{z} \). Given our tie-breaking rule, the contract \( \bar{z} \) attracts the bank with probability of at most \( f_0/2 \). Now, suppose by way contradiction that at this contract profits are strictly positive. Consider a deviation from \( \bar{z} \) to the contract \( \bar{z}' = (x_h, t_h + \epsilon, x_l, t_l + \epsilon) \) for some small \( \epsilon > 0 \) where \( \bar{z} = (x_h, t_h, x_l, t_l) \). The profits conditional on this offer being better than the contract offered by the other buyer are arbitrarily close to conditional profits under \( \bar{z} \). Since this contract is strictly preferred to \( \bar{z} \) by both types, the probability that the deviating contract is accepted is at least \( f_0 \). Since the probability that \( \bar{z} \) is accepted is at most \( f_0/2 \), it follows that expected profits strictly increase with this deviation. We have established a contradiction so profits are zero.

4. Suppose that \( t_l < \bar{\pi}v \) for some contract offered in equilibrium. A deviation to a contract \( \tilde{z} = (1, \bar{\pi}v - \epsilon, 1, \bar{\pi}v - \epsilon) \) for \( \epsilon \) sufficiently small makes positive profits from low-quality banks by attracting them with positive probability and possibly the high-quality banks as well.

Q.E.D.

### A.3 Proof of Proposition 1.

Consider the case where \( \mu < \bar{\mu} \). We begin by showing that in any pure strategy equilibrium, \( t_l = \bar{\pi}v \). This result together with the four key properties implies that any pure strategy
outcome must coincide with the least cost separating outcome. Suppose, by way of con-
diction, that a pure strategy equilibrium has \( t_l > \pi v \). Clearly if \( x_h = 0 \), this contract makes
negative profits so \( x_h > 0 \). Consider a deviating contract \( \hat{z} = (x_h - \varepsilon, t_h - \varepsilon(\pi v - c), 1, t_l) \)
with \( \varepsilon \) small. This deviating contract keeps the high-quality bank indifferent to the supposed
equilibrium contract and relaxes the low-quality bank’s incentive constraint. It attracts the
high-quality bank with probability 1/2 under our tie-breaking rule. Conditional on attracting
the high-quality bank profits from this bank rise by \( \varepsilon c \) and profits from the low-quality
bank are unchanged. Thus profits are strictly positive so we have a contradiction.

To show that the least-cost separating outcome is a pure strategy equilibrium, we es-
tablish a preliminary result regarding the high-quality bank’s payoffs. Consider the set of
offers \( (x_h, t_h) \), to the high quality bank implied by a binding incentive constraint for the
low-quality bank and zero profits as a function of \( t_l \). Since both the profit function and the
incentive constraint are linear functions of \( x_h, t_h, \) and \( t_l \), it follows that \( x_h \) and \( t_h \) are linear
functions of \( t_l \). Since the payoff function to the high quality bank is linear in \( x_h \) and \( t_h \), this
payoff, \( \hat{u}_h(t_l) \), is also linear in \( t_l \). Clearly \( \hat{u}_h(\pi v) \) is equal to the high quality bank’s payoff in
the least cost separating outcome and \( \hat{u}_h(\hat{p}(\mu)) = \hat{p}(\mu) \). Since \( \mu < \bar{\mu}, \hat{u}_h(\pi v) > \hat{u}_h(\hat{p}(\mu)) \).
Since \( \hat{u}_h(t_h) \) is a linear function of \( t_h \), it follows that \( \hat{u}_h(\pi v) > \hat{u}_h(t_l) \) so that the high-quality
bank prefers the least-cost separating outcome to any associated with \( t_l > \pi v \) which breaks
even and in which the incentive constraint for the low-quality bank holds with equality.

Given this preliminary result, suppose that buyer 2 offers the least-cost separating outcome.
Consider a deviation by buyer 1 to contracts of the form \( \hat{z} = (\hat{x}_h, \hat{t}_h, 1, \hat{t}_l) \) with \( \hat{t} > \pi v \).
Since any such contract that breaks even does not attract the high-quality bank, it is clear that
any contract which does attract the high-quality bank must yield negative profits. Clearly
since any deviation which has \( \hat{x}_l < 1 \) is dominated by a further deviation which sets the
amount sold by the low-quality bank to 1 and keeps it indifferent to the original deviation,
the restriction to deviations which have \( \hat{x}_l = 1 \) is without loss of generality.

To show that when \( \mu < \bar{\mu}, \) our model has no mixed strategy equilibrium, suppose by way
of contradiction that it did. Note that the preliminary result established above implies that
the high-quality bank strictly prefers the least-cost separating outcome to any other offer
in the support of the offer distribution and the low-quality bank strictly prefers any offer
greater than \( \pi v \) to the least-cost separating outcome. Thus a deviation by one of the buyers
to a pure strategy contract close to the least-cost separating outcome, and more profitable
when accepted, attracts high-quality banks with probability close to 1 and low-quality banks
with probability close to zero.

Suppose now that \( \mu \geq \bar{\mu} \). We will show a mixed strategy equilibrium exists where the
distribution over contracts \( F(z) \) is given by (9). Since \( t_l \geq \pi v \) and profits are zero at every
contract in the support of \( F \), it follows that \( \hat{x}_h \pi v \geq \hat{t}_h \). Next we show that the best deviations \( \hat{z} \) have the property that for some \( z \) in the support of \( F \), the high-quality bank is indifferent between \( z \) and \( \hat{z} \), or

\[
\hat{t}_h + (1 - \hat{x}_h) (\pi v - c) = t_h + (1 - x_h) (\pi v - c) .
\]  

(A.2)

To see this result, notice that if for all \( z \), the left side of (A.2) is strictly greater than the right side, then \( \hat{t}_h \) can be reduced and profits can be raised. If the left side of (A.2) is strictly below the right side for all \( z \), then \( \delta_1 (\hat{z}, z; \pi) = 0 \) for all \( z \) and profits cannot be strictly positive.

Clearly, the best deviation satisfies the incentive constraint of the low-quality bank with equality, or

\[
\hat{t}_l + (1 - \hat{x}_l) (\pi v - c) = \hat{t}_h + (1 - \hat{x}_h) (\pi v - c) ,
\]  

(A.3)

and \( \hat{x}_l = 1 \).

We now evaluate profits for a deviation of the form \( \hat{x}_h = x_h + \varepsilon \) and \( \hat{t}_h \) and \( \hat{t}_l \) given by (A.2) and (A.3). For any contract \( z \) in the support of \( F \), the probability of attracting either a high- or low-quality bank to contract \( z \) is given by \( F(t_l) \). Since the high-quality bank is indifferent between \( z \) and \( \hat{z} \), the probability of attracting the high quality bank to \( \hat{z} \) is given by \( F(\hat{t}_l) \). It is straightforward to show that the deviation contract has \( \hat{t}_l \) satisfying

\[
\hat{t}_l = t_l + (\pi - \bar{\pi}) v \varepsilon.
\]

Thus, the probability of attracting the low-quality bank to the deviation is given by \( F(\hat{t}_l) = F(t_l + (\pi - \bar{\pi}) v \varepsilon) \). Holding fixed \( (x_h, t_h) \), we then define profits from this deviation as a function of \( \varepsilon \) as

\[
\Pi (\varepsilon) = (1 - \mu) F(t_l + (\pi - \bar{\pi}) v \varepsilon)(\pi v - t_l - (\bar{\pi} - \pi) v \varepsilon) + \mu F(t_l)((x_h + \varepsilon) \pi v - t_h - (\bar{\pi} v - c) \varepsilon)
\]  

(A.4)

Next we prove that profits are globally concave and attain a maximum at \( \varepsilon = 0 \). First, note that

\[
\Pi'(0) = (1 - \mu)(\pi - \bar{\pi}) v f(t_l)(\pi v - t_l) - (1 - \mu) F(t_l)(\pi - \bar{\pi}) v + \mu F(t_l) c \quad (A.5)
\]

\[= 0\]

Suppose now that the distribution \( F \) is continuous, has no mass point, has a connected support which is a subset of \([\pi v, \hat{p}(\mu)]\), and \( F(t_l) < 1 \) if \( t_l < \hat{p}(\mu) \). Then solving the differential equation (A.5), we obtain that it has a unique solution given by (9).

We now prove that \( \Pi''(\varepsilon) < 0 \) for all \( \varepsilon \) so that \( \varepsilon = 0 \) is a global maximum. Differentiating
Now, note that by construction the high type is indifferent between these properties.

Now, note that

$$f'(t_l) = \frac{f(t_l)}{t_l - \overline{\pi}v} \left[ \frac{\mu c}{(1 - \mu)(\overline{\pi} - \pi)v} - 2 \right].$$

Hence,

$$\Pi''(\epsilon) = (1 - \mu) ((\overline{\pi} - \pi)v)^2 f(t_l + (\overline{\pi} - \pi)v) \\
	\times \left[ -2 + (\overline{\pi}v - t_l - (\overline{\pi} - \pi)v\epsilon) \left( \frac{\mu c}{(1 - \mu)(\overline{\pi} - \pi)v} - 2 \right) \left( \frac{1}{t_l + (\overline{\pi} - \pi)v\epsilon - \pi v} \right) \right] \\
= -(1 - \mu) ((\overline{\pi} - \pi)v)^2 f(t_l + (\overline{\pi} - \pi)v) \frac{\mu c}{(1 - \mu)(\overline{\pi} - \pi)v} \frac{1}{t_l + (\overline{\pi} - \pi)v}.$$ 

<0.

Consider the solution to the differential equation in (A.5) with the boundary condition that $F(\hat{\rho}(\mu)) = 1$. This solution coincides with (9).

Next, we show that the above mixed strategy equilibrium is unique in the class of monotone equilibria. Since the solution to (A.5) is unique in the class of distributions which satisfy continuity, have no mass point, has a connected support which is a subset of $[\overline{\pi}v, \hat{\rho}(\mu)]$, and $F(t_l) < 1$ if $t_l < \hat{\rho}(\mu)$, we need only to show that any mixed strategy distribution must have these properties.

To see that $t_l \leq \hat{\rho}(\mu)$, let $\hat{x}_h(t_l)$ and $\hat{t}_h(t_l)$ denote the set of offers $(x_h, t_h)$, to the high quality bank implied by a binding incentive constraint for the low-quality bank and zero profits as a function of $t_l$. These solutions have the properties that $\hat{x}_h(t_l)$ is increasing and $\hat{x}_h(\hat{\rho}(\mu)) = 1$. Since $\hat{x}_h(t_l) \leq 1$, it follows that $t_l \leq \hat{\rho}(\mu)$.

To show that $F$ has no mass points, suppose there exists a $t_l \in Supp(F)$ and that $F(t_l+) > F(t_l-)$ with $t_l > \overline{\pi}v$. Then, consider a deviation by one of the buyers to $z' = (\hat{x}(t_l) - \epsilon, \hat{t}_h(t_l) - \epsilon (\pi v - c), 1, t_l - \epsilon (\overline{\pi} - \pi)v).$

Note that by construction the high type is indifferent between $z'$ and $\hat{z}(t_l)$ while the low type strictly prefers $\hat{z}(t_l)$. Now suppose that $F(t_l+) - F(t_l-) = f(t_l) > 0$, i.e., $F$ puts mass $f(t_l)$ on $t_l$. Clearly, the deviation contract $z'$ is incentive compatible. Hence, as $\epsilon$ converges to 0, the fraction of high-quality banks attracted to $z'$, $\int \delta_j(z', z_{-j}; \pi)dF(z_{-j})$ approaches $F(t_l-) + f(t_l)/2$ (the 1/2 follows from the uniform tie breaking rule). The fraction of low-quality banks attracted to $z'$, $\int \delta_j(z', z_{-j}; \pi)dF(z_{-j})$ approaches $F(t_l-)$. Hence, the profits
for this deviation approach
\[
\mu \left( F(t_{l}^{-}) + f/2 \left( \tilde{x}_{h}(t_{l})\pi v - \hat{t}_{h}(t_{l}) \right) \right) + (1 - \mu) F(t_{l}^{-}) (\pi v - t_{l}) =
\]
\[
F(t_{l}^{-}) \left[ \mu \left( \tilde{x}_{h}(t_{l})\pi v - \hat{t}_{h}(t_{l}) \right) \right] + (1 - \mu) (\pi v - t_{l}) + \mu f(t_{l})/2 \left( \hat{x}_{h}(t_{l})\pi v - \hat{t}_{h}(t_{l}) \right) > 0
\]
where the inequality follows from the fact that (A.1) holds with equality and \( t_{l} > \pi v \).

Hence, for \( \varepsilon \) sufficiently small enough, \( z' \) is a profitable deviation which yields the necessary contradiction.

To see that the upper bound of the support of \( F \) is \( \hat{p}(\mu) \), suppose for some \( t_{l} < \hat{p}(\mu) \), \( F(t_{l}) = 1 \). We prove that there exists a profitable deviation. Consider the deviation \( z' = (1, \hat{p}(\mu) - \varepsilon, 1, \hat{p}(\mu) - \varepsilon) \). Straightforward algebra can be used to show that
\[
\hat{t}_{h}(t_{l}) + (1 - \hat{x}_{h}(t_{l})) (\pi v - c) = \hat{x}_{h}(t_{l}) \hat{p}(\mu) + (1 - \hat{x}_{h}(t_{l})) (\pi v - \mu c).
\]
Since \( t_{l} < \hat{p}(\mu) \), \( \hat{x}_{h}(t_{l}) < 1 \) and \( \mu > \bar{\mu} \), it follows that \( \hat{p}(\mu) > \pi v - \mu c \) so that
\[
\hat{p}(\mu) > \hat{t}_{h}(t_{l}) + (1 - \hat{x}_{h}(t_{l})) (\pi v - c).
\]
Hence, for \( \varepsilon > 0 \) and sufficiently small, the deviation contract \( z' \) attracts both high- and low-quality banks with probability 1 against the equilibrium, or \( \int \delta_{j}(z'; z_{-j}; \pi)\,dF(z_{-j}) = 1 \) for \( \pi = \pi, \tilde{\pi} \). But, profits at \( z' = \varepsilon > 0 \), which implies a profitable deviation would exist. Thus, for all \( t_{l} < \hat{p}(\mu) \), \( F(t_{l}) < 1 \).

Suppose next that \( F(t_{l}) \) does not have connected support. That is, suppose there exists an interval \( [t_{1}, t_{2}] \) such that \( F(t_{l}) = F(t_{1}), \forall t_{l} \in [t_{1}, t_{2}] \). Since \( F(t_{1}) < 1 \) and for all \( t < \hat{p}(\mu) \), \( F(t) < 1 \), it must be that \( t_{2} < \hat{p}(\mu) \). Consider then a deviation of the form \( z' = (\hat{x}_{h}(t_{2}), \hat{t}_{h}(t_{2}) - \varepsilon, 1, t_{2}) \). Since \( t_{2} < \hat{p}(\mu) \) the incentive constraint for the high-quality bank is slack at \( \hat{z}(t_{2}) \) so that if \( \varepsilon > 0 \) is chosen sufficiently small enough, \( z' \) is incentive compatible. Furthermore, for \( \varepsilon \) sufficiently small the fraction of high- and low-quality banks attracted to \( z' \) is the same as \( \hat{z}(t_{2}) \), since \( F(t_{l}) \) is constant in a neighborhood below \( t_{2} \). This implies that \( z' \) makes higher profit than \( \hat{z}(t_{2}) \) and hence \( z' \) is a profitable deviation.

Since the distribution has the desired properties, it must coincide with the distribution \( F(\cdot) \) in (9).

Q.E.D.

A.4 Proof of Lemma 1

Our construction of out of equilibrium beliefs is similar for both \( \mu \geq \mu^{*} \) and \( \mu \leq \mu^{*} \). Letting \( z = (x_{h}, t_{h}, x_{l}, t_{l}) \) be the equilibrium contract, let the out of belief function following
acceptances of offers which are not part of the equilibrium contract be
\[
\mu'(\hat{x}, \hat{t}) = \begin{cases} 
0 & \text{if } \hat{t} + (1 - \hat{x})(\bar{\pi}v - c) \leq \max \{ \hat{x}\hat{p}(\mu) + (1 - \hat{x})(\bar{\pi}v - c), t_h + (1 - x_h)(\bar{\pi}v - c) \} \\
1 & \text{o.w.} 
\end{cases} 
\]  \tag{A.6}

Note that these beliefs say that future buyers believe that the bank is high-quality if the bank accepts an offer which is both statically more favorable to the high-quality bank than the equilibrium and above the market-odds line.

We prove the following extension of lemma 1. This extension characterizes all complete pooling equilibria for \( \mu \geq \mu^* \).

**Lemma 4** Suppose \( \mu \in [\mu^*, 1] \) and banks are sufficiently patient. Then the two period model has an equilibrium for any value of \( x \in [\underline{x}(\mu), 1] \) where \( \underline{x}(\mu) = 1 \) when \( \mu \leq \bar{\mu} \) and otherwise it satisfies
\[
\hat{p}(\mu) + \beta V(0; \bar{\pi}) = \underline{x}(\mu)\hat{p}(\mu) + (1 - \underline{x}(\mu))(\bar{\pi}v - c) + \beta V(\mu; \bar{\pi}) \tag{A.7}
\]

Note that \( \underline{x}(\mu) \) is a lower bound on the set of complete pooling equilibria. Values of \( x < \underline{x}(\mu) \) cannot be supported as equilibria because one of the buyers can deviate to a pooling outcome in which both types sell their entire loan portfolios at a price slightly less than \( \hat{p}(\mu) \), attract banks of both quality levels and make strictly positive profits.

**Proof.** The proof is by contradiction and has three steps. Let \( (x, x\hat{p}(\mu)) \) denote a pooling contract that satisfies (A.7) and let \( \hat{z} = (\hat{x}_h, \hat{t}_h, \hat{x}_l, \hat{t}_l) \) denote a deviating contract which makes positive profits. Let \( \hat{\mu}_h, \hat{\mu}_l \) denote the reputation levels associated with acceptance of the deviating offers. Clearly, any contract that makes positive profits must attract the high quality bank and make strictly positive profits from such banks. We prove the claim in three steps. The first step is to show that \( \hat{t}_h \geq \hat{x}_h\hat{p}(\mu) \). The second step is to show that \( \hat{t}_l \geq \hat{x}_l\hat{p}(\mu) \). The key part of this step is to show that if \( (\hat{x}_h, \hat{t}_h) \) attracts the high quality bank, it also attracts the low quality bank. The third step shows that \( \hat{z} \) makes non-positive profits.

Step 1. Given our belief function in (A.6), any offer to the high-quality bank that makes it statically worse off also makes it dynamically worse off and cannot attract it. Thus \( (\hat{x}_h, \hat{t}_h) \) must satisfy
\[
\hat{t}_h + (1 - \hat{x}_h)(\bar{\pi}v - c) \geq t_h + (1 - x)(\bar{\pi}v - c) \tag{A.8}
\]
where \( t_h = x\hat{p}(\mu) \). In addition, if \( \hat{t}_h < \hat{x}_h\hat{p}(\mu) \), then (A.6) implies that \( \hat{\mu}_h = 0 \), so that the payoffs associated with an acceptance of \( \hat{x}_h, \hat{t}_h \) for the high-quality bank are less than the left side of (A.7). The payoffs associated with the equilibrium contract are at least as large as the right side of (A.7). If the deviation is to attract high-quality banks, it must offer them
a payoff higher than the equilibrium payoff so that \( \hat{t}_h \geq \hat{x}_h \hat{p}(\mu) \). Note for later that from (A.6), it follows that the beliefs following an acceptance of \((\hat{x}_h, \hat{t}_h)\) are given by \( \hat{\mu}_h = 1 \).

Step 2. Since \((\hat{x}_h, \hat{t}_h)\) makes positive profits from high-quality banks, we have \( \hat{t}_h < \hat{x}_h \bar{\pi} v \). Using this inequality in (A.8), we obtain
\[
\hat{x}_h c > t_h = x \hat{p}(\mu) - x (\bar{\pi} v - c) . 
\]
(A.9)

Using the results that the beliefs following an acceptance of \((\hat{x}_h, \hat{t}_h)\) are given by \( \hat{\mu}_h = 1 \), we claim that the low-quality bank gets a higher utility by accepting \((\hat{x}_h, \hat{t}_h)\) than \((x, t_h)\), or that
\[
\hat{t}_h + (1 - \hat{x}_h) (\bar{\pi} v - c) + \beta V(1; \bar{\pi}) > x \hat{p}(\mu) + (1 - x) (\bar{\pi} v - c) + \beta V(\mu; \bar{\pi}) \quad \text{(A.10)}
\]

We show this result by substituting for \( \hat{t}_h \) from (A.8) into (A.10) and simplifying to obtain
\[
\beta V(1; \bar{\pi}) - \beta V(\mu; \bar{\pi}) > (x - \hat{x}_h)(\bar{\pi} v - \bar{\pi} v) \quad \text{(A.11)}
\]

Using (A.9) to substitute for \( \hat{x}_h \) in (A.11), and \( d = (\bar{\pi} v - \bar{\pi} v)/c \), we have that (A.11) is satisfied if
\[
\beta V(1; \bar{\pi}) - \beta V(\mu; \bar{\pi}) > x(1 - \mu)d(\bar{\pi} v - \bar{\pi} v) \quad \text{(A.12)}
\]

Suppose that \( \mu \geq \bar{\mu} \). Substituting for the continuation payoffs from (10), (A.12) is satisfied if
\[
\beta > x d . \quad \text{(A.13)}
\]

Since \( \bar{\mu} = d/(1 + d) \), and \( \beta \geq (1 + d)/d \), the left side of (A.13), is greater than \((1 + d)^2/d \). Since the right side of (A.13) is less than \( d \), the needed inequality is satisfied. If \( \mu < \bar{\mu} \), substituting from (10) (A.12) becomes \( \beta > x(1 - \mu)d \). This inequality is satisfied because \( \mu \geq \mu^* = (1 - 1/d) \) and \( \beta \geq (1 + d)/d \). Thus, the low quality bank is attracted to the deviating offer to the high-quality bank.

Suppose now that \( \hat{x}_l \hat{p}(\mu) > \hat{t}_l \). We show that the low quality bank weakly prefers the equilibrium allocation to \((\hat{x}_l, \hat{t}_l)\). Together with the result that the low-quality bank is attracted to the high-quality offer, we have a contradiction. Since from (A.6), beliefs are zero for all accepted offers with \( \hat{x} \hat{p}(\mu) > \hat{t}_l \), the highest possible payoff for the low quality bank from such a contract are given by \( \hat{p}(\mu) + \beta V(0; \bar{\pi}) \). We show that
\[
\hat{p}(\mu) + \beta V(0; \bar{\pi}) \leq x \hat{p}(\mu) + (1 - x)(\bar{\pi} v - c) + \beta V(\mu; \bar{\pi}) . \quad \text{(A.14)}
\]

If \( \bar{\mu} \geq \mu \geq \mu^* \), \( V(0; \bar{\pi}) = V(\mu; \bar{\pi}) \), and (A.14) clearly holds. Suppose \( \mu \geq \bar{\mu} \). Using (10), we
can rewrite this needed inequality as

\[(1 - x) (\hat{p} (\mu) - \pi v + c) \leq \beta \frac{\mu - \bar{\mu}}{1 - \bar{\mu}} (\bar{\pi} - \pi) v\]

Equation (A.7) implies that left side of (A.7) is less than the payoff to the high-quality bank in the equilibrium pooling contract is greater so that

\[\hat{p} (\mu) + \beta V (0; \bar{\pi}) \leq x \hat{p} (\mu) + (1 - x) (\bar{\pi} v - c) + \beta V (\mu; \bar{\pi})\]

which using (10) can be rewritten as

\[(1 - x) (\hat{p} (\mu) - \pi v + c) \leq \beta \frac{(\mu - \bar{\mu})^2}{\mu (1 - \bar{\mu})} (\bar{\pi} - \pi) v\]  \hspace{2cm} (A.15)

Thus, (A.14) holds if

\[(1 - x) (\hat{p} (\mu) - \pi v + c) \leq \frac{\mu}{\mu - \bar{\mu}} (1 - x) (\hat{p} (\mu) - \pi v + c)\]  \hspace{2cm} (A.16)

Substituting for \(\hat{p} (\mu)\) from (7), using \(d = (\bar{\pi} - \pi) v / c\), and simplifying, (A.16) holds if

\[\mu d + 1 \leq \frac{\mu}{\mu - \bar{\mu}} (- (1 - \mu) d + 1)\]  \hspace{2cm} (A.17)

Using \(\bar{\mu} = d / (1 + d)\) and simplifying it is straightforward to check that (A.17) holds so (A.14) does as well.

Step 3. Adding the incentive compatibility constraints (16) and (17), and simplifying we have that

\[(\hat{x}_l - \hat{x}_h) (\bar{\pi} - \pi) v \geq \beta \left[ (V (1; \bar{\pi}) - V (\mu; \bar{\pi})) - (V (1; \pi) - V (\mu; \pi)) \right] \geq 0,\]

where \(\hat{\mu}\) is the belief associated with \((\hat{x}_l, \hat{\mu})\) so that \(\hat{x}_l \geq \hat{x}_h\). The profits from \(\hat{\pi}\) are given by

\[\Pi = \mu (\hat{x}_h \bar{\pi} v - \hat{t}_h) + (1 - \mu) (\hat{x}_l \bar{\pi} v - \hat{t}_l)\]

\[\leq \mu (\hat{x}_h \bar{\pi} v - \hat{x}_h \hat{\mu} (\mu)) + (1 - \mu) (\hat{x}_l \bar{\pi} v - \hat{x}_l \hat{\mu} (\mu))\]

\[= \mu (1 - \mu) (\bar{\pi} - \pi) v (\hat{x}_h - \hat{x}_l).\]

Here, the first inequality follows from step 2. Since \(\hat{x}_l \geq \hat{x}_h\), profit \(\Pi\) is non-positive.

Q.E.D.

A.5 Proof of Lemma 2

Here, we prove the following extension of lemma 2:

**Lemma 5** Suppose that banks are sufficiently patient and that \(\mu \leq \mu^*\). Any allocation such that \(\alpha_h = 1, \mu_h' \geq \mu\), the incentive constraint for the low quality bank is satisfied with equality, (19) equals 0, and (20) and (23) are satisfied can be supported as an equilibrium. In addition, any equilibrium of the game has this form.
**Proof.** The proof is by contradiction. Suppose first that a deviation contract attracts high-quality banks and makes strictly positive profits on such banks so that $\hat{x}_h > 0$. As in the proof of the preceding lemma, attracting such banks requires that the analog of (A.8) hold. In section A.6.1 below, we show that (20) implies that

$$x_h \leq \frac{\beta}{(\bar{\pi} - \pi)v} [V(1; \pi) - V(\mu_h'; \pi)]$$

(A.18)

so that

$$\beta [V(1; \pi) - V(\mu_h'; \pi)] > (x_h - \hat{x}_h)(\bar{\pi} - \pi)v$$

(A.19)

since $\hat{x}_h > 0$. Adding (A.8) and (A.19) we obtain

$$\hat{t}_h + (1 - \hat{x}_h)(\bar{\pi}v - c) + \beta V(1; \pi) - \beta V(\mu_h'; \pi) > t_h + (1 - x_h)(\pi v - c) + (x_h - \hat{x}_h)(\bar{\pi} - \pi)v.$$  

(A.20)

Simplifying (A.20), we get

$$\hat{t}_h + (1 - \hat{x}_h)(\bar{\pi}v - c) + \beta V(1; \pi) > t_h + (1 - x)(\bar{\pi}v - c) + \beta V(\mu; \pi)$$

(A.21)

which implies that low-quality banks are attracted to the offer intended for the high-quality bank. The argument that the profits to the deviating buyer are then negative taking account of the offer intended for the low-quality banks is the same as in the preceding lemma.

Next we show that a deviation offer which attracts low-quality banks is not profitable. Clearly, it is impossible to attract only low quality banks and make positive profits. What is left to be shown is that a deviation to a contract of the form $z' = (x_h, t_h, x_l', t_l')$ cannot make positive profits. Note that the contract that maximizes buyers payoff and attracts the low quality bank is given by

$$(x_h, t_h, 1, t_l + (1 - x_l)(\bar{\pi}v - c)).$$

This contract attracts the high quality bank with probability $1/2$ and attracts the low quality bank with probability 1. Hence, the profits from this deviation are given by the left side of (23) and are, therefore, non-positive.

To show that any equilibrium must have the specified form, it suffices to show that there is no equilibrium in which the high quality bank uses mixed strategies. Suppose, to the contrary, that an equilibrium exists in which the high-quality bank mixes. Then, it must be that the low quality bank is also mixing. To see this, suppose that only the high quality bank uses mixed strategies. Then on-path continuation beliefs are given by 1 and $\mu' < \mu$. Since $V(0; \pi) = V(\mu'; \pi)$ and that banks are sufficiently patient, a proof similar to proposition 4 implies that no incentive compatible contract can exist. Hence, we focus on contracts in which both banks mix. Such an equilibrium can be described by $(x_h, t_h, x_l, t_l, \alpha_h, \alpha_l)$ with $\alpha_h, \alpha_l \in (0, 1)$ and $\alpha_h > 1 - \alpha_l$. From (18), we have that $\mu' > \mu$. Since $\mu < \mu$, $V(\mu'; \pi) = V(0; \pi)$. Since both types are mixing, it must be that both types are indifferent.
between the two contracts. Subtracting (16) from (17), we obtain
\[ x_h (\bar{\pi} - \pi) v + \beta (\Delta - \bar{\Delta}) = x_l (\bar{\pi} - \pi) v \]
where \( \bar{\Delta} = V (\mu'_h; \bar{\pi}) - V (0; \bar{\pi}) \) and \( \Delta \) is defined similarly for the low type. Since \( V (\cdot; \cdot) \) is sub-modular, \( \Delta > \bar{\Delta} \) and therefore, \( x_l > x_h \). Furthermore, it follows that \( t_l > t_h \). We can rewrite the above as
\[ x_l = x_h + \beta \left[ \frac{\mu'_h - \bar{\mu}}{1 - \bar{\mu}} \left( 1 - \frac{\mu'_h - \bar{\mu}}{\mu'_h} \right) \right] = x_h + \beta \frac{\mu'_h - \bar{\mu}}{1 - \bar{\mu}} \frac{1 - \mu'_h - \bar{\mu}}{\mu'_h} \]
Hence,
\[ t_h = t_l + (x_h - x_l) (\bar{\pi} v - c) - \beta \bar{\Delta} \]
\[ = t_l - \beta \frac{\mu'_h - \bar{\mu}}{\mu'_h} \left( \bar{\mu} (\bar{\pi} v - c) + (\bar{\pi} - \pi) v (\mu'_h - \bar{\mu}) \right) \]
\[ = t_l - \beta \frac{\mu'_h - \bar{\mu}}{\mu'_h} \left[ (\bar{\pi} - \pi) v \mu'_h + \bar{\mu} (\bar{\pi} v - c) \right] = t_l - a \]
Extensive algebra shows that \( t_l - x_l \hat{p} (\mu) \leq 0 \) (this derivation is available upon request). Therefore, \( t_l - (\bar{\pi} v - c) x_l < 0 \). That is the high type strictly prefers holding to trading at \((x_l, t_l)\) – this holds true statically and since \( V (\mu'_l; \bar{\pi}) = V (0; \bar{\pi}) \) it must also hold dynamically. This implies that an offer of the form \((\varepsilon, \hat{p} (\mu) \varepsilon - \delta)\) for small enough \( \varepsilon \) and \( \delta \) would attract the high quality bank and makes positive profits. Hence, the original allocation cannot be an equilibrium.
Q.E.D.

A.6 Basic Properties of Partial Pooling Allocations

Lemma 6 Consider the partial pooling allocation described in section 3.2 with \( x_l = 1 \). Then we have the following:

1. The constraints (16)-(17), the expression in (19) is non-negative, (20) and (23) can be simplified to \( x_h \leq \bar{x}_h (\mu_h, v) \), and \( x_h \geq \bar{x}_h (\mu_h, v) \) where \( \bar{x}_h (\mu_h, v) \) is given in (25) and \( \bar{x}_h (\mu_h, v) \) is given in (26).

2. The upper bound implied by (20) is decreasing in \( \mu'_h \) when \( \mu'_h \geq \bar{\mu} \) and increasing
3. The lower bound $\bar{x}_h$, when positive, is decreasing in $\mu'_h$.

4. Aggregate volume implied by the upper bound in (20) is maximized at $\mu'_h = \tilde{\mu}$

5. Ex-ante welfare implied by the upper bound in (20) is maximized at $\mu'_h = \tilde{\mu}$.

6. When $\mu'_h$ is large enough, $\bar{x}_h \leq \bar{x}_h$.

**Proof.**

1. Using (17) and zero profits, we solve for $t_h$ and $t_l$ to obtain

\[
\begin{align*}
t_h &= \left( \hat{\mu}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right) \right) x_h + \left( \frac{\mu}{\mu'_h} - 1 \right) (\beta \Delta (\mu'_h; \bar{\pi}) - c - c \Delta (\mu'_h; \bar{\pi})) \quad (A.22) \\
t_l &= \bar{\pi} v + \frac{\mu}{\mu'_h} (\mu'_h (\bar{\pi} - \bar{\pi}) v + c) x_h + \frac{\mu}{\mu'_h} (\beta \Delta (\mu'_h; \bar{\pi}) - c)
\end{align*}
\]

Substituting for $t_h$ in (20), we get

\[
\left( \hat{\mu}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right) \right) x_h + \left( \frac{\mu}{\mu'_h} - 1 \right) (\beta \Delta (\mu'_h; \bar{\pi}) - c) - x_h (\bar{\pi} v - c) + \beta \Delta (\mu'_h; \bar{\pi}) \geq 0
\]

\[
\left( \frac{\mu}{\mu'_h} - 1 \right) (\beta \Delta (\mu'_h; \bar{\pi}) - c) + \beta \Delta (\mu'_h; \bar{\pi}) \geq \left[ \bar{\pi} v - c - \left( \hat{\mu}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right) \right) \right] x_h
\]

Note that since $\mu \leq \mu^*$ and that $\mu \leq \mu'_h$, $\bar{\pi} v - c - \hat{\mu}(\mu) \geq 0$ and $-c (\mu/\mu'_h - 1) \geq 0$. Hence, we have

\[
\bar{x}_h = \left( \frac{\mu}{\mu'_h} - 1 \right) (\beta \Delta (\mu'_h; \bar{\pi}) - c) + \beta \Delta (\mu'_h; \bar{\pi}) \geq \left( \frac{\mu}{\mu'_h} - 1 \right) \left( \bar{\pi} v - c - \left( \hat{\mu}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right) \right) \right) x_h \geq x_h. \quad (A.23)
\]

Next, substituting for $t_h$ and $t_l$ from (A.22) into (23), we have that

\[
\begin{align*}
\frac{1}{2} \mu \left[ (1 - \mu) (\bar{\pi} - \bar{\pi}) v x_h + c \left( 1 - \frac{\mu}{\mu'_h} \right) x_h + \left( 1 - \frac{\mu}{\mu'_h} \right) (\beta \Delta (\mu'_h; \bar{\pi}) - c) \right] &\leq \\
\left( 1 - \mu \right) \left[ \frac{\mu}{\mu'_h} \left( (\bar{\pi} - \bar{\pi}) v + c \right) x_h + \frac{\mu}{\mu'_h} (\beta \Delta (\mu'_h; \bar{\pi}) - c) \right]
\end{align*}
\]

After simplification this inequality can be written as

\[
\frac{2 - \mu - \mu'_h}{\mu'_h} \left( c - \beta \Delta (\mu'_h; \bar{\pi}) \right) \leq x_h \left[ (1 - \mu) (\bar{\pi} - \bar{\pi}) v + \frac{2 - \mu - \mu'_h}{\mu'_h} c \right]
\]

The right side of this inequality is positive so that dividing both sides by the expression in brackets on the right side we obtain the expression for the lower bound

\[
\bar{x}_h = \frac{2 - \mu - \mu'_h}{\mu'_h} \left( c - \beta \Delta (\mu'_h; \bar{\pi}) \right) \left( \bar{\pi} - \bar{\pi} v + \frac{2 - \mu - \mu'_h}{\mu'_h} c \right) \leq x_h. \quad (A.24)
\]
2. Suppose that \(\mu_h' \geq \bar{\mu}\). The denominator of (A.23) is an increasing function of \(\mu_h'\). Hence, it suffices to show that the numerator is decreasing in \(\mu_h'\). Using (10), we can write the numerator as

\[
\left(\frac{\mu}{\mu_h'} - 1\right) \left(\beta \frac{\mu_h' - \bar{\mu}}{1 - \mu} (\bar{\pi} - \pi) v - c\right) + \beta \frac{(\mu_h' - \bar{\mu})^2}{\mu_h' (1 - \mu)} (\bar{\pi} - \pi) v =
\beta \frac{(\mu_h' - \bar{\mu})}{\mu_h' (1 - \bar{\mu})} (\bar{\pi} - \pi) v [\mu_h' - \bar{\mu} + \mu - \mu_h'] + c \left(1 - \frac{\mu}{\mu_h'}\right) =
\beta \frac{(\bar{\pi} - \pi) v (\mu - \bar{\mu})}{1 - \bar{\mu}} \left[1 - \frac{\bar{\mu}}{\mu_h'}\right] + c \left(1 - \frac{\mu}{\mu_h'}\right)
\] (A.25)

Using the property that \(\bar{\mu}/(1 - \bar{\mu}) = (\bar{\pi} - \pi) v/c\), we obtain that the derivative of the last expression with respect to \(\mu_h'\) is given by

\[-\frac{c}{(\mu_h')^2} \left[\beta \left(\frac{(\bar{\pi} - \pi) v}{c}\right)^2 (\bar{\mu} - \mu) - \mu\right]\]

Using the definition of the adverse selection discount, \(d\), we can write this expression as

\[-\frac{c}{(\mu_h')^2} \left[\beta d^2 \left(\frac{d}{1 + d} - \mu\right) - \mu\right]\]

Here, we show that the term in bracket is positive and hence the derivative of the numerator is negative:

\[\beta d^2 \left(\frac{d}{1 + d} - \mu\right) - \mu \geq (1 + d) d \left(\frac{d}{1 + d} - \mu\right) - \mu\]
\[\geq d^2 - \left(1 - \frac{1}{d}\right) (d^2 + d + 1)\]
\[= \frac{1}{d^3} \geq 0\]

where the first inequality follows from the assumption that banks are sufficiently patient and the second inequality follows from \(\mu \leq \mu^* = 1 - 1/d\).

To see that \(\bar{x}_h\) is increasing in \(\mu_h'\) when \(\mu_h' \leq \bar{\mu}\), note that when \(\mu_h' \leq \bar{\mu}\), \(\Delta (\mu_h' ; \pi) = 0\), so that we can write the upper bound as

\[\bar{x}_h = \frac{c}{\bar{\pi} v - c - \bar{p}(\mu)} + c\]

Clearly this function is increasing in \(\mu_h'\).

3. Suppose that \(\bar{x}_h\) is positive. Note that the expression \(c - \beta \Delta (\mu_h' ; \pi)\) is decreasing in \(\mu_h'\). In addition:

\[
\frac{2 - \mu - \mu_h'}{\mu_h' (1 - \mu)} (\bar{\pi} - \pi) v + \frac{2 - \mu - \mu_h'}{\mu_h' (1 - \mu)} c = \frac{1}{c + \frac{(\bar{\pi} - \pi) v}{\mu_h' (1 - \mu)}} (A.26)
\]
The function
\[
\frac{2 - \mu - \mu_h'}{\mu_h' (1 - \mu)} = \frac{1}{c + \frac{(\pi - \bar{\pi}) v}{2 - \mu - \mu_h'} \frac{2 - \mu - \mu_h'}{\mu_h'(1 - \mu)c}}
\]  
(A.27)

is decreasing in \(\mu_h'\) and hence the expression in (A.26) is decreasing in \(\mu_h'\). Thus \(x_h\) is the product of two decreasing functions. Since both are positive functions, \(x_h\) must be decreasing.

4 and 5. Note that aggregate welfare is given by
\[
\hat{p} (\mu) - (1 - T) c + \beta \mu V (\mu_h'; \bar{\pi}) + \beta \left( \frac{\mu}{\mu_h'} - \mu \right) V (\mu_h'; \bar{\pi}) + \beta \left( 1 - \frac{\mu}{\mu_h'} \right) V (0; \bar{\pi})
\]  
(A.28)

where \(T\) is aggregate volume. To prove our claims, we first obtain a simplified version of the continuation value
\[
\beta \mu V (\mu_h'; \bar{\pi}) + \beta \left( \frac{\mu}{\mu_h'} - \mu \right) V (\mu_h'; \bar{\pi}) + \beta \left( 1 - \frac{\mu}{\mu_h'} \right) V (0; \bar{\pi})
\]
and show for later use that this continuation value is increasing in \(\mu_h'\). Note that when \(\mu_h' \leq \bar{\mu}\), the continuation value is constant since \(V (\mu_h'; \pi) = V (0; \pi)\) and hence it is weakly increasing. For values of \(\mu_h' \geq \bar{\mu}\), the continuation value is given by
\[
\beta \mu [V (\mu_h'; \bar{\pi}) - V (0; \bar{\pi})] + \beta \left( \frac{\mu}{\mu_h'} - \mu \right) [V (\mu_h'; \bar{\pi}) - V (0; \bar{\pi})] + \beta (1 - \mu) V (0; \bar{\pi}) + \beta \mu V (0; \bar{\pi}) =
\]
\[
\beta \mu (\bar{\pi} - \pi) v \frac{(\mu_h' - \bar{\mu})^2}{(1 - \bar{\mu}) \mu_h'} + \beta \left( \frac{\mu}{\mu_h'} - \mu \right) \left( \bar{\pi} - \pi \right) v \frac{\mu_h' - \bar{\mu}}{1 - \mu} + \beta (1 - \mu) V (0; \bar{\pi}) + \beta \mu V (0; \bar{\pi}) =
\]
\[
\beta \mu_h' \frac{\bar{\mu}}{1 - \mu_h'} (\bar{\pi} - \pi) v \left[ \frac{\mu_h' - \bar{\mu}}{\mu_h'} + \frac{\mu}{\mu_h'} - \mu \right] + \beta (1 - \mu) V (0; \bar{\pi}) + \beta \mu V (0; \bar{\pi}) =
\]
\[
\beta \mu_h' \frac{\bar{\mu}}{1 - \mu_h'} (\bar{\pi} - \pi) v \mu - \beta (1 - \mu) V (0; \bar{\pi}) + \beta \mu V (0; \bar{\pi})
\]  
(A.29)

The above function is increasing in \(\mu_h'\).

In what follows, we first show that ex-ante welfare in (A.28) is maximized at \(\mu_h' = \bar{\mu}\). This establishes claim 5. In addition, we show that aggregate volume \(T\) is increasing in \(\mu_h' \leq \bar{\mu}\). Since continuation values are increasing in \(\mu_h'\), that welfare is maximized at \(\mu_h' = \bar{\mu}\), implies that volume is also maximized at \(\mu_h' = \bar{\mu}\).

Before studying properties of welfare, note that if we set \(\mu_h' = 1\), given that banks are sufficiently patient, an argument similar to that of proposition 4, shows that \(x_h\) must be negative. Hence, in order to have a partial pooling allocation \(\mu_h'\) must be bounded above. In other words, admissible values of \(\mu_h'\) belong to the interval \([\mu, \bar{\mu}]\) where \(\bar{\mu}\) is such that when
we set $\mu'_{h} = \bar{\mu}$, $\bar{x}_{h}$ is equal to 0.

To study ex-ante welfare, suppose that $\mu'_{h} \geq \bar{\mu}$ and recall that total volume is given by

$$T = \frac{\mu}{\mu'_{h}} (\bar{x}_{h} - 1) + 1$$

and hence, ex-ante welfare can be written as

$$\kappa' + \frac{\mu}{\mu'_{h}} \left( \bar{x}_{h} - 1 \right) c + \beta \frac{\mu'_{h} - \bar{\mu}}{\mu'_{h}} (\bar{\pi} - \bar{\pi}) v \mu$$

where $\kappa$ is independent of $\mu'_{h}$ and we have used (A.29) to rewrite the continuation utilities. We can rewrite the above as

$$\kappa' + c \frac{\mu}{\mu'_{h}} \left( \bar{x}_{h} - 1 - \frac{\bar{\mu} (\bar{\pi} - \bar{\pi}) v}{c} \right) = \kappa' + c \frac{\mu}{\mu'_{h}} \left( \bar{x}_{h} - 1 - \beta \frac{d^{2}}{1 + d} \right) \tag{A.30}$$

where $\kappa'$ is independent of $\mu'_{h}$. As we have shown in part 2, specifically in (A.25), both numerator and denominator of $\bar{x}_{h}$ are linear functions of $y = \frac{1}{\mu_{h}}$. This implies that $\bar{x}_{h} - 1 - \beta d^{2} / (1 + d)$ has the same property. That is $a, a', b, b'$ exists such that

$$\bar{x}_{h} - 1 - \beta d^{2} = \frac{-a + a' \mu}{b - b' \mu} = \frac{-a + a'y}{b - b'y}$$

where $b = \bar{\pi} v - \bar{p} (\mu)$ is positive. Note that $\bar{x}_{h}$ is a decreasing function of $\mu'_{h}$ and hence an increasing function of $y$. This implies that the derivative of the above expression with respect to $y$ is positive, or

$$\frac{b' a - a' b}{(b - b'y)^{2}} \geq 0 \tag{A.31}$$

Now, if we let (A.30) as a function $y$ be called $f (y)$, then we can write

$$f (y) = \kappa' + \mu y \frac{-a + a'y}{b - b'y}$$

and we have

$$f' (y) = \mu \frac{-a + a'y}{b - b'y} + \mu y \frac{b' a - a' b}{(b - b'y)^{2}}$$

$$f'' (y) = \mu \frac{b' a - a' b}{(b - b'y)^{2}} + \mu \frac{b' a - a' b}{(b - b'y)^{2}} + 2 \mu y \frac{(b' a - a' b) b'}{(b - b'y)^{3}}$$

$$= 2 \mu \frac{(b' a - a' b) (b - b'y) + y (b' a - a' b) b'}{(b - b'y)^{3}}$$

$$= 2 \mu b \frac{b' a - a' b}{(b - b'y)^{3}}$$

Note that from (A.31) and the fact that $b > 0$, the above expression is convex. This implies that ex-ante welfare as a function of $\mu'_{h}$, given by $f \left( \frac{1}{\mu_{h}} \right)$ is convex $- f (\cdot)$ as well as $\frac{1}{\mu_{h}}$ are convex functions. Hence, in order to show that welfare is maximized at $\mu'_{h} = \bar{\mu}$, it is sufficient
to show that \( f \left( \frac{1}{\hat{\mu}} \right) \geq f \left( \frac{1}{\bar{\mu}} \right) \) where \( \bar{\mu} \) is such that \( \bar{x}_h = 0 \). Extensive and cumbersome algebra (available upon request) establishes that since banks are sufficiently patient, this inequality is satisfied. Hence, among values of \( \mu'_h \) for which \( \bar{x}_h \geq 0 \), welfare is maximized at \( \mu'_h = \hat{\mu} \).

As we have shown before, when \( \mu'_h \leq \hat{\mu} \), \( \bar{x}_h \) is increasing in \( \mu'_h \). This implies that aggregate volume is increasing in \( \mu'_h \) for values of \( \mu'_h \leq \hat{\mu} \) and hence so is welfare. This establishes the claim that welfare is maximized at \( \mu'_h = \hat{\mu} \).

Finally, note that the above establishes that for values of \( \mu'_h \geq \hat{\mu} \), volume is maximized at \( \mu'_h = \hat{\mu} \).

6. In order to prove the claim, let \( \mu'_h \) be such that \( \Delta (\mu'_h; \pi) = c \). This is the value of \( \mu'_h \) for which \( x_h = 0 \). At this value, \( \bar{x}_h \) is given by

\[
\bar{x}_h = \frac{\left( \frac{\mu}{\mu'_h} - 1 \right) \beta (\mu'_h; \pi) - c + \beta (\mu'_h; \bar{\pi})}{\pi v - c - (\hat{p}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right))} = \frac{\beta (\mu'_h; \bar{\pi})}{\pi v - c - (\hat{p}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right))} > 0
\]

This establishes the claim.

Q.E.D.

A.6.1 Establishing the constraint (A.18)

We establish the result by showing that \( \bar{x}_h (\mu'_h, v) \) is less than the right side of (A.18). Using (A.25), we have that

\[
\bar{x}_h (\mu'_h, v) = \frac{\beta \left( \frac{\pi - \bar{\pi} (\pi - \bar{\pi})}{1 - \mu} \right) \left( 1 - \frac{\mu}{\mu'_h} \right) + c \left( 1 - \frac{\mu}{\mu'_h} \right)}{\pi v - c - (\hat{p}(\mu) + c \left( \frac{\mu}{\mu'_h} - 1 \right))}
\]

so that using (21) and simplifying

\[
\bar{x}_h (\mu'_h, v) = \frac{\beta (\mu - \bar{\mu}) \frac{\mu}{\mu'_h} (\pi - \bar{\pi}) v + (\mu'_h - \mu) c}{\mu' (\mu^* - \mu) (\pi - \bar{\pi}) v + (\mu' - \mu) c}
\]

Further algebra shows that \( \bar{x}_h (\mu'_h, v) \) is increasing in \( \mu \). Since the right side of (A.18) equals \( \beta \frac{1 - \mu'_h}{1 - \mu} \), it is sufficient to show the following inequality holds.

\[
\frac{\beta (\mu^* - \bar{\mu}) \frac{\mu}{(1 - \mu) (\pi - \bar{\pi}) v + (\mu' - \mu^*) c}{(\mu' - \mu^*) c} \leq \beta \frac{1 - \mu'}{1 - \mu}. \tag{A.32}
\]

Straightforward algebra shows that the left side of (A.32) is concave and decreasing in \( \mu' \) (all the derivations are available upon request). Since the right side of (A.32) is linear in \( \mu' \),
it is sufficient to show that the above inequality is satisfied at $\mu' = \bar{\mu}$ (the lowest possible value of $\mu'$) and the derivative of the left side is also lower than the derivative of the right side. This result implies that the inequality in (A.32) holds for all values of $\mu' \geq \bar{\mu}$. The value of the left side at $\mu' = \bar{\mu}$ is given by 1 while the value of the right side is given by $\beta$.

By the assumption that banks are sufficiently patient, $\beta \geq 1$. The derivative of the left side is given by

$$-\beta \frac{(\bar{\pi} - \pi) v}{c} \frac{1}{1 - \bar{\mu}} \frac{(\bar{\mu} - \mu^*)^2}{(\mu' - \mu^*)^2}$$

Evaluating the derivatives of the left and right sides of (A.32) at $\mu' = \bar{\mu}$, we obtain the following needed inequality,

$$-\beta \frac{(\bar{\pi} - \pi) v}{1 - \bar{\mu}} \frac{1}{c} \leq -\frac{\beta}{1 - \bar{\mu}}$$

which holds under our assumption that $(\bar{\pi} - \pi) v/c = d > 1$.

Q.E.D.

A.7 Proof of Lemma 3

Lemma 6 in section A.6, establishes that if (26) is not binding, volume is maximized when $\mu'_h = \bar{\mu}$ and $x_h = \bar{x}_h$. Thus, if (26) is binding, then $\bar{x}(\bar{\mu}, v) < \bar{x}(\bar{\mu}, v)$. Also, in lemma 6 above, we have shown that $\bar{x}(\mu'_h, v)$ is a decreasing function of $\mu'_h$ and $\bar{x}(\mu'_h, v)$ is increasing when $\mu'_h \leq \bar{\mu}$. Thus for all values $\mu'_h \leq \bar{\mu}$, $\bar{x}(\mu'_h, v) < \bar{x}(\mu'_h, v)$. Thus, if (26) is binding, $\mu'_h > \bar{\mu}$.

Q.E.D.

A.8 Proof of Proposition 5

Note that the upper bound in (25) is decreasing in $v$. If (26) is not binding, $\mu'_h = \bar{\mu}$ and does not change with $v$, so that an increase in $v$ lowers total volume. Suppose (26) is binding. As it is clear from the formula in (26), the lower bound on $x_h$ is a decreasing function of $v$. Furthermore, as we have established in lemma 6 above, an increase in $v$ decreases the upper bound in (25). Since the upper bound is always binding in the maximal trade equilibrium, $\mu'_h$ should change in order to satisfy both the upper and the lower bound. In particular, $\mu'_h$ should increase so as to satisfy both (25) and (26). As we show in lemma 6, aggregate volume is decreasing in $\mu'_h$. This implies that the change in $\mu'_h$ decreases volume even further.

Q.E.D.

53
A.9 Basic Properties of Mixed Strategy Allocations

Lemma 7 Consider an allocation \((x_h, t_h, x_l, t_l)\) with mixed strategies \((\alpha_l, \alpha_h)\) in which both types mix. Then,

1. If \(\tilde{\mu} \leq \mu'_l \leq \mu \leq \mu_h\), then ex-ante welfare is given by:
\[
\kappa - (1 - x_l)c - \beta d^2 \frac{\frac{1}{\mu'_l} - \frac{1}{\mu'_h}}{1 + dc} \mu_h \mu
\]
where \(\kappa\) is a constant and independent of \(x_l, \mu'_l\) and \(\mu'_h\).

2. If \(\mu'_l \leq \tilde{\mu} \leq \mu'_h\), then ex-ante welfare is given by
\[
\kappa' - c (1 - x_l) - \beta dc \frac{\mu'_h - \tilde{\mu}}{\mu'_h} (1 - \mu) (1 - \alpha_l)
\]
where \(\kappa'\) is a constant and independent of \(x_l, \mu'_l\) and \(\mu'_h\).

Proof. Substituting for \(t_h\) and \(t_l\) from the incentive constraints with equality and the zero profit constraint, we obtain that, ex-ante welfare is given by
\[
\hat{\mu}(\mu) - \mu (\alpha_h (1 - x_h) c + (1 - \alpha_h) (1 - x_l) c) - (1 - \mu) (\alpha_l (1 - x_l) c + (1 - \alpha_l) (1 - x_h) c)
+ \beta \mu [\alpha_h \hat{V}(\mu'_h; \bar{\pi}) + (1 - \alpha_h) \hat{V}(\mu'_l; \bar{\pi})] + \beta (1 - \mu) [\alpha_l \hat{V}(\mu'_l; \bar{\pi}) + (1 - \alpha_l) \hat{V}(\mu'_h; \bar{\pi})]
\] (A.33)

Adding the incentive constraints for the banks we obtain
\[
x_h - x_l = \beta [\hat{V}(\mu'_h; \bar{\pi}) - \hat{V}(\mu'_l; \bar{\pi})] - \beta [\hat{V}(\mu'_h; \bar{\pi}) - \hat{V}(\mu'_l; \bar{\pi})]
\]
Now consider case 1 where \(\mu'_l \geq \tilde{\mu}\). Then, we can use the above to solve for \(x_h\) and replace in the (A.33). Using (10), we arrive at the formula stated above. A similar procedure provides the formula for the case with \(\mu'_l \leq \tilde{\mu} \leq \mu'_h\).

Q.E.D.

A.10 Proof of Proposition 6

We first show that any feasible allocation with both types mixing yields lower ex-ante utility than the utility in the full trade allocation. With both types mixing, without loss of generality \(\mu'_h \geq \mu \geq \mu'_l\). Suppose that \(\mu'_l = \mu'_h\). Then continuation values must satisfy \(V(\mu'_l, \pi) = V(\mu'_l, \pi)\) so that (16), and (17) imply \(x_h = x_l\) as well as \(t_h = t_l\). Hence, ex-ante welfare is highest at the full trade allocation.

Suppose, next, that \(\mu'_l \geq \tilde{\mu}\). From Lemma 7 we have that welfare is given by
\[
\kappa - c (1 - x_l) + \beta \frac{d^2 \mu_h}{1 + dc} \frac{1}{\mu'_l} \left( \frac{1}{\mu'_l} - \frac{1}{\mu} \right)
\]

54
Since $\mu'_{l} \leq \mu$, welfare is maximized at $\mu'_{l} = \mu$ and $x_{l} = 1$ which from (18) implies that $\mu'_{h} = \mu$. This allocation coincides with the full trade allocation.

Now suppose that $\mu'_{l} \leq \tilde{\mu}$. Here welfare is given from Lemma 7 by

$$\kappa' - c \left(1 - x_{h}\right) \frac{\mu}{\mu'_{h}} + \beta \Delta \left(\mu'_{h}; \bar{\pi}\right) \geq x_{h} \left(\bar{\pi} - \bar{\pi}\right) v. \ (A.35)$$

Adding the two incentive compatibility constraints, we have the following upper bound for $x_{h}$,

$$\left(\bar{\pi} - \bar{\pi}\right) v + \beta \left(\Delta \left(\mu'_{h}; \bar{\pi}\right) - \Delta \left(\mu'_{h}; \bar{\pi}\right)\right) \geq x_{h} \left(\bar{\pi} - \bar{\pi}\right) v. \ (A.35)$$

Using the upper bound on $x_{h}$ implied by (A.35) and grouping terms, we have:

$$W \leq \kappa + \beta \left(\frac{\mu'_{l} - \bar{\pi}}{1 - \bar{\pi}} \mu\right) \frac{\mu}{\mu'_{h}} \left[-c \frac{\mu}{\mu'_{h}} - \left(\bar{\pi} - \bar{\pi}\right) v \frac{\bar{\pi}}{\mu'_{h}} + \left(\bar{\pi} - \bar{\pi}\right) v\right]$$

where $\kappa = \hat{p} \left(\mu\right) + \beta \mu V \left(0; \bar{\pi}\right) + \beta \left(1 - \mu\right) V \left(0; \bar{\pi}\right)$. Since $\bar{\mu} = (\bar{\pi} - \bar{\pi}) v / (c + (\bar{\pi} - \bar{\pi}) v)$ and $\mu'_{h} \leq 1$, it follows that

$$W \leq \hat{p} \left(\mu\right) + \beta \mu V \left(0; \bar{\pi}\right) + \beta \left(1 - \mu\right) V \left(0; \bar{\pi}\right),$$

$$\leq \hat{p} \left(\mu\right) + \beta \mu V \left(\mu; \bar{\pi}\right) + \beta \left(1 - \mu\right) V \left(\mu; \bar{\pi}\right)$$

where the last inequality follows because $V \left(\mu; \pi\right)$ is an increasing function of $\mu$.

Q.E.D.

A.11 Proof of Proposition 7

In section A.11.1 below, we show that high type does not mix in the efficient allocation. Hence, we only need to consider allocations in which the low type mixes. As shown in lemma 6, when (20) is binding, aggregate welfare as a function of $\mu'_{l}$ is maximized at $\mu'_{h} = \tilde{\mu}$. Since in any ex-ante efficient allocation with $\mu \leq \mu^{*}$, (20) must be binding, we have established that any efficient allocation must have $\mu'_{h} = \tilde{\mu}$. Since the continuation value functions are
constant for $\mu \leq \tilde{\mu}$, it follows that (26) is satisfied in the efficient allocation if

$$\frac{c_{\mu}(1-\mu)}{\bar{\pi} - \bar{\pi}} v + c_{\tilde{\mu}(\mu - \mu)} \leq \frac{c_{\tilde{\mu}(\mu - \mu)}(1-\mu)}{\bar{\pi} - \bar{\pi}} v + c_{\tilde{\mu}(\mu - \mu)}$$

Straightforward algebra show that the above inequality is equivalent to

$$\frac{2}{1 - \mu} - \frac{1}{1 - \tilde{\mu}} \leq \frac{1}{1 - \mu}$$

or, substituting for $\mu^*$ and $\tilde{\mu}$ in terms of $d$, we obtain

$$d - 1 \leq \frac{1}{1 - \mu} \quad \text{(A.36)}$$

The inequality in (A.36) holds for all values of $\mu$ when $d \leq 2$. If $d \geq 2$, then there is a threshold $\hat{\mu} (d)$ defined by equality in (A.36) such that the maximal trade equilibrium is efficient for $\mu \geq \hat{\mu} (d)$ and inefficient for $\mu \leq \hat{\mu} (d)$.

Q.E.D.

A.11.1 No Mixed Strategy by the High Quality Bank

Here we show that, when $\mu \leq \mu^*$, the high-quality bank does not mix in the efficient allocation. Consider an allocation where $\alpha \in (0, 1)$ so that the high quality bank uses a mixed strategy. We show that this allocation can be perturbed so that ex-ante welfare is higher and hence, cannot be an ex-ante efficient. First notice that an argument similar to that of proposition 4 implies that the low quality bank should be mixing as well. Furthermore, note that the allocation cannot be a pooling allocation since a pooling allocation does not satisfy (20). Given that the allocation is not pooling, we must have $\alpha_h > 1 - \alpha_l$. Furthermore, we must have $\mu^*_h > \tilde{\mu} > \mu > \mu^*_l$. If not, $V (\mu^*_h; \bar{\pi}) = V (\mu^*_l; \pi)$ for $\pi = \bar{\pi}, \pi$ and the incentive compatibility constraints imply that $x_h = x_l$ and $t_h = t_l$ - a pooling allocation.

Before showing how to perturb the allocation, it is useful to write the payoff for the buyers as

$$0 = \mu \alpha_h (\bar{\pi} v x - t_h) + (1 - \mu) (1 - \alpha_l) (\bar{\pi} v x - t_h) + (1 - \mu) (1 - \alpha_l) + \mu (1 - \alpha_h) (\bar{\pi} v x - t_l) = (\mu + (1 - \mu) \alpha_l) (\hat{\mu} x - t_h) + (\mu (1 - \alpha_l) + (1 - \mu) \alpha_l) (\hat{\mu} x - t_l) \quad \text{(A.37)}$$

Now we consider three possible cases:

1. Suppose that $\hat{\mu} (\mu^*_h) x_h - t_h > \hat{\mu} (\mu^*_l) x_l - t_l$. Consider the following allocation: $(x_h, t_h, x_l, t_l)$ with mixing probabilities given by $\alpha_h (1 + \varepsilon)$ and $\alpha_l - \varepsilon (1 - \alpha_l)$ for high and
low quality types respectively and for a small value of \( \varepsilon > 0 \). Given this allocation, future beliefs are given by
\[
\tilde{\mu}_h = \frac{\mu \alpha_h (1 + \varepsilon)}{\mu \alpha_h (1 + \varepsilon) + (1 - \mu) (1 - \alpha_l) (1 + \varepsilon)} = \mu'_{h},
\]
\[
\tilde{\mu}_l = \frac{\mu (1 - \alpha_h (1 + \varepsilon))}{\mu (1 - \alpha_h (1 + \varepsilon)) + (1 - \mu) (\alpha_l - \varepsilon (1 - \alpha_l))} = \frac{\mu (1 - \alpha_l) - \varepsilon \mu \alpha_h - \varepsilon (1 - \mu) (1 - \alpha_l)}{\mu (1 - \alpha_h) + (1 - \mu) \alpha_l - \varepsilon \mu \alpha_h - \varepsilon (1 - \mu) (1 - \alpha_l)} < \mu'_{l} \leq \mu \leq \mu^{*} < \tilde{\mu}
\]
where the last inequality follows from the fact that \( 1 - \alpha_l < \alpha_h \). Therefore, we have
\[
V(\mu'_{l};\pi) = V(\tilde{\mu};\pi)
\]
so that the payoffs to the banks are the same as in the original allocation. The buyer’s payoff given the new allocation is given by
\[
(1 + \varepsilon) (\mu \alpha_h + (1 - \mu) (1 - \alpha_l)) (\hat{p}(\mu'_{h}) x_h - t_h)
+ (\mu (1 - \alpha_h) + (1 - \mu) \alpha_l - \varepsilon \mu \alpha_h - \varepsilon (1 - \mu) (1 - \alpha_l)) (\hat{p}(\mu'_{l}) x_l - t_l)
\]
Since
\[
\hat{p}(\mu'_{h}) x_h - t_h > \hat{p}(\mu'_{l}) x_l - t_l,
\]
the above expression must be higher than that in A.37 since it puts a higher weight on \( \hat{p}(\mu'_{h}) x_h - t_h \). This implies that this is a pareto improving allocation—we can use the extra revenues by buyers and increase transfers.

2. Suppose that \( \hat{p}(\mu'_{h}) x_h - t_h < \hat{p}(\mu'_{l}) x_l - t_l \). In this case, a perturbation of the form \((x_h, t_h, x_l, t_l, \alpha_h (1 - \varepsilon), \alpha_l + \varepsilon (1 - \alpha_l))\) Pareto improves the original allocation. The proof is similar to the first case.

3. Finally, suppose that \( \hat{p}(\mu'_{h}) x_h - t_h = \hat{p}(\mu'_{l}) x_l - t_l = a \geq 0 \). Then, we have the following incentive constraints:
\[
t_h + (1 - x_h) (\bar{\pi}_v - c) + \beta V(\hat{\mu}_h;\bar{\pi}) = t_l + (1 - x_l) (\bar{\pi}_v - c) + \beta V(\hat{\mu}_l;\bar{\pi})
\]
Using the assumption above, the constraint becomes
\[
\hat{p}(\mu'_{h}) x_h - a + (1 - x_h) (\bar{\pi}_v - c) + \beta V(\mu'_{h};\bar{\pi}) = \hat{p}(\mu'_{l}) x_l - a + (1 - x_l) (\bar{\pi}_v - c) + \beta V(\mu'_{l};\bar{\pi})
\]
After some algebra we have
\[
(\hat{p}(\mu'_{h}) - \bar{\pi}_v + c) x_h + \beta (V(\mu'_{h};\bar{\pi}) - V(\mu'_{l};\bar{\pi})) = x_l (\hat{p}(\mu'_{l}) - \bar{\pi}_v + c)
\]
Note that \( \mu'_{l} < \mu \leq \mu^{*} < \tilde{\mu} < \mu'_{h} \). Therefore, \( \hat{p}(\mu'_{h}) > \bar{\pi}_v - c > \hat{p}(\mu'_{l}) \). Therefore, the left side of the above equality is positive while the right side is negative which is a contradiction.
Q.E.D.

**A.12 Proof of Proposition 9**

Consider the pooling equilibrium \((x, x\hat{p}(\mu))\) where \( x \in (\underline{x}(\mu), 1) \). Furthermore, suppose that beliefs following a sale of loans to the government is given by \( \mu' = 0 \). As we have shown in
the proof of Lemma 1, (A.7) implies that neither type chooses \((1, \hat{p}(\mu))\) when its associated belief is given by \(\mu' = 0\).

Q.E.D.

### A.13 Proof of Proposition 11

This equilibrium is supported by beliefs which specify that if any bank accepts a contract \((x, xp)\) where \(x \in \{0, 1\}\) and

\[
\mu'(\hat{x}, \hat{x}p) = \begin{cases} 
1 & \text{if } \hat{x}\hat{p} + (1 - \hat{x})(\pi v - c) \geq x^*(\mu, v)\hat{p}(\mu, v) + (1 - x^*(\mu, v))(\pi v - c) \\
0 & \text{o.w.}
\end{cases}
\]

where \(x^*(\mu, v)\) specifies the equilibrium action of the high-quality bank, which is either 1 if \(\mu \geq \mu^*(v)\) or 0 if \(\mu \leq \mu^*(v)\).

To see that this is an equilibrium, we start by showing that \(\mu_h\) exists and that the value functions for each bank quality are increasing in reputation. Clearly, if \(\mu = 0\), then

\[
V(0; \pi) = (\pi Ev)/(1 - \beta(1 - \lambda)) + \beta\lambda W.
\]

Suppose \(\mu \leq \mu_h\) and define \(v^*(\mu) = (1 - \mu)/(\pi - \pi)\) (note that \((\mu^*)^{-1}(\mu) = v^*(\mu)\)) so that if \(v \leq v(\mu)\) then \(\mu \geq \mu^*(v)\). Recall that when \(v\) is below \(v(\mu)\) so that \(\mu \geq \mu^*(v)\), the bank sells its loan portfolio at the pooling price and when \(v\) is above \(v(\mu)\) so that \(\mu \leq \mu^*(v)\) (and \(\mu \leq \mu_h\)) the bank is indifferent between selling at price \(\pi v\) and receiving a reputation of 0 and holding its loan and receiving a reputation of \(\mu_h\). The value function for the low-quality bank when \(\mu \leq \mu_h\) is then given by

\[
V(\mu; \pi) = \int_{v_l}^{v^*(\mu)} [\hat{p}(\mu, v) + \beta(1 - \lambda) V(\mu; \pi)] dG(v) + \int_{v^*(\mu)}^{v_{\text{max}}} [\pi v + \beta(1 - \lambda) V(0; \pi)] dG(v) + \beta\lambda W.
\]

(A.38)

Straightforward algebra then implies that for \(\mu \leq \mu_h\),

\[
V(\mu; \pi) = \frac{\pi Ev}{1 - \beta(1 - \lambda)} + \frac{G(v^*(\mu))}{1 - \beta(1 - \lambda) G(v^*(\mu))} \mu_h (\pi - \pi) E[v|v \leq v^*(\mu)] \quad (A.39)
\]

A consequence of (A.39) is that in this equilibrium, the value \(\mu_h\) always exists. To see this, substitute for \(V(\mu; \pi)\) into (A.38) which implies that \(\mu_h\) must be chosen so that

\[
c = \frac{\beta(1 - \lambda) G(v^*(\mu_h))}{1 - \beta(1 - \lambda) G(v^*(\mu_h))} \mu_h (\pi - \pi) E[v|v \leq v^*(\mu_h)].
\]

(A.40)

Note that \(v^*(1) = \infty\), \(v^*(0) = v_l\), and the right side of (A.40) is strictly increasing in \(\mu_h\). At \(\mu_h = 0\), the right side is equal to \(0 < c\) while at \(\mu_h = 1\), the right side is given by

\[
\beta(1 - \lambda)/(1 - \beta(1 - \lambda)) E_v \pi > c
\]

where the inequality follows from the assumption that \(\beta(1 - \lambda)/(1 - \beta(1 - \lambda)) > c + (\pi - \pi) v_{\text{max}}/(\pi - \pi) E_v\). Hence, \(\mu_h\) exists and is unique.
To finish the expression of the low-quality bank’s value function, recall that when \( \mu > \mu_h \), when \( v \) is below \( v^*(\mu) \), the bank sells its loan portfolio at the pooling price and when \( v \) is above \( v^*(\mu) \) the bank holds its loan portfolio. The value function of the low-quality bank for \( \mu > \mu_h \) then satisfies

\[
V(\mu; \pi) = \int_{v_l}^{v^*(\mu)} [\hat{p}(\mu, v) + \beta (1 - \lambda) V(\mu; \pi)] dG(v) + \int_{v^*(\mu)}^{v_{\max}} [\pi v + c + \beta (1 - \lambda) V(\mu; \pi)] dG(v) + \beta \lambda W.
\]

Straightforward algebra implies that for such \( \mu \),

\[
V(\mu; \pi) = \frac{1}{1 - \beta (1 - \lambda)} [G(v^*(\mu)) [\mu (\pi - \pi) E[v|v \leq v^*(\mu)] + c] + \pi Ev - c + \beta \lambda W]
\]

which is clearly increasing in \( \mu \). Moreover, since for \( \mu < \mu_h \) we can express the value function as

\[
V(\mu; \pi) = \int_{v_l}^{v(\mu)} [\hat{p}(\mu, v) + \beta (1 - \lambda) V(\mu; \pi)] dG(v) + \int_{v(\mu)}^{v_{\max}} [\pi v + c + \beta (1 - \lambda) V(\mu_h; \pi)] dG(v) + \beta \lambda W
\]

it is immediate that \( V(\mu; \pi) \) is increasing in \( \mu \).

Next consider the value for the high-quality bank and show that it is also increasing in \( \mu \). When \( \mu \leq \mu_h \),

\[
V(\mu; \pi) = \int_{v_l}^{v(\mu)} [\hat{p}(\mu, v) + \beta (1 - \lambda) V(\mu; \pi)] dG(v) + \int_{v(\mu)}^{v_{\max}} [\pi v + c + \beta (1 - \lambda) V(\mu_h; \pi)] dG(v) + \beta \lambda W
\]

when \( \mu > \mu_h \),

\[
V(\mu; \pi) = \int_{v_l}^{v(\mu)} [\hat{p}(\mu, v) + \beta (1 - \lambda) V(\mu; \pi)] dG(v) + \int_{v(\mu)}^{v_{\max}} [\pi v + c + \beta (1 - \lambda) V(\mu_h; \pi)] dG(v) + \beta \lambda W.
\]

It is immediate from these expressions that \( V(\mu; \pi) \) is also increasing in \( \mu \).

We have established that \( \mu_h \) exists and the value functions for both quality types are increasing in reputation \( \mu \). Consider now possible deviations by buyers when facing a bank with reputation \( \mu \) and current collateral value \( v \) and show for all \( \mu \) and \( v \), there are no profitable deviation offers by buyers. Clearly, if a profitable deviation exists, it must satisfy \( p' \leq \hat{p}(\mu, v) \) since for all \( p' > \hat{p}(\mu, v) \) such deviations at best attract both high- and low-quality banks and earn negative profits. We partition the \((\mu, v)\) space into cases. First suppose that \( \mu \geq \mu^*(v) \). In this equilibrium outcome, both banks sell at price \( \hat{p}(\mu, v) \). At any price \( p' \) higher than \( \hat{p}(\mu, v) \), since \( \mu'(1, p') = 1 \), both banks would accept the offer but such an offer would then earn negative profits. At any price \( p' \) below \( \hat{p}(\mu, v) \), both banks would obtain less static payoff and a reputation of 0 so such offers attract no banks. Thus, when \( \mu \geq \mu^*(v) \), there are no profitable deviations by buyers.

Next suppose that \( \mu < \mu^*(v) \). For any price \( p' \leq \hat{p}(\mu, v) \), the high type prefers the
equilibrium since
\[
\bar{\pi} v - c + \beta (1 - \lambda) V(\mu; \bar{\pi}) > \hat{p}(\mu, v) + \beta (1 - \lambda) V(\mu; \bar{\pi}) \geq p' + \beta (1 - \lambda) V(0; \bar{\pi})
\]
since \(\bar{\pi} v - c > \hat{p}(\mu, v)\) and \(V(\mu; \bar{\pi})\) is increasing in \(\mu\). Thus, when \(\mu < \mu^*(v)\), if the deviation satisfies \(\bar{\pi} v < p' \leq \hat{p}(\mu)\), it must earn non-positive profits since at best it attracts no banks and earns zero profits.

Suppose further that \(\mu_h < \mu < \mu^*(v)\). We now show that no deviation with \(p' \leq \bar{\pi} v\) attracts low-quality banks since if they accept such a deviation offer they would obtain a reputation of 0. To see this, note that at any price \(p' \leq \bar{\pi} v\), the low-quality bank prefers the equilibrium since
\[
\bar{\pi} v - c + \beta (1 - \lambda) V(\mu_h; \bar{\pi}) = \bar{\pi} v + \beta (1 - \lambda) V(0; \bar{\pi})
\]
Finally, suppose instead that \(\mu \leq \min \{\mu_h, \mu^*(v)\}\). We again show that no deviation with \(p' \leq \bar{\pi} v\) attracts the low-quality bank. In this case, the low-quality bank obtains utility equal to \(\bar{\pi} v + \beta (1 - \lambda) V(0; \bar{\pi}) + \beta \lambda W\). Hence, at price any \(p' \leq \bar{\pi} v\), the deviating buyer earns at most zero profits.

Q.E.D.

A.14 Infinite Horizon Model with Divisible Assets

In this section, we show that the equilibrium constructed in section 5 is also an equilibrium if we allow for trade of a fraction of bank portfolios. To do so, we make the following additional assumption:
\[
\frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} \geq \frac{(\bar{\pi} - \pi) v_{\text{max}}^2}{c \hat{E} v}
\]
As in the indivisible asset case, we investigate whether the following is an equilibrium:

if \(\mu_t < \min \{\mu^*(v_t), \mu_h\}\), \((x_t, t_t) = (0, (1, \bar{\pi} v_t))\)

if \(\mu_t \geq \mu^*(v_t), (x_t, t_t) = (x_t, t_t) = (1, \hat{p}(\mu_t) v_t)\)

if \(\mu_t \in (\mu_h, \mu^*(v))\), \((x_t, t_t) = (x_t, t_t) = (0, 0)\)

where \(\hat{p}(\mu_t)\) with the little abuse of notation is given by \(\bar{\pi} \mu_t + \pi (1 - \mu_t)\) and \(\mu^*(v) = 1 - \frac{c}{(\bar{\pi} - \pi) v}\). Since the allocations are identical, the value function is identical as well. We let the beliefs be
\[
\mu'(x, t; v_t, \mu_t) = \left\{ \begin{array}{ll}
0 & t - x (\bar{\pi} v_t - c) \leq \max \{\hat{x} \hat{p}(\mu_t) v_t - \hat{x} (\bar{\pi} v_t - c), t_h - x_h (\bar{\pi} v_t - c)\} \\
1 & \text{o.w.}
\end{array} \right.
\]
In what follows, we check that the above strategies as well as the beliefs constitute an equilibrium.
1. We start from the case where $\mu^*(v_i) \leq \mu_h$. Suppose that $\mu < \mu^*(v)$. Consider some deviation, $z' = (x'_h, t'_h, x'_l, t'_l)$. As in the proof of lemma 2 and 1, both of the contracts cannot be preferred statically to $(0, 0)$ by the high quality bank. Suppose $(x'_h, t'_h)$ is statically preferred to $(0, 0)$ by the high quality bank. We first show that this contract must also attract the low type. To see this note that we must have

$$\beta (1-\lambda) \left( \frac{\pi E v}{1-\beta (1-\lambda)} - V (\mu; \bar{\pi}) \right) \geq x'_h (\pi v - c) - t'_h$$

which implies that the low type also prefers $(x'_h, t'_h)$ to $(0, 0)$. As in the proof of lemma 2 and 1, we cannot have $\mu' (x'_l, t'_l) = \mu' (x'_l, t'_l) = 1$. In addition, if $\mu' (x'_l, t'_l) = 0$, we can assume that $x'_l = 1$ and we must have

$$t'_l + \beta (1-\lambda) V (0; \bar{\pi}) \geq t'_h + (1 - x'_h) (\pi v - c) + \beta (1-\lambda) V (1; \bar{\pi})$$

So the profits from this deviation are given by

$$\mu (x'_h \pi v - t'_h) + (1 - \mu) (\pi v - t'_l) \geq \mu (x'_h \pi v - t'_h) + (1 - \mu) (\pi v - t'_h - (1 - x'_h) (\pi v - c) - c)$$

$$\mu (x'_h \pi v - t'_h) + (1 - \mu) (x'_h (\pi v - c) - t'_h)$$

$$= x'_h \hat{p} (\mu) v - t'_h - c (1 - \mu) x'_h$$

Since $\mu^*(v) > \mu$ and $(x'_h, t'_h)$ is statically preferred to $(0, 0)$ by the high quality bank, it must be that $\hat{p} (\mu) x'_h \leq t'_h$ and hence the above expression is negative.

Now, suppose that $\mu \geq \mu^*(v)$. As we have done before, it is sufficient to show that any contract that attracts the high type also attracts the low type. To do this, consider the offer $(x', t')$ that gives the lowest static payoff the bank of low quality and is statically preferred to $(1, \hat{p} (\mu))$ by the high quality bank. Such offer satisfies:

$$t' = \bar{\pi} v x'$$

$$x' \pi v + (1 - x') (\pi v - c) = \hat{p} (\mu) v$$

$$x' c = c - (1 - \mu) (\bar{\pi} - \bar{\pi}) v$$
It is sufficient to show that the low quality bank prefers this offer to the pooling outcome:

\[
\begin{align*}
    x'\bar{\pi}v + (1 - x')(\pi v - c) + \beta V (1; \bar{\pi}) & \geq \mu (\bar{\pi} - \pi) v + c + \beta V (\mu; \bar{\pi}) \\
    1 - (1 - \mu) \left( \frac{(\bar{\pi} - \pi)}{c} v \right) & \geq \mu (\bar{\pi} - \pi) v + c + \beta V (\mu; \bar{\pi}) - \frac{\beta}{1 - \beta} \bar{\pi} Ev \\
    (\bar{\pi} - \pi) v (1 - \mu) - (1 - \mu) (\bar{\pi} - \pi) v & \frac{(\bar{\pi} - \pi)}{c} v + c \geq \beta V (\mu; \bar{\pi}) - \frac{\beta}{1 - \beta} \bar{\pi} Ev \\
    \frac{(1 - \mu) ((\bar{\pi} - \pi) v)^2}{c} & \leq \frac{\beta}{1 - \beta} \bar{\pi} Ev - \beta V (\mu; \bar{\pi}) , \forall \mu \in [\mu^* (v) , 1]
\end{align*}
\]

We will show later that this inequality is satisfied.

2. Now, suppose that \( \mu^* (v) \geq \mu_h \). For \( \mu^* (v) < \mu \), the proof is the same as before. So we only need to consider the case with \( \mu_h \leq \mu < \mu^* (v) \). It is clear that no offer below the market odds line attracts the high quality bank. Furthermore, as before we can show that an offer of \((x'_h, t'_h)\) that is above the market odds line attracts the low type as well and hence the accompanying contract \((1, t'_l)\) must be at least as good as \((x'_h, t'_h)\). That is

\[
    t'_l + \beta V (0; \bar{\pi}) \geq t'_h + (1 - x'_h) (\pi v - c) + \beta V (1; \bar{\pi})
\]

So as before, we can show that this contract cannot make positive profits.

To conclude the proof, we show that the following inequality is satisfied:

\[
\frac{(1 - \mu) ((\bar{\pi} - \pi) v)^2}{c} \leq \frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} \bar{\pi} Ev - \beta (1 - \lambda) V (\mu; \bar{\pi}) , \forall v, \forall \mu \in [\mu^* (v) , 1]
\]

Note that the RHS of the above inequality is independent of \( v \), so we have to check the above inequality at highest possible value of \( v \). Now there are two possibilities. Either, \( \mu > \mu^* (v) \) in which case we only have to show that the inequality holds for \( v = \bar{v} \) or \( \mu \leq \mu^* (v) \) in which case we have to show the inequality for \( v = v^* (\mu) \). Suppose that \( v = \bar{v} \) and that \( \mu > \mu^* (v) \). Note that since \( v^* (\mu) > \bar{v} \), the equilibrium calls for selling at pooling price all the time. Hence, the above inequality becomes

\[
\frac{(1 - \mu) ((\bar{\pi} - \pi) v)^2}{c} \leq \frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} (1 - \mu) (\bar{\pi} - \pi) Ev
\]

Given our assumption above, this inequality is satisfied. Now, let us turn our attention to the second case where \( \mu \leq \mu^* (v) \). In this case, the inequality becomes

\[
\frac{c}{1 - \mu} \leq \frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} \bar{\pi} Ev - \beta (1 - \lambda) V (\mu; \bar{\pi}) , \forall v, \forall \mu \in [\mu^* (v) , 1]
\]

where the LHS is evaluated at \( v = v^* (\mu) \). We can re-arrange the terms and write the above inequality as

\[
\frac{c}{1 - \mu} + \beta (1 - \lambda) V (\mu; \bar{\pi}) \leq \frac{\beta (1 - \lambda)}{1 - \beta (1 - \lambda)} \bar{\pi} Ev
\]
Since the LHS is an increasing function of $\mu$, we need to show this inequality to be satisfied only at $\mu = \mu^*(\bar{v})$. Note that for $\mu = \mu^*(\bar{v})$, the equilibrium involves selling by all types all the times, hence $V(\mu; \pi) = \frac{(\mu\pi + (1-\mu)\bar{\pi})Ev}{1-\beta(1-\lambda)}$ and we can write

$$\frac{c}{1 - \mu^*(\bar{v})} = \frac{(1 - \mu^*(\bar{v}))((\bar{\pi} - \pi)\bar{v})^2}{c}$$

Hence, the above inequality can be written as

$$\frac{(1 - \mu^*(\bar{v}))((\bar{\pi} - \pi)\bar{v})^2}{c} \leq \beta (1 - \lambda) \frac{(1 - \mu^*(\bar{v}))((\bar{\pi} - \pi)\bar{v})^2}{1 - \beta (1 - \lambda)} (1 - \mu^*(\bar{v})) (\bar{\pi} - \pi) Ev$$

which is satisfied by our assumption above.

Q.E.D.