PA, and whose well-formed formulae are formed out of the closed formulae of **PA** by use of the connectives &, ¬, and □. (I say '&' and '¬' to indicate that I am using the same conjunction and negation as in **PA** itself, not introducing new ones. See footnote 16, p. 71.) Then **E** is called a reflexive extension of **PA** iff: (1) It is an inessential extension of **PA**; (2) $\Box A$ is provable in **E** iff A is; (3) there is a valuation α, mapping the closed formulae of **E** into the set {**T, F**}, such that conjunction and negation obey the usual truth tables, all the true closed formulae of **PA** get the value **T**, $\alpha(\Box A) =$ **T** iff A is provable in **E**, and all the theorems of **E** get the value **T**. It can be shown that there are reflexive extensions of **PA** containing the axioms of S4 or even S4.1, but none containing S5.

Finally, we remark that, using the usual mapping of intuitionistic logic into S4, we can get a model theory for the intuitionistic predicate calculus. We will not give this model theory here, but instead will mention, for propositional calculus only, a particular useful interpretation of intuitionistic logic that results from the model theory. Let **E** be any consistent extension of **PA**. We say a formula P of **PA** is *verified* in **E** iff it is provable in **E**. We take the closed wffs P of **PA** as atomic, and build formulae out of them using the intuitionistic connectives $\wedge$, $\vee$, ¬, and $\supset$. We then stipulate inductively: $A \wedge B$ is verified in **E** iff A and B are; $A \vee B$ is verified in **E** iff A or B is; $\neg A$ is verified in **E** iff there is no consistent extension of **E** verifying A; $A \supset B$ is verified in **E** iff every consistent extension **E′** of **E** verifying A also verifies B.

Then every instance of a law of intuitionistic logic is verified in **PA**; but, e.g., $A \vee \neg A$ is not, if A is the Gödel undecidable formula. In future work, we will extend this interpretation further, and show that using it we can find an interpretation for Kreisel's system FC of absolutely free choice sequences.[18] It is clear, incidentally, that **PA** can be replaced in the provability interpretations of S4 and S5 by any truth functional system (i.e., by any system whose models determine each closed formula as true or false); while the interpretation of intuitionism applies to any formal system whatsoever.

[18] G. Kreisel, 'A Remark on Free Choice Sequences and the Topological Completeness' Proofs, *Journal of Symbolic Logic*, 23 (1958), 369–88.

VI

ESSENTIALISM AND QUANTIFIED MODAL LOGIC[1]

TERENCE PARSONS

PROBLEMS involving essentialism are now receiving a great deal of attention from modal logicians and philosophers. Even a cursory glance at work in this field, however, soon reveals that there are many doctrines which go by this title. I will isolate and discuss one such doctrine. In particular, after isolating one version of essentialism (Sections I and II), I will argue that work in quantified modal logic can be and is independent of the acceptance of the truth of this doctrine (Sections III–V). In the last section (Section VI) I will attempt to show, on the basis of facts established in Sections III–V, just why this particular form of essentialism is a philosophically suspect doctrine. I will also argue that work in quantified modal logic need not even presuppose the *meaningfulness* of essentialist claims *in any objectionable sense*.

My arguments aim at (a) a clarification of one sort of essentialism, and (b) a partial vindication of quantified modal logic.

I. PRELIMINARY CLARIFICATION

To begin, let us dichotomize essentialist doctrines into two kinds. One kind has to do with what I shall call *individual* essences and the other with what I shall call *general* essences. The former doctrine makes some claim to the effect that some or all objects have characteristics (or properties) which are so intimately associated with the object that nothing else *could* (with emphasis on the 'could') have precisely those characteristics without being that object. This is meant to be a stronger thesis than the Identity of Indiscernibles, which holds merely that no two objects can simultaneously exist while sharing all properties. It is stronger in two ways: (1) it prohibits the simultaneous existence of two objects which share the same individual essence (even when they could differ in other of their properties), and (2) it makes a claim about what *might have been*: had

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[1] In addition to the authors cited in the paper, I am particularly indebted to John Vickers and to Kathryn Pyne Parsons for comments on earlier drafts, and to the referee of *The Philosophical Review* for help in improving the final draft.

the world been different, and had there been an object, b, in it, where b had the individual essence which object a has in this world, then b (in the world which might have been) would have *been* object a. This doctrine of individual essences comes in for discussion in modal logic in the problem of identifying objects in one possible world with objects in other possible worlds.[2]

The doctrine of *general* essences, on the other hand, simply singles out certain characteristics as being necessarily true of certain objects. Distinct objects are not prohibited from sharing the same general essence, as is the case with individual essences. While individual essences completely individuate their bearers, general essences do not (although they may help). The doctrine of general essences is a natural, though not inevitable, extension of the metaphysical doctrine of natural kinds (where natural-kind properties and properties definitional to them are taken as the general essences).

My discussion is concerned wholly with the doctrine of general essences. This doctrine also may take many forms, and the next section is devoted to an exposition of one such form.

II. FORMULATING THE 'TROUBLESOME' KIND OF ESSENTIALISM

Part of the motivation for studying essentialism is a suspicion that two claims about it are true:

(A) that quantified modal logic is committed to essentialism, and
(B) that essentialism is a false, or at least philosophically suspect, doctrine.

If both (A) and (B) are true, they constitute a strong argument against the significance of quantified modal logic. This argument is mainly due to Quine.[3] It is intended to apply specifically to *quantified* modal logic—that is, the kind of essentialism involved is supposed to arise *only* when quantifiers are added to modal logic and allowed to intermingle with modal operators.[4]

In fact, there are many different forms of essentialism and many different forms of modal logic, and until these are made specific, the argument implicit in (A) and (B) cannot be evaluated. Previous attempts to find a precise formulation of a form of essentialism which would make both (A) and (B) true (for some interesting modal logic) have ended with negative conclusions[5]—the forms of essentialism to which quantified modal logic is committed are the forms which are most innocuous. This article is in part another step in that search.

Previous searching has been for a purely syntactical characterization of essentialism. In particular, it has been a search for a schema, S, such that any quantified modal logic endorsing any instance of S would be committed to a suspect version of essentialism. Two interesting schemata were these:

(1) $(\exists x_1)...(\exists x_n)(\Box F \;\;\&\;\; -\Box G)$

and

(2) $(\exists x_1)...(\exists x_n)(\Box F) \;\;\&\;\; (\exists x_1)...(\exists x_n)(-\Box F)$.[6]

Definition (1), which is essentially due to Quine,[7] is not intrinsically more suspect than is the division of closed sentences into necessary and contingent. That a system of modal logic is essential in sense (1) does not entail that it will endorse controversial claims like 'something is necessarily greater than seven'. It may only endorse 'essential' claims like 'something is necessarily either-bald-or-not-bald,' claims whose truth-conditions can be made perfectly precise.[8]

[2] Roughly, an object in one world is identified with an object in another world just in case they have the same individual essence. Cf. J. Hintikka, *Knowledge and Belief* (Ithaca: Cornell Univ. Press, 1962); R. Chisholm, 'Identity Through Possible Worlds: Some Questions', *Nous*, I (1967). This issue was also the central topic of a symposium sponsored jointly by the Western Division A.P.A. and the Association for Symbolic Logic in May, 1967, with D. Kaplan (principal speaker), J. Hintikka, and T. Parsons.

[3] W. Quine, 'Two Dogmas of Empiricism' and 'Reference and Modality' (esp. pp. 143–56), in *From a Logical Point of View* (New York: Harper & Row, 1961) [See Essay I, above]; also 'Three Grades of Modal Involvement', *The Ways of Paradox* (New York, 1966).

[4] i.e., it applies specifically to Quine's *third* grade of modal involvement. Cf. 'Grades of Modal Involvement', Section III.

[5] Cf. R. Marcus, 'Essentialism in Modal Logic', *Nous*, I (1967); also T. Parsons, 'Grades of Essentialism in Quantified Modal Logic', *Nous*, I (1967).

[6] In each case F and G are to be formulas whose only free variables are among $x_1 \ldots x_n$. We also assume that F and G contain no constants. If they contain constants, a more detailed formulation is necessary; see Parsons, op. cit., for details. It should be stressed that schemata (1) and (2) provide a 'syntactic' characterization of forms of essentialism only on the assumption that the quantifiers are interpreted to range over individuals, and not over individual concepts or descriptions. For interpretations of the latter sort, see footnote 12.

[7] 'Grades of Modal Involvement', p. 174. Quine adds an additional clause within the scope of the quantifiers, the clause 'Gx'. The point is that G is a property which objects have, but have contingently. The assumption that some objects have some properties contingently is shared by *all* views under discussion. This particular assumption is not an issue here—cf. footnote 8.

[8] See Parsons, op cit., for a way to do this for one system. The important point is that the 'essentialist' difficulties at issue are supposed to be difficulties *in addition to* the difficulties encountered in making sense of modal operators preceding closed sentences.

Version (2), roughly due to Marcus,[9] is an attempt to formulate a more troublesome kind of essentialism. While version (1) merely says (for case $n = 1$) that an object has some properties necessarily and some contingently, version (2) says that an object has a property necessarily, which other objects *do not* have necessarily. Version (1) will be true as soon as properties are divided into contingent ones and necessary ones, a division that can be made with no more difficulty than the division of propositions (or sentences) into necessary and contingent; but for version (2) to be satisfied we need a more complex categorization. Properties cannot be just necessary or not necessary; they must be necessary *for* this object and not necessary *for* that one.

This notion—that properties can be necessary for some objects but not necessary for others—seems to be precisely that doctrine which is responsible for the troublesomeness of the examples which Quine offers. What worries us about some things being necessarily two-legged[10] is that other things are *not* necessarily two-legged, and thus we cannot attribute the necessity of the two-leggedness to the predicate or property in question. There must be something about the *object* which gives rise to the necessity. But what could this be? The lack, so far, of a satisfactory answer to this question is what makes this version of essentialism a real source of philosophical perplexity.

Unfortunately, version (2), *as stated*, is not a completely adequate syntactic formulation of the 'suspect' version of essentialism. For there are a *few* instances of version (2), which are as untroublesome as instances of version (1). For example, consider the following instance of schema (2):

$$(\exists x)(\exists y)\Box(Fx \vee -Fy) \quad \& \quad (\exists x)(\exists y)-\Box(Fx \vee -Fy).$$

Whenever there exists more than one object, this sentence will be true.[11] It will be true because the two quantifiers in the first conjunct can range over the same objects. Keeping this in mind, it is clear that this instance of schema (2) is true, and unobjectionably so, and thus 'being an instance of (2)' cannot be taken as a criterion for being essential in a clearly troublesome sense.

[9] Op. cit., p. 93.
[10] Cf. 'Two Dogmas of Empiricism', p. 22.
[11] i.e., the sentence will be true in most interesting modal logics; for example, in the systems in S. Kripke, 'Semantic Considerations on Modal Logics', *Acta Philosophica Fennica*, 16 (1963) [V above]. Another sentence which illustrates the same point is (given one usual treatment of '='):

$$(\exists x)(\exists y)\Box(x = y) \quad \& \quad (\exists x)(\exists y)-\Box(x = y)$$

which will be true in any domain of more than one individual.

Nor is the general schema (2) an adequate formalization of the troublesome doctrine discussed above in rough terms. We have discussed only essential properties, but (2) treats relations as well as properties. And here is where the special examples arise. For (2) treats a relation as essential even if it applies necessarily to a single object in relation to itself, while not applying necessarily to distinct objects in relation to each other.

I propose, then, to modify (2) so as to rule out these special cases. I think the following formulation does it: I shall call a sentence *essential* (in the sense under discussion hereafter) if it is an instance of the schema:

$$(3)\quad (\exists x_1)\ldots(\exists x_n)(\pi_n x_n \quad \& \quad \Box F) \quad \& \quad (\exists x_1)\ldots(\exists x_n)(\pi_n x_n \quad \& \quad -\Box F)$$[12]

where F is an open formula whose free variables are included in $x_1, \ldots, x_n$, and where $\pi_n x_n$ is any conjunction of formulas of the form $x_i = x_j$ or $x_i \neq x_j$ for every $1 \leq i < j \leq n$, but not including both $x_i = x_j$ and $x_i \neq x_j$ for any i, j.

Schema (3) differs from schema (2) only in the insertion of the clause '$\pi_n x_n$' following the quantifiers; this (hopefully) rules out exactly the trivial and unobjectionable instances of schema (2), and leaves us with only examples of the *troublesome* essentialist doctrine we have been trying to characterize. (In most cases the additional complexity of schema (3) can be ignored—that is, except for the 'special' cases, schema (2) is an adequate formulation.)

III. A NONESSENTIAL MODAL LOGIC

Having isolated a notion of essentialism which qualifies for the title 'philosophically suspect' (this will be argued in more detail in Section VI), let me turn to question (A) above: *is* quantified modal logic committed to this doctrine? The answer, of course, will depend both on which version of modal logic is at issue, and on what 'commitment' means.

Taking the latter issue first, there seem to be three relevant notions of

[12] Again the assumption is that the variables are to range over individuals and not over individual concepts or descriptions. In some of these other systems, equivalent formulations can be given. In particular, suppose that $x_1, x_2, \ldots$ range over individuals, and $\alpha_1, \alpha_2, \ldots$ range over individual concepts. Suppose also that $\alpha_i \Delta x_j$ means 'α_i is a concept of x_j'. We also suppose that we quantify into modal contexts by use of variables $\alpha_1, \alpha_2, \ldots$ but not with variables $x_1, x_2, \ldots$ (otherwise (3) is adequate as it stands). We can now define an essential sentence as any instance of:

$$(3')\quad (\exists x_1)\ldots(\exists x_n)(\pi_n x_n \;\&\; (\alpha_1)\ldots(\alpha_n)(\alpha_1 \Delta x_1 \;\&\ldots\&\; \alpha_n \Delta x_n \supset \Box F)) \;\&\; (\exists x_1)\ldots(\exists x_n)(\pi_n x_n \;\&\; -(\alpha_1)\ldots(\alpha_n)(\alpha_1 \Delta x_1 \;\&\ldots\&\; \alpha_n \Delta x_n \supset \Box F))$$

(note that F will contain some of $\alpha_1, \ldots, \alpha_n$ but none of $x_1, \ldots, x_n$).

commitment to essentialism. We will say that a system of quantified modal logic is committed to essentialism if

(*i*) it has some essential sentence as a theorem,

or

(*ii*) it has no essential sentence as a theorem, but nevertheless requires that some essential sentence be true—in the sense that the system, together with some obvious and uncontroversial non-modal facts, entails that some such sentence be true,[13]

or

(*iii*) the system allows the formulation of (and thus presupposes the *meaningfulness* of) some essential sentence.

I will discuss (*i*) and (*ii*) in Sections III–V; discussion of (*iii*) is postponed until Section VI.

As for the particular version of quantified modal logic, I will treat the class of systems discussed by S. Kripke (op. cit.). These systems have come in for considerable discussion recently, and the results presented here will extend to a great deal of related work.

The idea behind Kripke's analysis is this: we begin with the notion of a set of *possible worlds* (or a set of *possible states of affairs*, or of *ways the world might have been*). Such a set is called a *model structure*.[14] We can remain neutral among various conceptions of the nature of possible worlds; for the purpose of modal semantics the only facts about possible worlds that are relevant are (*i*) which things exist in each possible world, and (*ii*) what the extensions of the predicates of the language are in each world. Given a *domain* for each world (i.e., given the set of things which exist in that world), and given an assignment of extensions to predicates of the language for each world, the truth values of formulas can be defined for each world. For a closed sentence, A, the definition of *necessity* is:

$\Box A$ is true in world H if and only if A is true in every alternative possible world H'.[15]

[13] I have in mind situations like the following. Essentialism of form (2) above is such that no essential sentence (of form (2)) is a theorem, but some modal logics (e.g., Kripke's) have the consequence that some essential sentence is true *whenever there exists more than one individual* (this is the 'obvious and uncontroversial non-modal fact').

[14] For complete details see Appendix A.

[15] I have omitted discussion of the alternativeness relation (of one world's being *possible relative to* another) for simplicity, and because it is irrelevant to the present issue. This relation enters into modal semantics in the following way: for some interpretations of the modal system we suppose that not all worlds are possible relative to a given world. For example, if we were to interpret '$\Box$' as physical necessity, rather than logical necessity, then we might require that a world not be possible relative to another unless all physical laws which hold in the latter also hold in the former. A world which *is* possible relative to a given world, H, is called an *alternative* to H. The formal structure of this alternativeness relation corresponds to the logical behaviour of iterations of modal operators (e.g., '$\Box A \supset \Box\Box A$' is valid if and only if the alternativeness relation is transitive). In the maximal models discussed in the text, every world is considered to be an alternative to every other world; this is the most general case (see also footnote 28).

A sentence is a *theorem* of quantified modal logic just in case it is true in every world in every model structure no matter how extensions are assigned to the predicates of the language. Let us call a model structure plus an assignment of extensions to the predicates of the language (for each world) a *model*. Then we can also state that a sentence is a theorem of quantified modal logic just in case it is true in every world in every model.

Now the following fact can be proved concerning Kripke's system:

Theorem 1: There are certain models, called *maximal models*, in which no essential sentence is true in any world in the model.

The exact statement of this theorem is given in Appendix A. Part of the significance of this theorem is that it shows conclusively that no essential sentence is a *theorem* of quantified modal logic, and therefore that Kripke's version of quantified modal logic is *not* committed to essentialism in the first sense defined above (in sense (*i*)).

But the theorem also has significance for evaluating commitment to essentialism in the second sense—that is, for deciding whether 'the facts' force some essential sentence to be true (according to the system). For, given any set of non-modal facts expressible in our symbolism (i.e., expressible in sentences without modal signs), these facts must be expressible by a *consistent* set of sentences.[16] And a maximal model is, by definition, a model that will contain, for every consistent set of non-modal sentences, a possible world in which they are all true. Thus, whatever the facts are, the sentences expressing them must all be true in some possible world in some maximal model. But no essential sentence is true in any world in any maximal model (Theorem 1). Thus there is a world in which all the 'facts' of this world hold, and in which no essential sentence is true. And therefore we are also free of commitment to essentialism in the second sense defined above.

IV. NONESSENTIALISM IN APPLIED MODAL LOGIC

The results of the preceding section have to do only with a relatively austere notion of necessity, roughly equivalent to Quine's notion of 'logical

[16] By 'consistent' here I simply mean consistent in the ordinary first-order predicate logic sense.

truth'.[17] The essentialism question re-arises when one goes on to 'apply' the modal logic. There are two natural ways in which this is frequently done. One is to extend the class of necessary sentences to include the truths of some a priori discipline—for example, mathematics. A second is to extend the class to include sentences which are analytic (but not theorems of logic).

First, what happens when we attempt to extend the interpretation of '$\Box$' to precede, say, truths of arithmetic as well as truths of logic? Will we *then* be stuck with essentialism? We are especially bothered by classic examples like:

$$\Box(9 > 7) \quad \& \quad -\Box(7 > 9)$$

which seem to lead to truths of the form:

$$(\exists x)(\exists y)(\Box(x > y)) \quad \& \quad (\exists x)(\exists y)(-\Box(x > y))$$

that is, to essentialism.[18]

The claim of this section is that in certain widespread and well-defined situations, essentialism *can* always be avoided. To be precise: suppose we wish to extend the range of the '$\Box$' sign by choosing a certain consistent set of sentences as axioms, and interpreting '$\Box F$' to be true just in case 'F' is true in every world in which the axioms hold. Then two requirements suffice to guarantee that we will find ourselves endorsing no essential sentences: (*i*) that the axioms all be closed and contain no constants, and (*ii*) that the axioms contain no modal operators, except on the front. Theorem 2, which shows this, is stated in Appendix B. We know that (*i*) is no real restriction, since any theory formulated with constants is replaceable by an equivalent theory without constants. Also, requirement (*ii*) can sometimes be relaxed (for example, in cases where modal operators appear within the sentence but only preceding closed formulas).

We might look at an example of an application of this theorem to arithmetic. What happens to

(a) Necessarily $9 > 7$

[17] Cf. Quine, 'Two Dogmas of Empiricism', p. 22.
[18] Strictly speaking, it is truths of the form

$$9 \neq 7 \;\&\; \Box(9 > 7) \;\&\; 7 \neq 9 \;\&\; -\Box(7 > 9)$$

which seem to lead to

$$(\exists x)(\exists y)(x \neq y \;\&\; \Box x > y) \;\&\; (\exists x)(\exists y)(x \neq y \;\&\; -\Box x > y),$$

i.e., our form of essentialism. I've omitted the non-identity clause for simplicity.

if we take this line? Formulated without constants, (a) has at least two plausible representations:

(b) $\Box(\exists x)(\exists y)(x \text{ is nine} \quad \& \quad y \text{ is seven} \quad \& \quad x > y)$

or

(c) $(\exists x)(\exists y)(x \text{ is nine} \quad \& \quad y \text{ is seven} \quad \& \quad \Box(x > y))$.[19]

In terms of the 'possible world' approach, the difference between (b) and (c) is this: (b) only requires that in each possible world there be some things or other such that the first 'is nine' and the second 'is seven' and the first is greater than the second. On the other hand, (c) requires that there be things, nine and seven, such that in every possible world *these* things are such that the first is greater than the second. Thus (c) requires that certain specific objects reappear in every world, retaining certain properties and relations in every world, while (b) does not.

Now if most of the motivation for accepting essentialism is that *some* analogue of (a) remain true, then that motivation is misguided. For (b) is such an analogue and is nonessential. (c), on the other hand, entails essentialism, and thus will be treated as false if we add axioms only in the manner indicated above. None of this proves that (c) *should* be false; only if it is to be true (i.e., if we are to accept essentialism) it cannot be because we are *forced* to, either by general considerations, or by examples like (a).

This nonessentialist option, incidentally, yields an *alternative* to Smullyan's escape from Quine's 'paradox'[20] based on (a) above. Quine suggests that both:

(d) Necessarily $9 > 7$

and

(e) $-$Necessarily (the number of planets > 7)

are true. But since 9 = the number of planets, (a) and (b) seem to contradict one another. Smullyan objects that if we rephrase (e) as

(e′) $-(\exists x)(x \text{ is the number of planets} \quad \& \quad \Box(x > 7))$

[19] Actually there are many other analogues of (a) as well; for example, we might include uniqueness claims, or in (c) the necessity sign might immediately follow the quantifiers. Or we might avoid the 'necessary existence' claim of (b) by altering it to

(*b′*) $\Box(x)(y)(x \text{ is nine} \quad \& \quad y \text{ is seven} \supset x > y)$.

(b) and (c) constitute one illustration among many of the *important* choice between essentialism and nonessentialism.

[20] Cf. the paragraph beginning at the bottom of p. 154 in 'Reference and Modality'. [Essay I, above, p. 30].

we no longer have any reason to assert (e) (so construed)—that is, it is just obviously *true* that

$$(\exists x)(x \text{ is the number of planets} \quad \& \quad \Box(x > 7)).$$

Thus Quine's 'paradox' for quantified modal logic is avoided.

I think we *do* have reason to assert (e′)—for (e′) is a denial of an instance of essentialism. And thus, any serious objections to essentialism (see Sections v and vi) will carry over to Smullyan's 'way out'.

But, contrary to claims by Quine,[21] there is *another* escape from the paradox, and one which avoids essentialism altogether. Consider both the essentialist and nonessentialist construals of (d) and (e) (where '*Px*' stands for '*x*, is the number of planets'):

(d′) $(\exists x)(\exists y)(x \text{ is nine} \quad \& \quad y \text{ is seven} \quad \& \quad \Box x > y)$
(d″) $\Box(\exists x)(\exists y)(x \text{ is nine} \quad \& \quad y \text{ is seven} \quad \& \quad x > y)$
(e′) $-(\exists x)(\exists y)(Px \quad \& \quad y \text{ is seven} \quad \& \quad \Box x > y)$
(e″) $-\Box(\exists x)(\exists y)(Px \quad \& \quad y \text{ is seven} \quad \& \quad x > y)$.

Maintaining a nonessentialist line, we can deny (d′), while accepting (d″), (e′), and (e″). The 'paradox' now has two construals, both of which are non-paradoxical. When construed as having (d′) and (e′) as premises, it simply has a false premise. On the other hand, when construed as having (d″) and (e″) as premises, no contradiction follows—for the familiar reason that interchanging contingently co-extensive predicates within modal contexts does not guarantee a preservation of truth value.

Thus there is an alternative to Smullyan's solution—an alternative which is consistent with nonessentialism, as Smullyan's is not. (This solution, again in distinction to Smullyan's, does not depend on a distinction between genuinely proper names and [overt and covert] descriptions; cf. Quine's discussion, footnote 20.)

What about analyticity? Well, Theorem 2 also shows that we can extend the interpretation of '□' to something like 'analytically true' in Quine's sense,[22] just provided that we do so by taking as axioms only synonymy relations which are closed non-modal sentences.

Thus even when '□' is extended so as to include arithmetical and non-logical analytic truths, commitment to essentialism in both senses (*i*) and (*ii*) can be avoided.

[21] The specific claim to which I take issue is '*The only hope lies in* accepting the situation illustrated by (32) and (33) and *insisting*, despite it, *that the object x in question is necessarily greater than* 7.' (Italics mine.) From p. 135, 'Reference and Modality' (I, above, p. 30].

[22] 'Two Dogmas of Empiricism', p. 22.

V. THE INDEPENDENCE OF *DE DICTO* AND *DE RE* MODALITIES

What is the relation between *de dicto* and *de re* modalities?[23]

Since essential sentences are paradigm cases of *de re* modalities, we have a partial answer to that question in the last section: some *de re* modalities (namely, all essential sentences) are not entailed by some *de dicto* modalities (namely, those which lack constants and which lack internal *de re* modalities). A converse result also holds; certain sorts of essential sentences do not entail any *de dicto* modalities of a certain sort.

Let us call a sentence *S* a *simple essential sentence* if *S* is an essential sentence—that is, of the form:

$$(\exists x_1)\ldots(\exists x_n)(\pi_n x_n \quad \& \quad \Box F) \quad \& \quad (\exists x_1)\ldots(\exists x_n)(\pi_n x_n \quad \& \quad -\Box F)$$

where F is non-modal, quantifier-free, and neither tautologous nor contradictory.

Let *R* be any non-modal closed sentence such that $\Box R$ is not already provable in our system. Then we can show:

Theorem 3: $\Box R$ is not entailed by any simple essential sentence.[24]

That is, accepting certain essential sentences (the *simple* ones) cannot force us to accept new *de dicto* modalities of the sort indicated.

Theorems 2 and 3 provide a partial independence of *de dicto* and *de re* modalities. A complete independence of *de dicto* and *de re* modalities is prevented by trivial counterexamples; for example, the essential sentence:

$$(\exists x)\Box(Fx \quad \& \quad G) \quad \& \quad (\exists x)-\Box(Fx \quad \& \quad G)$$

where *G* is closed and nonessential, entails the nonessential *de dicto* sentence:

$$\Box G;$$

Conversely, the nonessential *de dicto* sentence:

$$-\Box G$$

entails the *de re* sentence:

$$-[(\exists x)\Box(Fx \quad \& \quad G) \quad \& \quad (\exists x)-\Box(Fx \quad \& \quad G)].$$

[23] I take a *de re* sentence to be one in which quantifiers outside of a modal context bind variables within. A *de dicto* sentence is a sentence without individual constants which is not *de re*. The classification of sentences which contain individual constants within modal contexts depends on the logical behaviour of the constants.

[24] T. Parsons, xeroxed, Chicago Circle, June, 1967.

These logical interrelations are of a trivial kind, however. I think there is an important sense in which essential and non-essential (or *de dicto* and *de re*) sentences are logically independent. This independence is partially formulated by Theorems 2 and 3; a fuller formulation of this independence (one which clearly distinguished the 'trivial' from the 'non-trivial' cases) would be desirable.[25]

VI. THE STATUS OF ESSENTIALISM; COMMITMENT IN SENSE (*iii*)

Given the logical independence of essential and nonessential sentences (as qualified above), we are now in a position to evaluate the status of essentialism—the view that some instances of schema (3) are true. I think two things begin to emerge. One is that if there are to be objections to essentialism, they cannot be made on the basis of claims that any essential sentence must deny facts which we normally regard as well established. Epistemologically, it seems that ordinary contingent truths, together (possibly) with certain nonessential *de dicto* truths, are basic; these are the truths for which we have evidence in the straightforward sense. And one thing that has been shown is that essentialism does not contradict *these*. Thus, I suggest that this general doctrine of essentialism is not the kind of doctrine for which (or against which) we can collect empirical evidence (or *one* sort of logical evidence; but see below).

The same logical independence, however, which frees essentialism from one kind of objection (an objection to its *truth*) opens the door to another: the claim that the truth-conditions of essential sentences are so indeterminate as to leave them devoid of any significance.

The consequence for quantified modal logic is this: although a system of quantified modal logic can assert, deny, or be neutral with respect to the *truth* of essentialism, it cannot be neutral concerning the *meaningfulness* of essentialism, for quantified modal logic simply *is* that symbolism within which essential sentences are formulable. Thus, in order to guarantee that all of its formulas are meaningful, any system of quantified modal logic must provide a meaning for each essential sentence. In short, quantified modal logic *is* committed, in the third sense, to essentialism—it is committed to the meaningfulness of essential sentences. But how bad is this?

Here is where prior discussion of the first two sorts of commitment is relevant. Suppose that a modal logician believes that essentialism is

[25] Theorems 2 and 3 are by no means maximal; that is, neither exhausts the categories of essential and nonessential sentences which fail to entail one another, nor the categories of *de dicto* and *de re* sentences which fail to entail one another.

true—that is, that some essential sentences are true. He must then face the problem of providing such sentences with a clear meaning. And although it is by no means obvious that this cannot be done in a clear and natural way, it is obvious that this is a *problem*, and that it is a problem *added on to* the problem of giving truth-conditions for nonessential sentences.

Suppose, however, that the modal logician disbelieves all essential sentences. He then has a simple method of assigning determinate (and natural) truth-conditions to all essential sentences. That is to make them all false in all possible worlds. In other words, freedom of commitment to essentialism in the first two senses *allows* a freedom of any *objectionable* commitment in the third sense.

Further, making essential sentences false in all possible worlds is not an *ad hoc* device adopted merely to avoid Quinean criticisms; it has an independent and natural motivation. As we mentioned earlier (Section II), it seems natural to locate the source of necessities in the logical character of predicates (on a conventionalist view) or properties (on a 'naturalist' view), *rather* than in the objects of which these are predicated. Yet essential sentences cannot be verified on such a view. So far Quine and I are in agreement. But while Quine wishes to infer from this that essential sentences are somehow deficient in meaning, it seems equally natural to conclude simply that they are false (and false 'for semantical reasons', and thus necessarily false).

In summary, there are an easy way and a hard way to free modal logic from any objectionable kind of commitment to essentialism. The easy way is to add the negation of schema (3) to the axioms for quantified modal logic—that is, to accept as *logically true* the schema:

$$(4)\ (\exists x_1)...(\exists x_n)(\pi_n x_n \ \ \& \ \ \Box F) \supset (x_1)...(x_n)(\pi_n x_n \supset \Box F).$$

The hard way is to provide and justify some *other* truth-conditions for essential sentences. One or the other of these ways must be chosen, in order for a system of quantified modal logic to be unobjectionable.[26]

[26] There is a stronger kind of essentialism than the one I have discussed in this paper. Instead of just requiring that an object have a property necessarily which other objects do not have necessarily, it requires that an object have a property necessarily which other objects *have*, although not necessarily. In symbols, it is given by the schema:

$$(3')\ (\exists x_1)...(\exists x_n)(\pi_n x_n \ \ \& \ \ \Box F) \ \ \& \\ (\exists x_1)...(\exists x_n)(\pi_n x_n \ \ \& \ \ F \ \ \& \ \ -\Box F)$$

Theorems 1 and 2 hold for (3′) as well as for (3); further, determining truth-conditions for essentialism of kind (3) would also determine truth-conditions for essentialism of kind (3′), so the formulability of (3′) in quantified modal logic adds no new problems.

APPENDIX A[27]

We define a *model structure* as a triple $\langle G, K, R\rangle$ together with a function, ψ, where K is a set (the set of 'possible worlds'), R a reflexive relation on K, G (the 'actual world') a member of K, and ψ(H) a set for each $H \in K$. Intuitively, ψ(H) represents the set of things which exist in world H. Now let $U = \cup_{H \in K} \psi(H)$, and let U^n be the nth Cartesian product of U with itself.

A *model* on a model structure $\langle G, K, R\rangle$ is a binary function $\phi(P^n, H)$, where the first variable ranges over n-adic predicate letters, H ranges over members of K, and $\phi(P^n, H) \subseteq U^n$. For the identity predicate, '=', we add the condition that $\phi(=, H) = \{\langle u, u\rangle : u \in U\}$ for every $H \in K$. We now extend ϕ to give a truth value for each formula A in each world H, relative to a given assignment of members of U to the free variables of A:

(*i*) For atomic A:

$\phi(P^n(x_1, \ldots, x_n), H) = T$ with respect to an assignment of $u_1, \ldots, u_n$ to $x_1, \ldots, x_n$ if and only if $\langle u_1, \ldots, u_n\rangle \in \phi(P^n, H)$.

(*ii*) $\phi(-A(x_1, \ldots, x_n), H) = T$ with respect to an assignment of $u_1, \ldots, u_n$ to $x_1, \ldots, x_n$ if and only if $\phi(A(x_1, \ldots, x_n), H) \neq T$ with respect to that assignment.

(*iii*) $\phi(A(x_1, \ldots, x_n) \mathbin{\&} B(y_1, \ldots, y_m), H) = T$ with respect to an assignment of $u_1, \ldots, u_n$ to $x_1, \ldots, x_n$, and of $v_1, \ldots, v_m$ to $y_1, \ldots, y_m$ if and only if both $\phi(A(x_1, \ldots, x_n), H) = T$ and $\phi(B(y_1, \ldots, y_m), H) = T$ with respect to that assignment.

(*iv*) $\phi((\exists x)A(x, y_1, \ldots, y_n), H) = T$ with respect to an assignment of $u_1, \ldots, u_n$ to $y_1, \ldots, y_n$ if and only if there is some $u \in \psi(H)$ such that $\phi(A(x, y_1, \ldots, y_n), H) = T$ with respect to an assignment of $u, u_1, \ldots, u_n$ to $x, y_1, \ldots, y_n$.

(*v*) $\phi(\Box A(x_1, \ldots, x_n), H) = T$ with respect to a given assignment if and only if $\phi(A(x_1, \ldots, x_n), H') = T$ with respect to that assignment, for every H′ such that HRH'.

The informal terminology used earlier can now be made precise: a closed sentence S is true in a given possible world H in the model ϕ on $\langle G, K, R\rangle$ just in case $\phi(S, H) = T$. A sentence S is a theorem of this system of modal logic just in case $\phi(S, H) = T$ for every world H in every model ϕ on every model structure $\langle G, K, R\rangle$.

[27] The following is a paraphrase of portions of pp. 84–7 of Kripke, op. cit. [Essay V above, pp. 64–67].

We are now in a position to define the sort of model referred to above as a 'maximal model':

We call ϕ a maximal model if ϕ is a model on a quantificational model structure $\langle G, K, R\rangle$ such that:

(*i*) $R = K \times K$[28]

(*ii*) $U = \cup_{H \in K} \psi(H)$ and $U \neq O$

(*iii*) For every function χ which maps the predicate symbols P^n of the language onto subsets of U^n, and for every subset U^* of U, there is an $H \in K$ such that $\psi(H) = U^*$ and such that $\phi(P^n, H) = \chi(P^n)$ for all P^n of the language other than =.

(*iv*) If $\psi(H_1) = \psi(H_2)$ and $\phi(P^n, H_1) = \phi(P^n, H_2)$ for all P^n of the language, then $H_1 = H_2$.

(Note: clause [*iv*] is added merely for technical reasons.) It is easy to verify that there are maximal models, and that the actual world is represented in each. The following theorem can be proved by straightforward model-theoretic reasoning:

Theorem 1: Every essential sentence is false in every world in every maximal model.[29]

APPENDIX B

Suppose ϕ is a maximal model on $\langle G, K, R\rangle$, and suppose that Δ is a consistent set of closed formulas with no modal operators (Δ is our set of axioms). Let K^Δ be the set of $H \in K$ such that $\phi(\Gamma, H) = T$ for every $\Gamma \in \Delta$. And let ϕ^Δ be the result of restricting ϕ to K^Δ. Then:

Theorem 2: Every essential sentence is false in every world in ϕ^Δ.[30]

[28] This clause requires that a maximal model be a model on an S_5 model structure. However, any S_5 model structure is also an S_4, *Br*-, and an *M*-model structure, so both of our conclusions, (*i*) that no essential sentence is a theorem, and (*ii*) that no essential sentence is entailed by any consistent set of non-modal sentences, carry over to these other systems automatically.

[29] T. Parsons, xeroxed, Chicago Circle, May, 1967. A similar theorem for a language including constants is given in T. Parsons, *The Elimination of Individual Concepts* (Stanford: Ph.D. Dissertation, 1966), p. 193.

[30] T. Parsons, xeroxed, Chicago Circle, May, 1967.