INDEX

atom

0, 8 1, 7 2, 7, 151 3, 8, 88

 \oplus

 \oplus

 \oplus

adjoint, 209-250 and orders, 221-223cocompletion as, 212 examples of, 217-223existence of, 227, 241, 250 exponential as, 217 freeness as, 209 image as, 222 in a poset, 222 left/right, 211limit as, 218 open interior as, 221 polynomial ring as, 220 preservation of limits, 227 product as, 211 quantifiers as, 223 rules of inference, 224 adjoint functor, 2Adjoint functor theorem, 241 special, 245 adjunction, 2, see adjoint counit of, 217 Hom-set definition, 217 preliminary definition, 210 unit of, 211 AFT, see Adjoint functor theorem algebra, 1, 4, 61 and logic, 223 boolean, see boolean algebra for a monad, 261 and endofunctors, 274 for an endofunctor, 268, 269and monads, 274 heyting, see heyting algebra initial, 270, 273 Lindenbaum-Tarski, 133 presentation, 68, 69, 255 arrow, 4 classifying, 201 associative law, 5

in a Boolean algebra, 179 automorphism, 12 inner, 167 Beck's theorem, 276 bifunctor, 161biHeyting algebra, 252 **BA**, 151 Boolean algebra, 34, 35, 51, 151, 153 finite, 179 boolean algebra, 15, 129 Cantor, 273, 295 Cat, 9, 24, 34, 147, 149 cartesian closed, 161 category, 2, 4 algebraic, 266 arrow, 16 cartesian closed, 11, 122, 122–128, 168 and lambda calculus, 135 equational, 134 cocomplete, 195 comma, 243 complete, 194, 244 concrete, 7, 14, 150 congruence, 84 constructions on, 14--17coslice, 17, 26 discrete, 11, 103, 149 dual, 15Eilenberg-Moore, 261 finite, 7 finitely presented, 85-87 free, 18–23, 27, 28 homomorphism of, 8 homomorphism theorem, 85 Kleisli, 265, 279 large, 24 locally cartesian closed, 233, 237 locally small, 25 monoid, 11 monoidal, 80, 168, 170 strict, 80

 \oplus

 \oplus

 \oplus

 \oplus

 \oplus

 \oplus

 \oplus

306

 \oplus

 \oplus

 \oplus

INDEX

category (cont.) of Boolean algebras, 151 of categories, 9, 147 of diagrams, 185, 202 of elements, 196, 238 of functors, see functor category of graphs, 166 of groups, 80--83of indexed sets, 143 of proofs, 10, 46 of propositions, 252of subobjects, 90 of types, 43, 136 opposite, 15 pointed sets, 17, 175 poset, 9, 17 preorder, 9 product, 15quotient, 84 skeletal, 183 slice, 16, 177 and indexed families, 234 small, 14, 24 with limits, 104 with products, 48, 46-48 category theory, 1-2, 8, 12, 23, 29, 38, 53, 147and adjoints, 209 history of, 1 Cayley's theorem, 13-14, 187 ${\rm CCC},\ see$ category, cartesian closed change of base, 233choice axiom of, 32, 51 function, see function, choice co-, 55 coalgebra for a comonad, 267 cocompletion of a category, 229 cocone, 105 codomain, 4, 55coequalizer, $\mathit{66},\,\mathit{65}\text{--}71$ of monoids, 70 of categories, 148 of monoids, 73 of posets, 148 of sets, 72of spaces, 73 cogenerating set, 245coherence theorem for monoidal categories, 170 coHeyting algebra, 252 boundary, 252negation, 252

cokernel of groups, 88 colimit, 105, 108-114 creation of, 111 ω -colimit, 273 colimits of posets, 114 comonad, 266, 266-267 and adjoints, 267 component, 156 composition, 4of functions, 3 computer science, 2, 10, 36 concatination, 18 cone, 101 as a natural transformation, 166congruence, 83-85, 88 constant, 36 C(X), 151contravariant, 107coproduct, 55, 55-61 of monoids, 59 in a CCC, 193in a posets, 58 injection, 55 of categories, 147 of groups, 60 of monoids, 56 of posets, 58 of sets, 56, 109 of spaces, 57 covariant, 48 ω CPO, 112, 145 map of, 113strict, 145 Curry-Howard correspondence, 45, 142 data type, 10 deduction, 10, 131 transcendental, 164 denotational semantics, 10, 138 diagram as a functor, 185 commutative, 3 hyperdoctrine, 253 of a group, 77 of type, 101 product, 39 pullback, 92 distributive law, 130 and adjoints, 228 domain, 4, 55

"13-Awodey-Index" — 2009/12/18 — 17:03 — page 307 — #3

INDEX

duality, 15, 53–55, 66, 71, 267
and equivalence, 178
conceptual, 54
formal, 54
in Sets, 160
of vector spaces, 159
principle, 53
stone, 153–155, 178
Eckmann-Hilton argument, 78, 88

 \oplus

 \oplus

 \oplus

88 element generalized, 35–39, 91, 93 global, 36 variable, 36 endofunctor, 249, 257, 268-276 epi, see epimorphism epimorphism, 29, 51, 67 of monoids, 31 split, 32 equalizer, 62, 62-65 of abelian groups, 65 of functors, 147 of monoids, 65 of posets, 65 of spaces, 65 equivalence and duality, 178 class, 65of categories, 172, 171-181 and isomorphism, 172relation, 65, 73, 90 equivalent categories, 172 evaluation, 121 exponent, 107 exponential, 121, 119-140 in a poset, 129 of categories, 161, 164 of diagrams, 199 of graphs, 125, 165 of groups, 167 of posets, 122 of ω CPOs, 123

Fib, 240 and presheaves, 240 fibration, 240 field, 151 filter, 35 maximal, 35 ultra-, see ultrafilter fixed point, 113, 271, 275 forgetful functor, see functor, forgetful 307

 \oplus

 \oplus

foundations, 23-25 free, 18 category, 20 cocompletion, 198 monoid, 18, 57, 209 monad, 264 free algebra, 245 Freyd, 241, 245 Freyd, P., 253 Fun, 158 function, 1–3 bijective, 12 characteristic, 63, 98, 152, 201 choice, 33 continuous, 6 equality of, 4 identity, 4 injective, 5, 29 monotone, 6recursive, 6, 248 surjective, 29, 50 functor, 2, 8, 32, 147-149 adjoint, see adjoint, see adjoint cocontinuous, 229codiagonal, 148 comonadic, 267 comparison for a monad, 266 composition of, 9constant, 166, 194 continuous, 106 contravariant, 107 coproduct, 61 embedding, 187 endo-, see endofunctor exponential, 126 factoring, 182 faithful, 148 forgetful, 16, 18, 21, 50, 148 adjoint, 246 and limits, 111 full, 148 identity, 9 injective, 148 monadic, 266morphism of, 155 of groups, 83 of posets, 10 polynomial, see polynomial functor, see polynomial functor precomposition, 231 product, 47 product preserving, 49 projection, 15 representable, see representable functor set-valued, 185

308

 \oplus

 \oplus

 \oplus

INDEX

functor category, 158, 164–168, 185 cocompleteness of, 195 completeness of, 194 of groups, 167 of posets, 167 Galois connection, 221 generator, 150 generators, 21, 68 for a category, 85, 88 for a CCC, 139 insertion of, 18, 157 graph, 6, 27 as a functor, 166, 186 directed, 20 of a function, 254 underlying, 21 **Graphs**, 166 graphs category of, 125 Grothendieck, 2 group, 5, 6, 12, 15, 75-88, 149 abelian, 60, 72 abstract, 75 as a category, 83–85, 87 factor, 81 homomorphism theorem, 81 in a category, 75–80, 87 in lambda calculus, 80 normal subgroup, 80 ordered, 78 permutation, 12 representation, 83, 149 structure, 150, 268 topological, 78 **Group**, 25 group theory, 1 groupoid, 168 groups direct limit of, 109 heyting algebra, $129 \!\!-\! 134$ and adjoints, 223

and adjoints, 223 Hom-set, 11, 48, 48–50 homomorphism category, 8 complete, 180 graph, 21 group, 13, 77, 149 monoid, 12 of algebras, 269 ring, 35 ideal, 17 adjoint, 233 identity arrow, 4 image direct, 222 dual, 222 inverse, 222indexed family, 177 and adjoints, 233–236 indexed sets, 143 $\int_{\mathbf{C}} P, 196, 238$ interpretation of propositional calculus, 134 invariant, 37 IPC, see propositional calculus, intuitionistic isomorphic, 12 isomorphism, 12, 12-14, 20, 25, 26, 31 natural, 158

 \oplus

 \oplus

 \oplus

junk, 18, 20

Kan extension, 232 Kan, Daniel, 209 kernel of a functor, 84 of a group homomorphism, 82 Kleene closure, 18 Kripke model, 240 of λ -calculus, 144 Kripke semantics for propositional calculus, 140

 $\lambda\text{-calculus},\ see$ lambda calculus lambda calculus, 10, 43, 80, 135-140, 194, 240and CCC, 139 completeness theorem, 138theory in, 137 Lambek's lemma, 271, 277 large category, see category, large lattice, 129 Lawvere, 2, 223, 253 Lawvere, F.W., 248 LCCC, see category, locally cartesian closed lifts, 33 limit, 102 creation of, 111direct, 109 equalizer as, 103 product as, 103 pullback as, 104 terminal object as, 103 lim, 109

"13-Awodey-Index" — 2009/12/18 — 17:03 — page 309 — #5

INDEX

lim, 102 limits, 100 and monomorphisms, 115 by products and equalizers, 104 of cardinality, 105 of categories, 147 of diagrams, 194 preservation of, 105 preserved by adjoints, 227 locally small category, see category, locally small logic, 2, 10, 36, 46, 69, 80, 135, 194, 252 algebraic, 223 and adjoints, 223 and topoi, 203 categorical, 91 first-order, 223 higher-order, 194, 203 intuitionistic, 131 linear, 79, 170 modal, 260, 267 positive, 133 Mac Lane, 1, 38, 61 Mac Lane's pentagon, 170 mapping, 6, 8 property, see universal mapping property model of propositional calculus, 134 **Mon**, 11 monad, 259, 255–279 algebra for, 261 and adjoints, 255, 257, 261 Eilenberg-Moore category, 261 on a poset, 260 powerset, 260 monadic, 266 mono, see monomorphism monoid, 11, 30, 59, 70, 271 and monad, 259 free, 19, 18-20 monomorphism, 29, 51, 64, 89 as a limit, 115 pullback of, 115 split, 32 morphism, 5of T-algebras, 261 of cocones, 109 of cones, 102

 \oplus

 \oplus

 \oplus

Natural numbers object, 253 natural numbers object, 248, 246–250, 271, 276 $\,$

309

 \oplus

 \oplus

natural transformation, 2, 156, 157–161, 164 naturality, 155–156 Newton's method, 113, 274 NNO, see natural numbers object noise, 18, 20 nonsense abstract, 147 numbers, 11, 12, 20, 247 ordinal, 171

object, 4 generating, see generator initial, 33–35 natural numbers, see natural numbers object projective, 33, 51 terminal, 33–36 objects family of, 33 **Ord**, 171

Par, 175 partial map, 175 $\Pi, 234$ point, 36, 175 points enough, 36 polynomial functor, 269, 279 and trees, 272 generalized, 270 Pos, 7, 25 not monadic, 266 poset, 6, 30, 35, 57, 149 complete, 130 fibration of, 240 preorder, 9, 90, 245 presheaf, 187 product, 39, 38–48 cartesian, 39 in a poset, 42 of categories, 42, 147 of functors, 157 of monoids, 42 of posets, 42 of sets, 41 of topological spaces, 42programming language, 10 projective, see object, projective propositional calculus, 131-134, 194 and adjoints, 223 completeness theorem, 134intuitionistic, 131

310

 \oplus

 \oplus

 \oplus

INDEX

propositional function, 98 in a topos, 203 pseudo-inverse, 172 pullback, 91, 91-100 and characteristic functions, 98 and extensions, 98 and reindexing, 99 by products and equalizers, 94 functor, 96, 178 inverse image as, 94 of sets, 94 properties of, 95-100 two, lemma, 95 pushout, 117 of sets, 108 quantifiers geometric interpretation, 225 quantifiers as adjoints, 203, 223-227 quotient, 65, 66 of sets, 73 RAPL, 227-233 recursion, 248, 271, 273 reindexing, 234 Rel, 7, 25 relation, 7, 68 composition, 7 identity, 7 local membership, 91 relative product, 7 representable functor, 48, 149, 187 and colimits, 107 and limits, 106 characterization of, 244 contravariant, 107, 150 representation, 13, 187 retract, 32retraction, 32, 32–33 ring, 30 of functions, 151 pointed, 220

SAFT, see Adjoint functor theorem, special
Scott, Dana, 146
section, 32, 32–33
semigroup, 11
set, 3, 11, 14
function, 120
pointed, 17, 175

polynomial, 220

power, 34, 90, 130, 152, 260, 270 as an adjoint, 251 structured, 6, 11, 30, 41, 188 and functors, 186 set theory, 23Sets_{*}, 17, 175 sets cumulative hierarchy, 111 family of, 99, 177, 233 Sets, 5, 24, 25 cartesian closed, $122\,$ **Sets**^C, 185 and adjoint functors, 231 cartesian closed, 201 cocomplete, 196 ${\rm complete},\,194$ locally cartesian closed, 239 slice category of, 238 topos, 202 $Sets_{fin}$, 5, 171, 179 $Sets^{I}$, 177 \mathbf{Sets}^{I} , 233 sieve, 202 Σ , 234 simplicial nerve, 186, 204 set, 186, 204 small category, see category, small solution set, 241 space, see topological space Stone duality, see duality, stone Stone duality theorem, 155 Stone representation, 155 Stone space, 181 string, 265structure, 1, 29, 37, 151, 258 finitary, 269 representable, 149–153 Sub, 90 subcategory, 147 full, 148 subobject, 89, 89-91 as an extension, 203 classifier, 201, 205 for presheaves, 202

 \oplus

 \oplus

 \oplus

 $\begin{array}{l} T\text{-algebra, } 245\\ T\text{-algebra, } 255\\ \text{for a monad, } 261\\ \text{test object, } 14, \, 36\\ \textbf{Top, } 25\\ \text{topological space, } 6, \, 42, \, 57, \, 130, \, 151, \, 267, \\ 278\\ \text{Alexandroff, } 182 \end{array}$

"13-Awodey-Index" — 2009/12/18 — 17:03 — page 311 — #7

INDEX

topological space (cont.)interior, 221 topology specialization order, 10, 26 topos, 202, 201-203, 254 and logic, 203 transformation natural, see natural transformation transpose, 121tree, 271, 272 triangle identities, 256, 255-257triple, see monad twist, 158 type theory, 43–45, 135–140, see lambda calculus dependent, 240

 \oplus

 \oplus

 \oplus

 \oplus

ultrafilter, 35, 51, 107, 153, 179 principal, 154 UMP, see universal mapping property unit law, 5 universal, 19, 119 subobject, 201 universal mapping property, 19, 21, 29, 39, 139 and adjoints, 210 of a congruence, 85 of coequalizers, 66 of coproducts, 55 of equalizers, 62 of exponentials, 121 of free categories, 23 of free monoids, 19 of function sets, 120 of initial objects, 33 of limits, 102 of natural numbers, 248 of polynomials, 221 of products, 39, 51 of pullbacks, 92 of terminal objects, 33 of Yoneda, 198, 229

vector space, 6, 30, 159 finite dimensional, 160

well powered, 245 word, 18, 60 empty, 18

Yoneda embedding, $187,\,187{-}188,\,192$ Yoneda lemma, 188, 193, 238 Yoneda principle, 193 311

 \oplus

 \oplus

 \oplus

 \oplus

"13-Awodey-Index" — 2009/12/18 — 17:03 — page 312 — #8

 \bigoplus

 \oplus

 \oplus

 \oplus

 \oplus

 \oplus

 \oplus

 \oplus