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# On the role of language from an epistemological point of view (1961)

## Zur Rolle der Sprache in erkenntnistheoretischer Hinsicht

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The book *The Logical Syntax of Language*[a] occupies a central place in the philosophy of Rudolph Carnap. The conception there developed of the logic of science as the investigation of the language of science together with its concepts forms is, so to speak, the initial framework for Carnap's further investigations. In the course of these investigations, he has significantly revised the views expressed in the *Logical Syntax*. The framework for the considerations itself and its associated concept formations have also undergone significant changes. Discussions with philosophers working in related areas have contributed substantially to this.

These stepwise revisions of Carnap's philosophy represent a successive dissociation from the exclusive and reductive tendencies of the initial program of the Vienna school. The *Logical Syntax* had already introduced significant corrections to its overly simplified theses. But there Carnap still defended the view that every epistemology, insofar as it claims to be scientific, has to be understood as being nothing other than the syntax of the language of science, resp. the language itself. Since then he has essentially extended the aim of scientific philosophy by adding semantics and pragmatics (following

[a] *Vide* [**?**].

C. W. Morris) and, furthermore, by comparing the distinction between the logical and the descriptive with that between theoretical and observational language. In the following, the significance of the introduction of these extensions of the methodological framework for the shaping of Carnap's philosophy, and also for its partial reconciliation with more familiar philosophical views, will be elucidated from several points of view; at the same time, certain questions that naturally suggest themselves in this context will be indicated.

# 1

The general tendency of the *Logical Syntax* can be said to be an extension of the approach of Hilbert's proof theory. For Hilbert the method of formalization is applied only to mathematics. However, in his lecture "Axiomatic thinking" Hilbert also said: "Everything at all that can be the object of scientific thinking falls under the axiomatic method, and thereby indirectly under mathematics, when it becomes mature enough to form into theory."[b] Carnap goes a step further in this direction in the *Logical Syntax*, by considering science as a whole as an axiomatic deductive system which becomes a mathematical object through formalization: the syntax of the language of science is metamathematics that is directed towards this object.

But the idealizing scheme of science that is used here is certainly not sufficient for epistemology. First of all, it of course represents only the finished result of science, not the entire process of scientific research. For the great mathematical theories an axiomatic deductive presentation of the finished disciplines might display sufficiently well what is significant in them. But the circumstances are already fundamentally different in theoretical physics, since here the supreme principles of the theory in their mathematically precise formulation are not the starting point of the research, but the final result.

Moreover, emphasizing the deductive form does violence to many areas of research. In these areas one does not proceed deductively at all; rather, logical reasoning is applied almost exclusively for *heuristic* considerations, which motivate the formulations of hypotheses or statements of fact.

By the addition of *pragmatics*, all of the above can be taken into account. It is clearly a task for pragmatics to consider the development of the sciences, not with regard to what is historical or biographical, to be sure, but in

[b] *Vide* [**?**], p. ■ .

the sense of working out the methodologically significant ideas. Thus, here heuristic considerations find their appropriate place.

Parenthetically I want to note that heuristics play a role not only in the empirical sciences, but also in purely mathematical research, as has been pointed out lately particularly emphatically by Georg Pólya. There exists a methodical analogy between research in mathematics and in the natural sciences, in the sense that also in mathematics there is a kind of empirical approach and guessing of laws based on a series of particular cases. But such a formulation of a law is only of provisional character in mathematics, as in number theory, where the individual case never can be singled out by inessential conditions (like place and time in physics), but rather each number has its own specific properties. That it is possible even in number theory to gain convictions based on our use of numbers, however, is shown by the example of the statement of the unique factorization into prime factors, which one tends to regard as completely self-evident (when one has not yet come across number theoretic proofs) from ones experiences with calculations. Only at an advanced level is the need for a proof for this statement acknowledged, which is then satisfied accordingly.

# 2

It is useful for the consideration of the relation between syntax and semantics to recall that, from the usual point of view, it is fundamental for a language as such that its words and sentences are directly connected to a sense. When the structure of a language is considered independently of the meaning of its expressions, this is an intended, modifying abstraction.

In Carnap's *Logical Syntax* the exclusion of what is meaningful is compensated for in part by stating "rules of transformation" as well as "rules of formation" as rules of the language. He not only counts those rules according to which a statement is transformed into a logically equivalent one as belonging to these transformation rules of a formalized theory, but, more generally, all those that determine logical dependencies, and moreover, also the stipulations of particular statements as logically universal propositions or *formalized axioms*.

Shortly afterwards, under the influence of Alfred Tarski's investigations and in connection with the extension of his methodological program, Carnap relocated the concept of logical consequence from syntax to semantics.

The logical symbols obtain their meaning in semantics through the "rules of truth," and the semantic concept of entailment is tied to these rules of truth. The formal deductions can be introduced from there by first noting the relations of consequence partly as propositions and partly as rules of inference, and then by axiomatizing the manifold of the obtained propositions and rules. In this way, the concept of rules of transformation as primary rules of the language becomes basically dispensable, while the "rules of truth" should be seen as belonging to the characterization of the language.

The suggestive contrast between the semantic and the syntactic concepts of entailment that is hereby obtained has great advantages for the presentation of mathematical logic—insofar as this is not directed towards a constructive methodology from the outset—and Heinrich Scholz in particular has emphasized this point of view.

It is often felt to be a shortcoming of semantics that it is based on non-constructive concept formation. But being non-constructive is not specific to semantics. One can in principle also pursue semantics within an elementary framework of concept formation. On the other hand, it will hardly be possible to avoid transcending elementary concepts, with or without a semantics, if one wants to fix a concept of "validity," as Carnap intends, such that for every purely logical proposition $A$ (i. e., a proposition without extra-logical components) not only the alternative "$A$ or not-$A$" is valid (in the sense of the principle of excluded middle), but in addition also that either the logical validity of $A$ or of not-$A$ holds.

Semantics is also criticized with regard to a different point, namely insofar as it goes beyond the logic of extensions and addresses questions regarding sense and in particular regarding the relation between the extensional and the intensional. In particular Willard Quine claims that a scientifically inadmissible hypostasis is performed by the introduction of senses (intensions) of expressions as objects, and that even by the reduction of questions about sense to those about sameness and difference of sense, one is still in a domain of terms that are difficult to make precise. In this discussion Quine agrees with Carnap by tending to explain the sameness of the sense of two statements as their logical equivalence, and accordingly to reduce the sameness of the sense of predicates and characterizations and definitions to logical equivalences. Thereby the concept of synonymy comes into close relation with the analytic.

But such a definition of synonymy yields unwanted consequences, provided, as Carnap and many contemporary philosophers do, that the matters of fact of pure mathematics are regarded as logical laws. From this point of view, any two valid statements of pure mathematics are logically equivalent and thus, if sameness of sense was the same as logical equivalence, any correct statements of pure mathematics, for example the statements that there exits infinitely many prime numbers and that the number $\pi$ is irrational, would have the same sense. Or, to take a simpler example: the statement $3 \times 7 = 21$ would have the same sense as the statement that 43 is a prime number.

For this consideration, however, we can even eliminate the dependence on the question of the purely logical character of arithmetic. Let us take an axiom system $A$ and two totally different theorems, $S$ and $T$, that are provable from these axioms. We would hardly be prepared to say that the claim "$S$ follows logically from $A$" has the same sense as "$T$ follows logically from $A$," even when both statements are true, thus both are logically valid, and so both are logically equivalent.

Therefore, the sameness of sense by no means always coincides with logical equivalence. On the other hand, in many cases, including mathematics, one surely would consider a logical transformation as not changing the sense. For example, one would consider the two statements "if $a, b, c, n$ are numbers of the sequence of numbers beginning with 1 and $a^n + b^n = c^n$, then either $n = 1$ or $n = 2$" and "there do not exist numbers $a, b, c, n$ of the sequence of numbers beginning with 1, such that $n > 2$ and $a^n + b^n = c^n$" to be formulations of the same mathematical claim (Fermat's theorem).

In these examples, we are confronted with the difficulty of determining what must be considered as having the same sense. But at the same time we notice that this difficulty is based on the distinction between the kinds of abstraction that are peculiar to different domains of inquiry. We will declare two theoretical physical assertions to have the same sense when one is obtained from the other by a conversion of a mathematical expression it contains; but this is not permissible in general with mathematical assertions. We will say of a formulation of a mathematical proposition that its sense is not changed by an elementary logical transformation; but this will no longer hold when the elementary logical relations themselves are considered. We have only considered the sameness of sense for statements; but the same can be said for predicates and definitions. Thereby the consideration of mathematical definitions yields many examples in which the contrast between extension and intension agrees with our usual scientific way of thinking. Let us take the representation of a positive real number by an expression of analysis, e. g., an infinite series or a definite integral. The extension of this definition is the real number itself, and the intension is a rule to determine this number, i. e., for it being contained in arbitrary small intervals. As is well-known, one and the same real number can be determined by very different such rules; then we have the same extension with different intensions.

To mention also a mathematical example of a predicate having the same extension with different intensions, the prime numbers among the numbers different from 1 can be characterized in two different ways: On the one hand, as those that have no proper divisor other than 1, on the other hand, as those that only divide a product if they divide at least one factor. This results in two different intensions of a predicate with the same extension: the extension is the class of prime numbers, the intensions are the two definitions of the concept "prime number" that correspond to the characterizations. Analogous examples can also be found in the empirical sciences, e. g., when it is possible to characterize an animal species in different ways, so that different definitions result in the same concept of species, and thus the name of the species has different intensions and the same extension.

On the one hand, our considerations show that there are large classes of cases in which the concept of intension has a natural scientific application. On the other hand, we have become aware of the difficulties with the concept of sameness of sense, which are related to different viewpoints in the different areas of research, whereby it does not suffice to contrast the logical with the extra-logical in order to account for the differences.

We can approach the relevant point by calling to mind the type of abstraction that is involved in the concept of intension. Here one does not start with the distinction between the form of the expressions of the language and their expressive function, but rather this latter is consciously retained. What is abstracted away are the particularities of the means of expression that are irrelevant for this function, and the variety of formulations that are based on them, which can be used for the same expressive purpose. This manifold of possibilities consists, from a conventional point of view, on the one hand, in the multiplicity of languages, and on the other hand, in conceptual and factual equivalences that can hold between definitions, properties, and relations. Such an equivalence warrants the substitution of an expression by another only if it is totally unproblematic in the framework of the exposition or investigation in which the expression is used, i. e., if it belongs to the domain that is not under discussion, but which is taken for granted. In fact, our search for knowledge, at least at the stage of developed reflections, is based on a certain supply (of which we are more or less conscious) of ideas, opinions, and beliefs, to which we, either consciously or instinctively, hold on in our questions, considerations, and methods. Following Ferdinand Gonseth's concept of "préalable," such ideas, opinions, and beliefs may be called "antecedent."

The assumption of certain antecedent ideas and premises for any scientific discipline and also for our natural attitude in day-to-day life, is not subject to the same problems as the assumption of *a priori* knowledge. It is not claimed that the antecedent ideas are irrevocable. A science that is initially based on a premise can lead us to abandon this premise in its further development, whereby we may be compelled to change the language of the science. The scientific method also requires that we make ourselves aware of the antecedent premises, and even make them the object of an investigation, resp., include them in the subject matter of an investigation.

These premises thereby lose their antecedent character for the research area in question. In the course of development of the theoretical sciences, this leads to the situation that more and more of the premises are subjected to investigation, so that the domain of what is antecedent becomes narrower and narrower. The specially formulated initial concepts and principles then take the place of the earlier, spontaneously formulated antecedents.

In contrast to the concept of the *a priori*, the concept of the antecedent is related either to a state of knowledge or to a discipline; nothing is assumed that is antecedent in an absolute sense.

Given the notion of antecedent, one can formulate the following definition of synonymy: two statements of a disciple have the same sense if the equivalence between them is antecedent to the discipline. The synonymy of predicates and definitions would have to be explained correspondingly. In the definition, the discipline can also be replaced by a state of knowledge, with respect to which one can speak of an antecedent in a sufficiently determinate way.

It appears that the observed difficulties with the determination of the sameness of sense can be removed in this way. To be sure, in this explanation one has to accept that the synonymy of sentences depends on the discipline or state of knowledge in which it is considered. But on closer inspection this turns out not to be so paradoxical.

# 3

Let us now turn to the extension of the methodological framework of the *Logical Syntax* that Carnap obtains by contrasting the theoretical language with the observation language.

When considering the method of natural science, we usually contrast theory and experiment. But in the initial form of logical empiricism, the theoretical aspect was not really recognized; only the course of discussions about the initial position, in which Karl Popper in particular was involved, has the preference emerged for the revised point of view of the *Logical Syntax*, in which formulations of laws of nature figure as proper sentences of the language of science.

One can understand that there was initially some resistance to this when it is realized that, with the acknowledgment of the role of statements of physical laws as proper sentences, the dualism of "relations of ideas" and "matters of fact," which was originally proposed by David Hume and which the Vienna school strived to maintain in a somewhat more precise form, turns out not to be exhaustive. On the one hand, the statements of laws of the natural sciences are not statements about "relations of ideas," i. e., not sentences of pure logic or pure mathematics; on the other hand, neither are they statements of matters of fact, since they have the form of general hypothetical sentences.

In Carnap's terminology, this result says that the domain of the descriptive (the extra-logical) does not coincide with that of the factual, but rather that the domain of the factual is narrower.

The same situation can be elucidated also from a different point of view. Carnap explains the concept of logical truth using "state description" in his book *Meaning and Necessity*.[c] He thereby follows Leibniz's idea of "possible worlds": what is necessary must hold in all possible worlds; and the "state descriptions" schematically represent the constitutions of the possible worlds. Thus, Carnap now defines: a sentence is logically true, if it holds for every "state description." In this approach, the concepts of necessity and possibility occur. But nowhere is it said that one can speak of necessity and possibility only in the logical sense. Carnap himself mentions the investigation of the non-extensional operators expressing physical and causal modalities under the open problems for semantics in the appendix to his *Introduction to*

[c] *Vide* [**?**].

*Semantics* (*vide* [**?**], § 38*d*, S. 243). Physical and causal modalities concern what is possible within natural laws and what is necessary in nature. Now if the laws of nature are stipulated as valid in the framework of the language of science, and, furthermore, it is acknowledged that the laws of nature are not logically necessary, then a distinction between between what is necessary and what is actual follows which is different from that between what is logical and what is descriptive. Then we can consider "state descriptions" in a narrower sense by admitting only those that conform to the laws of nature, and we thereby obtain a narrower range of possible worlds.

Thus, not only are factual statements contrasted with the statements of logical laws, but more generally with any statements of laws. We can now express this more general contrast using the concept of the theoretical, by comparing the statements about what is actual with theoretical statements. Then the domain of what is theoretical contains what is logical as a proper subset.

What is specific to the theoretical surely consists not only in the totality of statements that are recognized as valid, but above all in a world of concepts, in the framework of which the theoretical statements occur. Within the language of science, the theoretical formation of concepts finds its place in what Carnap calls the "theoretical language."[1]

Let us now take a closer look at the role that Carnap assigns to the theoretical language. In his view the theoretical language is not immediately interpreted, but rather the theoretical terms obtain their significance only in connection with the "correspondence postulates," which establish the relations between the theoretical terms and the observational terms. However, these relations are not thought to be so extensive that they define all the theoretical terms in the observational language. Instead, Carnap agrees with the view that the requirement that every theoretical term be experimentally definable and its use be bound by such a definition is too restrictive for theoretical research, and that it is not in accordance with actual practice in the theoretical sciences, as has been expressed in neo-positivist circles, in particular by Herbert Feigl and Carl Hempel.

[1]By following Carnap in speaking here simply of "the theoretical language," the idea of a universal science should not be implied. Nor is this the case in Carnap's own discussions of the theoretical language. He speaks for instance of "methodological problems, that are connected to the construction of a theoretical system, like one for theoretical physics" (*vide* [**?**], pp. 241–242).

A fundamental requirement for the freedom of the theoretical formation of ideas is hereby acknowledged. But it remains the fact that the theory is not seen as a world of ideas, but only as a linguistic apparatus, so to speak. The reduction to the purely mathematical is added as another characteristic feature to this more technical aspect that Carnap attributes to the theoretical language. Whenever possible Carnap strives to reduce theoretical entities to mathematical ones. This possibility is shown in the domain of physics in a particular way by the presentation of field theory, whereby physical events consist of a succession of states in the space-time continuum. The determination of states is given by scalars, vectors, and tensors.

For example, the description of physical state in the pure field theory of gravitation and electricity employs both the symmetric tensor of the metric field, from which the measurement of length and time and the forces of inertia and gravitation are determined, as well as the antisymmetric electro-magnetic tensor, which determines the electrical and magnetic forces. Particles of matter, either charged or uncharged, are understood here as particularly concentrated distributions of the field magnitudes in a spatially confined part of the world. The components of the tensors are functions of space-time positions, and when a coordinate system is introduced and units are chosen, the magnitudes of the components become mathematical functions of the space-time coordinates;[2] let us call them "field-functions." The physical laws of the field are formulated by the differential equations for these mathematical functions (in a way that is invariant with respect to the coordinate system), and the field-functions which represent the sequence of states of the system form a solution for this system of differential equations.

The connection between the theory and the world of actual experience is given through various kinds of relations:

1. those on which the introduction of space-time coordinate systems and the possibilities of the values of the field-functions are based,

2. those which concern the effects of system states partly on our direct perceptions and partly on our experimental observations,

3. those which yield the instructions for the theoretical translation of a case that is observationally given (either only schematically or in a precise experimental determination) and is to be investigated using the theory.

Carnap thinks that all these relations are axiomatizable by the corre-

[2][1] Initially the components of the metric field are unspecified numbers.

spondence postulates in which the links between the field-functions and our observations are expressed. Such a system of correspondence postulates can only be formulated if the manifold of possible applications of field theory (of the differential equations of the field) to observations is axiomatizable.

With these reservations, the possibility is therefore given to wholly restrict the theoretical language of physics to mathematical concepts, transferring everything that is specific to physics either into the observational language or into the correspondence postulates. Then the theory of physics no longer makes statements about something that exists in the physical nature, it does not even state anything at all by itself, but only yields a mathematical tool for the predictions of observations on the basis of given observations. Strictly speaking, one should not talk here of a theoretical language at all.

But a kind of theoretical language can still be regained by introducing suitable physical names for certain often recurring mathematical relations and expressions according to the meanings that they have in the interpreted theory; then the procedure is analogous to interpreting geometrically the arithmetical relations and objects of an (analytical) geometry that is constituted purely arithmetically.

What is perplexing in the described method of eliminating theoretical entities is the fact that it can be applied to any treatment of natural objects. If the assumption of natural objects is appropriate in the familiar cases of daily life and, furthermore, if we extrapolate familiar methods of orientation in place and time to the notion of the four-dimensional space-time manifold, then it does not seem appropriate to discontinue, so to speak, the treatment of natural objects at a certain point and to replace the objects there by their mathematical descriptions.

However, Carnap can reply to this consideration that the difference of the methodical treatment does not relate to the differences of positions in the space-time manifold, but rather to the different theoretical levels. What is meant by such a difference of level can be exemplified with the distinction between macro- and micro-physics. In general, a further theoretical level is present in the treatment of a domain of knowledge where the formation of concepts forces a further remove from the intuitively familiar. Such a step of increased theoretical character can be successful and prove satisfactory, and a practical confidence can arise in the course of applying the initially unfamiliar concept. But here the difference remains between what is methodologically more or less elementary, i. e., between what is closer and farther away from concrete observations.

It is obvious that quantum physics involves an increased theoretization, in the sense described above, compared with the previous "classical" physics. But the method for the elimination of entities described above cannot be applied directly to quantum physics, since here the idea of a definite sequence of states in the space-time manifold that is determined objectively and independently of experiments is lost. In a different respect quantum physics is well-suited to the aims of the method of elimination, since here the idea of objecthood is already weaker, and mathematical considerations dominate the concept formations. Quantum physics also shows us how the different methodological treatments of diverse theoretical entities can be implemented without objectionable disruptions, by giving the role of the observation language, so to speak, to the theoretical language of the previous physics.

At the same time it is hereby suggested that it is reasonable to relate the distinction between the observation and theoretical languages to the level of the concept formations, instead of taking it to be absolute. If we consider the role of the observation language in scientific practice this idea is confirmed. When physicists talk about their experiments they surely do not speak only of objects of immediate perception. One talks maybe of a piece of wood, of an iron rod, of a rubber band or of a mercury column. But the language in which physicists report their experiments goes much farther in this respect.[3] It is also noteworthy that many terms for the concepts of physics (like "barometric pressure," "electrical current") have entered into everyday language.

On the whole, the situation can be characterized by saying that an observational language of a science that is at a certain level refers to an antecedent world of ideas and concepts—"antecedent" in the sense introduced in our section 2. The antecedent theoretical concepts also obtain their terms in the observational language at this level. We do not need to separate the observational language from the colloquial language at all. The observational language can instead be understood as a colloquial language that has been

[3][1] Indeed, the claim has been made that all experiments in physics turn out to be statements about coincidences. But surely this claim has to be taken only *cum grano salis*: The statement of coincidence (or non-coincidence) is in each case only the last decisive step in the overall process of an experiment. Moreover this requires that the person conducting the experiment understands the equipment and handles it correctly, also that this apparatus has been set up appropriately. Moreover the scientist should have sufficiently confirmed that no interferences occur, etc. It is hardly the case that everything that has to be understood and done in order to achieve this can be reduced to simple statements about coincidences. But this is surely not intended by that thesis.

augmented by a larger set of terms.

The relativization of the observational language to a conceptual level is also appropriate to the kind of opposition between what is empirical and what is theoretical that is intended by Ferdinand Gonseth's principle of duality. What is meant here is that there are no distinct empirical and theoretical domains, but that both aspects come into play in every domain and in every stage of knowledge. The different points of view in the above considerations: the elimination of abstract entities, the distinction between theoretical levels, and the relativization of the observational language to a conceptual level, all have their application in particular to mathematical proof theory. The latter assumes a distinction between the "classic" method of mathematics that is applied in analysis, set theory, and the newer abstract mathematical disciplines, and the more elementary methods that are characterized as "finitist," "constructive," or "predicative," depending on the restrictions at issue. In the proof-theoretic investigation of classical mathematics, an elimination of abstract entities is made possible by the method of formalizing propositions and proofs using logical symbolism. One attempts to use this elimination to prove the formal consistency of classical theories from one of the more elementary standpoints. So far formal consistency proofs using constructive methods have been obtained only for such formal systems as can be interpreted at least predicatively. Recently, it appears that a consistency proof for formalized impredicative analysis is possible from a broad view of the constructive standpoint, by a method developed by Clifford Spector.

The elementary "meta-language" in which such a consistency proof is carried out has the role of an observation language, as has been noted by Carnap. Originally it was Hilbert's idea that this language should remain totally within the framework of concrete considerations, i. e., should be an observation language in the absolute sense. But gradually one has been forced to include more and more theoretical terms. The "finitist standpoint" already uses strictly more than Hilbert originally wanted to allow; but even this standpoint turned out to be insufficient for the intended purpose, according to the results of Kurt Gödel. The consequences of this result appear not to be as fatal to proof theory as it initially seemed, if one accepts the idea of relating the observation language to a conceptual level. The acknowledgment of the methodological importance of proof-theoretic investigations, and in particular those concerning formal consistency, is not tied to the view that conventional, classical mathematics is dubious, or to the standpoint of "formalism" according to which classical mathematics is justified only as a purely formal technique. Hilbert essentially never thought of this, despite some remarks that point in this direction. The task of constructive consistency proofs is motivated by the high theoretical level that is present in classical mathematics.

In any case, an adherent of the constructive proof-theoretic direction of research can very well have the point of view also favored by Carnap, that the concept formations of classical mathematics have justified application when taken as interpreted. But whether it is reasonable to accept all entities that are introduced by set theory as real is open for discussion from this standpoint as well. Nor is one inclined to withhold the positive status from the theoretical concepts, awarding that privilege only to mathematical concept formation; what is just for mathematical classes and functions is equitable for the entities of the natural sciences, insofar as they are used in a way that promotes understanding.