

IAN HACKING. **Why Is There Philosophy of Mathematics at All?** Cambridge University Press, Cambridge, 2014 xv + 290 pp.

According to its foreword, the book under review is a collection of “philosophical thoughts about proofs, applications, and other mathematical activities.” It is not a conventional work of philosophy in which an author stakes out and defends philosophical claims, but rather, a journey through a landscape of ideas related to the practice of mathematics.

The initial answer to the question posed by the title is twofold: there is philosophy of mathematics, first, “because of the *experience* of some demonstrative proofs” and the powerful hold this experience exerts on us; and, second, “because of the richness of applications of mathematics, often derived by thinking at a desk and toying with a pencil,” a phenomenon which calls for explanation.

The book aims to challenge and deflate many of our intuitions on these matters. For example, after drawing a distinction between a “leibnizian” view of proof, on which a proof is a mechanically-checkable artifact, and a “cartesian” view, on which a proof is something that delivers a certain experience of understanding, Hacking argues, surprisingly, that neither notion of demonstrative proof is essential: “deep mathematics could have developed without what we call proofs at all.” Among the supports for this are the fact that the Greek ideal of proof did not figure as centrally into early Chinese, Persian, and Arab traditions. Historical counterfactuals are curious beasts, however, and while Hacking challenges us to imagine that the role of proof in mathematics could have been different today, he leaves us wondering whether or not that would be a good thing.

In a similar way, a chapter on the applicability of mathematics argues that the contemporary distinction between pure and applied mathematics is also a historical accident. Canvassing historical views as to the role of mathematics vis-à-vis practical pursuits shows a more nuanced relationship, and by exploring a number of senses in which a piece of mathematics can be said to be “applied,” Hacking argues, convincingly, that overly naïve models of how that works — we abstract a model, reason about it, and then read off empirical predictions — is too simplistic.

The bulk of the text is devoted to presenting the views of others, including a battery of Fields medalists (such as Michael Atiyah, Alain Connes, Tim Gowers, Alexander Grothendieck, Atle Selberg, Willam Thurston, and Vladimir Voevodsky) and great mathematicians and philosophers of the past (including Bacon, Berkeley, Bernays, Cantor, Dedekind, Descartes, Frege, Gödel, Hamilton, Hardy, Hilbert, Kant, Kronecker, Kuhn, Lakatos, Leibniz, Littlewood, Maxwell, Newton, Pascal, Peirce, Polya, Poncelet, Pythagoras, Quine, Russell, Weil, Weyl, Wigner, and Wittgenstein). These are supplemented by the views of a host of contemporary mathematicians and philosophers, as well as anecdotal factoids, such as the location of the largest tensegrity structure in the world and the contents of a telephone company error message in Cape Town (“the number that you have called does not exist”).

Most of the transplanted views are not discussed in any depth, however, re-

ducing the exploration to a hodge-podge of slogans, catchphrases, and taglines. The lack of detailed analysis makes the work feel ungrounded and impressionistic. Hacking's own views are notably absent, and when they do intrude, it is with the disclaimer that his "personal opinion is as worthless as almost everyone else's." The strategy seems to be to throw everything against the wall and see what sticks, leaving the reader wondering what it all amounts to.

The final pages, though, bring a number of discursive threads together with a startling conclusion: it really *doesn't* amount to much. After professing a "lack of interest in a contemporary philosophical problematic," Hacking answers the book's central question by endorsing a quotation from Thurston: "mathematics, and not only its basic contents, exists independently of us. This is a notion that is hard to credit, but hard for a professional mathematician to do without." When push comes to shove, core philosophical questions are largely irrelevant; but it is important to the practice that we *think* that they matter, and that we experience mathematics as more than a "barren formalism."

This conclusion, if warranted, is depressing. Some of our greatest minds devote their lives to the study of mathematics; we subject our offspring to countless hours of mathematical instruction from kindergarten to graduate school; and we let mathematical results guide our decisions in practical endeavors of all sorts, from industrial engineering to medicine to public and fiscal policy. Surely, one would think, sustained thought as to what it means to do mathematics should provide an understanding that can help us do it better, not just make us feel better about doing it. In that respect, Hacking has done us a service: by presenting us with an extensive catalog of philosophical ideas that do not seem to matter much, the book challenges us to do a better job of finding the ones that do.

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