Limited Stock Market Participation and Goods Market Frictions: A Potential Resolution for Puzzles in International Finance

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Abstract

We study asset prices, exchange rates, and consumption dynamics in a general equilibrium two-county macro-finance model that features limited stock market participation as well as non-traded goods and distribution cost. The model generates a high price of risk, smooth exchange rates, and makes substantial progress towards explaining the empirically observed low consumption growth correlation between countries. We find that distribution cost play a central role for reducing international consumption co-movement while also amplifying risk premia.

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\section{Introduction}

It has long been a challenge for macro-finance models to jointly explain i) the high equity premium, ii) relatively smooth exchange rates, and iii) the low international correlation of consumption growth.\footnote{See, for example, Brandt et al. (2006).} We propose a general equilibrium two-county macroeconomic model that features limited stock market participation as well as non-traded goods and distribution cost to address these salient features of the data.

Our model brings together two strands of literature. Firstly, we build on work showing that the nature of goods markets is an essential determinant of the correlation of consumption growth between countries in general equilibrium models. In particular, Corsetti et al. (2008) show that modeling goods markets to feature non-tradable goods and distribution cost helps decrease the strong international correlation of consumption. These features moderate the international risk sharing mechanism shown in Cole and Obstfeld (1991), in which a country hit by an adverse productivity shock benefits from a natural hedge, as its goods become more scarce and appreciate in price. We model distribution services that are produced with the intensive use of local inputs, allowing the model to generate deviations from the law of one price. The role of such distribution cost in explaining real exchange rate movements has been emphasized by a growing empirical literature, e.g. Crucini et al. (2005).

Secondly, we draw from the literature on limited stock market participation. We adopt the asset market structure of Guvenen (2009), where only a fraction of the population has access to the stock market and the remainder of agents are restricted to trade in a bond.\footnote{See also Vissing-Jørgensen (2002).} This type of setup has two appealing features for our analysis. First, it is known to generate a realistic price of risk, i.e. Sharpe ratio, which we need in order to jointly study asset prices, international consumption co-movements, and exchange rates. Secondly, it introduces market incompleteness. Brandt et al. (2006) argue that the equilibrium condition linking marginal utility growth to the rate of depreciation in the exchange rate that results in complete market models makes it impossible to generate high risk premia, smooth exchange rates, and moderately correlated consumption growth simultaneously. Allowing for market incompleteness in the form of limited stock market participation breaks this link, making it feasible - at least in principle - for the model to match the three stylized data facts.

There have been previous attempts in the literature to generate the joint dynamics of asset prices, consumption, and exchange rates. Colacito and Croce (2011) study a model that combines cross-country-correlated long-run risk with Epstein-Zin preferences. In their paper,
the two countries’ exogenous consumption growth processes are calibrated to be moderately correlated but include a persistent predictable component that is highly correlated across counties. This leads to moderate consumption growth correlation and high pricing kernel correlation, allowing the model to successfully match the stylized data facts. Stathopoulos (2012) uses preferences with external habit formation and home-bias in consumption to address the puzzle. The present paper differs from previous work in that we do not resort to non-standard preferences. Rather, we ask whether goods market frictions that have been studied in the international macroeconomics literature can generate a similar result. Our results also do not rely on exogenous driving processes such as long-run risk or external habits that are empirically difficult to observe.

2 The Role of Asset and Goods Markets

The stylized fact in the data that our model attempts to rationalize is the co-existence of moderately correlated consumption growth between countries with smooth exchange rates and high equity risk premia. Our model features two main ingredients that help it address these data facts. On the asset market side, we assume that only a limited fraction of agents in each country can participate in equity markets. On the goods market side, we model a traded and non-traded sector in each country featuring distribution cost for the consumption of tradables.

In this section, we motivate the choice of these two key model components and provide intuition for why they help the model come closer to the data. Since an analytical solution is not available for the full model, we conduct this analysis with respect to two benchmarks. First, we study a complete markets model. This analysis highlights the importance of introducing some form of market incompleteness - in our case limited stock market participation - as a necessary condition for models of a wide class to be able to fit the data. Then we study another benchmark, a model without financial markets and with just traded goods. In that setup, we revisit the result from Cole and Obstfeld (1991) that consumption tends to co-move strongly between countries even in the absence of financial markets and provide intuition for how non-traded goods with distribution cost weaken this effect.

2.1 Asset Markets

To motivate our modeling of asset markets, we first consider a representative agent model with complete financial markets in which agents have power utility $U(C)$. In this type of framework, one of the equilibrium conditions requires that exchange rates depreciate by the
difference between foreign and marginal utility grow,

$$\ln \frac{Q_{t+1}}{Q_t} = \ln U' \left( \frac{C_{t+1}^*}{C_t^*} \right) - \ln U' \left( \frac{C_{t+1}}{C_t} \right). \quad (1)$$

Here, $C_t$ denotes the home consumption index at time $t$ and the * superscript indicates that a variable refers to the foreign country (as we will adopt throughout the paper). The real exchange rate is $Q = \frac{P_F}{P_H}$, where $P_H$ denotes the consumer price index for the aggregate domestic consumption basket and $P_F$ denotes the same index for the foreign consumption basket.

Brandt et al. (2006) use equation 1 to document a tension that exists between the data and the theory. On the one hand, we know that observed high risk premia in financial markets require marginal utility growth to be highly volatile at home and in the foreign country. On the other hand, observed exchange rates are comparatively smooth. With regards to equation 1, the only way that exchange rates on the left hand side can exhibit low volatility while the two marginal utility terms on the right hand side can exhibit high volatility is if the marginal utilities are highly correlated. This implication of theory is not borne out by the data, however, as the correlation of consumption growth across countries is around 0.6 (depending on the measure and country pair), much lower than required to satisfy equation 1.

In order to break the tight link between exchange rates and consumption growth in equation 1, we assume that only a fraction of the population in each country has access to the stock market. Hence, we introduce a particular type of market incompleteness in which stockholders have access to more complete markets than non-stockholders. In addition, using a limited stock market participation setup has proven successful in explaining high equity risk premia. The model in Guvenen (2009) represents the current state-of-the-art framework for asset pricing with limited stock market participation and we will adopt his specification of the asset market in our full model below.

While this risk sharing condition in equation 1 only arises if financial markets are complete, it turns out that it still holds approximately true even in incomplete markets. The goods market frictions that we turn to next play an important role in that respect.

### 2.2 Goods Markets

While having incomplete markets is necessary to reconcile asset prices with consumption and exchange rate dynamics, it is not sufficient. In particular, Cole and Obstfeld (1991) famously point out that consumption growth tends to be highly correlated across countries even without financial markets. We will in turn summarize their argument in a model with
no financial markets and only traded goods. Note that the exact relationships derived below will not hold in our model. However, analyzing the role of distribution cost in a simple framework that is analytically tractable is useful to build intuition for the results from our full model that we present in the next section.

There are two symmetrical countries, called “home” and “foreign”. Aggregate consumption (also referred to as the consumption index) in the home county is given by the constant elasticity of substitution (CES) goods aggregator

$$C = C_T = \left[ a_D^{1-\rho} (c_H)\rho + (1 - a_D)^{1-\rho} (c_F)\rho \right]^{1/\rho}, \quad \rho < 1, \quad (2)$$

where $c_H$ denotes domestic consumption of the home tradable good and $c_F$ denotes domestic consumption of the foreign tradable good. The elasticity of substitution between the two varieties of traded goods is given by $\omega = \frac{1}{1-\rho}$ and the weight of home tradables in aggregate consumption is $a_D$. Further, define the terms of trade $\tau = \frac{p_F}{p_H}$ as the ratio of the price of the foreign tradable $p_F$ to that of the domestic tradable $p_H$. Corsetti et al. (2008) show that the response of domestic demand for the home good to a fall in its price (increase in $\tau$) can be decomposed into a substitution effect (SE) and income effect (IE), such that

$$\frac{\partial C_H}{\partial \tau} = \omega \left( \frac{a_H (1-a_H)}{[a_H + (1-a_H)\tau^{1-\omega}]^2} \right) Y^Y \left( \frac{a_H (1-a_H)}{[a_H + (1-a_H)\tau^{1-\omega}]^2} \right) Y^T, \quad (3)$$

where $Y^T$ is the home endowment of the domestic tradable good.

Equation 3 allows us to analyze how supply shocks propagate between countries. If the elasticity of substitution between domestic and foreign tradables $\omega$ is larger than 1, then the substitution effect dominates the income effect. This is the case studied by Cole and Obstfeld (1991). If the home country is hit with a negative supply shock, the home good needs to become more expensive ($\tau$ decreases, the terms of trade improve) for domestic demand to fall and match supply. This partially compensates domestic agents for the adverse supply shock by raising the value of their tradable endowment in international goods markets. Hence, with a large trade elasticity, goods market prices endogenously adjust to provide insurance against negative supply shocks. For this reason, consumption growth will be highly correlated across countries even in the absence of financial markets. This mechanism is amplified by the fact that foreign demand for the home good unambiguously decreases in its price, i.e. $\frac{\partial C_H}{\partial \tau}$, as the substitution and income effects are both positive regardless of the trade elasticity.

Now consider the case where the trade elasticity is below one. Then, the income effect dominates. If the home country is hit with a negative shock, the price of the home tradable has to fall in order to induce home agents to reduce their demand to match the restricted
supply. Hence, the value of their income drops further, amplifying the effect of the negative endowment shock on their consumption. This is the key mechanism that pushes the consumption growth correlation below unity. Note, however, that the domestic income effect not only has to be stronger than the domestic substitution effect - it also has to outweigh the positive income and substitution effects of the foreign agents.

In the next section, we develop the full model whose goods market side also includes non-tradable goods and distribution cost. It turns out that distribution cost amplify the size of the income effect relative to the substitution effect and play a quantitatively important role for our results. We will discuss the role of the distribution cost in more detail in Section 6.2. Further, recall that equation 3 only holds without financial markets. The full model will have nearly complete asset markets, allowing agents to share consumption risk more effectively between countries. The quantitative results from the full model will hence shed light on the question of how much of the income effect discussed in this section survives after adding financial markets.

3 Model

This section presents the full model featuring limited stock market participation and distribution cost with non-traded goods. Each country is now endowed with two Lucas trees, producing a country-specific tradable and non-tradable good, respectively. Furthermore, each country is inhabited by two types of agents according to the set of financial securities they are allowed to hold. A fraction \( \mu \) of agents in each country has access to the home and foreign stocks as well as the one international bond, a fraction \( 1 - \mu \) can only hold the international bond. In the interest of brevity, we focus our presentation on the domestic economy with the understanding that the foreign counterparts are defined symmetrically.

3.1 Preferences

As above, the consumption index for tradable consumption \( C_T \) is given by equation 2. Aggregate consumption now consists of traded and non-traded goods with CES aggregator

\[
C \equiv \left[ a_T^{1-\phi} (C_T)^\phi + (1 - a_T)^{1-\phi} (c_N)^\phi \right]^{1/\phi}, \quad \phi < 1,
\]

where \( c_N \) is domestic consumption of the home non-tradable good. The elasticity of substitution between tradables and non-tradables is \( \frac{1}{1-\phi} \) and agents assign a weight of \( a_T \) to tradables in aggregate consumption.

Agents have power utility and maximize
where $\gamma$ controls relative risk aversion. The discount factor $\theta_t$ is endogenous and evolves as $\theta_{t+1} = \theta_t \omega C_t^{-\eta}$, with $0 \leq \eta \leq \gamma$ and $0 < \omega \bar{C}^{-\eta} < 1$ and where $\bar{C}$ denotes the steady-state value of consumption.  

3.2 Distribution Cost and Goods Prices

In addition to distinguishing between tradable and non-tradable goods, our economy features distribution cost such that for every unit of either the home or foreign tradable good consumed in the home (foreign) country, $\nu$ units of the home (foreign) non-tradable good are needed to distribute the tradable good to consumers. This drives a wedge between prices for tradable goods at the producer and consumer level. Taking consumption of the home-tradable in the home country as an example,

$$p_H = \bar{p}_H + \nu p_N$$

gives the relation between the producer price of the home tradable, $\bar{p}_H$, its consumer price, $p_H$, and the price of the home non-tradable, $p_N$.

The utility-based consumer price indices for the home basket of tradable goods is

$$P_T = \left[a_D (p_H)^{\rho/(\rho-1)} + (1 - a_D) (p_F)^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho}.$$  

Similarly, the utility-based consumer price index for the aggregate home consumption basket is

$$P_H = \left[a_T (P_T)^{\phi/(\phi-1)} + (1 - a_T) (p_N)^{\phi/(\phi-1)}\right]^{(\phi-1)/\phi}.$$  

We chose the home consumption basket as the numeraire, so that $P_H \equiv 1$. The exchange rate is given by $Q = \frac{P_F}{P_H}$, where $P_F$ denotes the consumer price index for the aggregate foreign consumption basket.

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3Endogenizing the discount factor in this way pins down a unique steady state for the distribution of wealth in the presence of incomplete financial markets. Otherwise, the model would exhibit a unit-root and not be amenable to standard numerical solution techniques.
3.3 Endowments and Asset Markets

The home country endowments of the tradable good, $Y_T$, and the non-tradable good, $Y_N$, evolve as

\[
\ln Y_T^t = \Psi^T \ln Y_T^{t-1} + \epsilon_T^t \\
\ln Y_N^t = \Psi^N \ln Y_N^{t-1} + \epsilon_N^t,
\]

where $\epsilon_T^t$ and $\epsilon_N^t$ are iid normally distributed disturbances.

Labor income $Y_L$ and capital income $Y_K$ are given by

\[
Y_{L,t} = \theta_L \left( \theta_T p_H^{T,t} Y^T + (1 - \theta_T) p_H^{N,t} Y^N \right) \\
Y_{K,t} = (1 - \theta_L) \left( \theta_T p_H^{T,t} Y^T + (1 - \theta_T) p_H^{N,t} Y^N \right),
\]

where the parameter $\theta_L$ controls the labor share and $\theta_T$ controls the share of traded goods.

Denoting asset prices by $Z$ with the appropriate sub and superscripts, the return to a claim to the home capital income is $r_{A,t+1} = \frac{Z_{A,t+1} + Y_{K,t+1}}{Z_{A,t}}$ and the corresponding return to the foreign capital income is $r_{A,t+1}^* = \frac{Z_{A,t+1} + Y_{K,t+1}}{Z_{A,t}}$. Furthermore, there is an international bond which pays off half a unit of the home tradable and half a unit of the foreign tradable with return $r_{B,t+1}^\ast = \frac{1}{2}(p_H^{t+1} + p_F^{t+1}) Z_{B,t}^{-1}$.

While both the stock market participants and non-participants receive labor income, the capital income is endowed only to the stock market participants.\(^4\) Recalling that the non-participants can only invest in the international bond, the budget constraints for the two types of home agents can be written as

\[
W_t^p = r_{b,t} W_{t-1}^p + \alpha_1^p (r_{A,t} - r_{b,t}) + \alpha_2^p (r_{A,t}^* - r_{b,t}) - C_t^p + \frac{1}{\mu} Y_{K,t} + Y_{L,t} \\
W_t^{np} = r_{b,t} W_{t-1}^{np} - C_t^{np} + Y_{L,t},
\]

where $W_t^p = \alpha_1^p + \alpha_2^p + \alpha_3^p$ and $W_t^{np} = \alpha_3^p$ denote net financial wealth of domestic participants and non-participants, respectively. The net amounts invested by an agent are denoted by $\alpha_1$ for the home stock, $\alpha_2$ for the foreign stock, and $\alpha_3^p$ for the international bond, with superscripts $p$ and $np$ referring to participants and non-participants, respectively.

\(^4\)As pointed out in Guvenen (2009), the 20% of US households who participate in the stock market own 90% of the economy’s wealth.
Note that since we defined wealth and asset positions as net positions\textsuperscript{5}, asset market clearing is given by

\[
\mu (W_t^p + W_t^{p*}) + (1 - \mu) (W_t^{np} + W_t^{np*}) = 0
\]

\[
\alpha_{1,t} + \alpha_{1,t}^{p*} = 0
\]

\[
\alpha_{2,t} + \alpha_{2,t}^{p*} = 0,
\]

where foreign variables are denoted by \(^*\).

4 Model Solution

The model is challenging to solve. It is not amenable to standard global solution techniques such as value function iteration because the incompleteness of financial markets requires that we solve for the decentralized equilibrium directly. Due to the large number of state variables, the computational burden of doing this is prohibitive.

The method proposed by Chien et al. (2011) who solve incomplete markets economies with heterogeneous trading technologies using stochastic Lagrange multipliers and measurability constraints is inapplicable in our setup as well. The reason for this is that their aggregation result for consumption fails due to the differentiated goods in our setup and the home bias in preferences.\textsuperscript{6}

For these reasons, we solve the model using second-order linearization techniques. This requires solving for the steady-state and first-order portfolio choice, for which we implement the method suggested in Devereux and Sutherland (2010). We rely on Dynare to implement this approach.

5 Data and Calibration

We follow a conservative calibration strategy in that we do not choose the structural parameters of our model to directly match the stylized facts for asset price, consumption, and exchange rate dynamics. Rather, we set the structural parameters to the values that are

\textsuperscript{5}As an example, consider the case where the home agents has a zero net position in both stocks and the bond \((\alpha_{1}^{p} = \alpha_{2}^{p} = W^p = 0)\). His gross position would correspond to holding all of the home equity and an amount \(l - 1\) times the size of his gross equity position in the bond. He would hold non of the foreign stock.

\textsuperscript{6}While extending their method to accommodate differentiated goods and heterogeneous specifications of goods aggregators might be possible, doing this would require a substantial methodological contribution.
Table 1: Data Summary

Panel A shows covariances between U.S. and foreign traded production (row 1), U.S. and foreign non-traded production (row 2), and U.S. traded and foreign non-traded production (row 3). The moments refer to real, per-capital production that has been logged and hp-filtered. Panel B shows the correlation of real per-capita consumption growth between the US and the foreign countries for non-durable consumption (row 1) and total consumption (durables plus non-durables and services, row 2). Panel C shows the volatility of the real exchange rate between the US and the foreign countries. Panel D shows the average share of a country’s trade with the US as a percentage of total US trade between our set of countries. The “Average” column uses the trade weights from Panel D.

Data is annual and covers 1970 to 2012 with the exception of non-durable consumption, which starts in the 1990's for some of our countries and is not available for the UK at all. See Appendix A for more details.

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Correlation of Output by Sector</strong></td>
<td></td>
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<tr>
<td>Traded</td>
<td>0.74</td>
<td>0.85</td>
<td>0.68</td>
<td>0.59</td>
<td>0.68</td>
<td>0.61</td>
<td>0.85</td>
</tr>
<tr>
<td>Non-Traded</td>
<td>0.61</td>
<td>0.75</td>
<td>0.78</td>
<td>0.24</td>
<td>0.43</td>
<td>0.47</td>
<td>0.71</td>
</tr>
<tr>
<td>Traded / Non-Traded</td>
<td>0.46</td>
<td>0.58</td>
<td>0.62</td>
<td>-0.05</td>
<td>0.09</td>
<td>0.38</td>
<td>0.84</td>
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<tr>
<td><strong>Panel B: Correlation of Consumption Growth</strong></td>
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<tr>
<td>Non-Durable</td>
<td>0.67</td>
<td>0.85</td>
<td>0.75</td>
<td>0.53</td>
<td>0.81</td>
<td>0.39</td>
<td>N/A</td>
</tr>
<tr>
<td>Total</td>
<td>0.52</td>
<td>0.61</td>
<td>0.49</td>
<td>0.39</td>
<td>0.29</td>
<td>0.39</td>
<td>0.72</td>
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<tr>
<td><strong>Panel C: Real Exchange Rate</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Volatility (%)</td>
<td>9.5</td>
<td>6.8</td>
<td>11.1</td>
<td>11.4</td>
<td>11.0</td>
<td>11.8</td>
<td>11.8</td>
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<tr>
<td><strong>Panel D: Trade Weights</strong></td>
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<tr>
<td></td>
<td>0.43</td>
<td>0.06</td>
<td>0.11</td>
<td>0.05</td>
<td>0.26</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

typically used in the literature. Similarly, the moments of traded and non-traded output are calibrated to directly match their empirical counterparts.

The next section describes the data and summarizes some empirical regularities regarding international production, consumption, and exchange rates. We then discuss the model calibration.

5.1 Data

Our main data sources are the National Accounts database provided by the OECD and the International Financial Statistics and Direction of Trade Statistics databases offered by the IMF. We draw on these international datasources rather than on national ones in order to have data measures that are comparable across countries. Furthermore, we use annual data which allows us to analyze time series that start early, ranging from 1970 to 2012. The only
time series that is not available for this time horizon is that for non-durable consumption, which starts in the 1990’s for most of our countries. Table 1 summarizes the data.

We analyze the data from the perspective of the US as the home country and focus on the other G7 economies, including Canada, France, Germany, Italy, Japan, and the UK, as the foreign countries. We then use the trade-weights reported in Panel D to average the moments with respect to the foreign countries.

While international co-movements in traded and non-traded production have been previously analyzed in the literature, (e.g. Stockman and Tesar (1995)) the data used in these studies only extends to 1990. Since then, rapid technological progress has facilitated international trade. We hence find it important to study more recent data.

We start by analyzing the international co-movement of output in the traded and non-traded sectors among the G7 countries. Following the methodology of Kravis et al. (1982) and Stockman and Tesar (1995), we assign output to be either tradable or non-tradable depending on the sector of production. We consider agriculture, fishing, mining, manufacturing, electricity and utilities, retail, hotels, and transportation to be tradable.\(^7\) The remaining categories, including construction, finance, real estate, and other services are assigned to the non-tradable sector.

Panel A shows the resulting correlations for real per-capita output in the two sectors that has been logged and hp-filtered. The first row shows the correlation between traded output in the US and traded output in the foreign country. The correlations range from 0.59 for Germany to 0.85 for Canada and the UK. The trade-weighted average, which takes into account the relative importance of a country for US trade, is quite high at 0.74. This is due in large part to the high correlation with Canada, which is responsible for nearly half of US trade among the G7 countries.

The second row of the panel shows the correlation of US non-traded output with foreign non-traded output. For all our countries, these correlations are lower than those for traded output, with a trade-weighted average of 0.61. Finally, while the correlations within the same sectors between countries tend to be quite high, the correlation of traded output in the US with non-traded output abroad reported in row 3 is significantly lower for most countries and even slightly negative for Germany. The trade-weighted average for this correlation between sectors is only 0.46. Overall, these results are quite comparable in magnitude to

\(^7\)While electricity and utilities are arguably non-tradable, in particular as they refer to the associated distribution services, they are reported together with manufacturing, which is a large component of tradable goods. Since electricity and utilities only make up a small fraction of output, classifying them as tradable rather than non-tradable does not have a significant bearing on the results.

One of our main moments of interest is the correlation of consumption growth between countries. Since all consumption in our model is non-durable, the appropriate moment to match is the correlation of real per-capita consumption growth between countries. From row 1 of Panel B, this correlation ranges from 0.85 with Canada to 0.39 with Japan, averaging 0.67. Since data on non-durable consumption is only available since 1990 for most countries and unavailable for the UK, we also compute the correlation from total household final consumption expenditure which is available over the period from 1970 to 2012. Row 2 of the panel shows that this correlation is significantly lower for all countries, averaging 0.52.

We are also interested in the volatility of real exchange rate growth. Panel C shows that this volatility is around 11% for all countries except for Canada, where it is almost half, at 6.8%. Since Canada is the most important trade partner for the US, we find the trade-weighted average volatility of real exchange rate growth to be 9.5%.

5.2 Calibration

We calibrate the endowment processes for tradable and non-tradable goods to their empirical counterparts in the data. For the within-country moments, we calibrate to US data. This leads us to setting the persistence parameter equal to $\Psi_T = 0.27$ for the tradable sector and $\Psi_N = 0.45$ to the non-tradable sector, matching the first order autocorrelation of hp-filtered US production. Similarly, we choose the volatility of the endowment shocks so that the implied standard deviation of traded output, $\text{std}(Y^T) = 0.023$, and non-traded output, $\text{std}(Y^N) = 0.010$, match that from US data. We calibrate the size of the traded sector to match the average share of traded goods in US production, leading to a tradables share of $\theta_T = 0.35$. Finally, we chose the covariance of the shocks to traded and non-traded goods within a country to match the correlation of traded and non-traded production in the US, setting $\text{cor}(Y^N, Y^T) = 0.64$.

We calibrate between-country moments of output to the average correlation between US and foreign production for a given sector. The moments we match are $\text{cor}(Y^T, Y^{T*}) = 0.74$ for the correlation of traded production between countries, $\text{cor}(Y^N, Y^{N*}) = 0.61$ for the correlation of non-traded production between countries, and $\text{cor}(Y^N, Y^{T*}) = 0.46$ for the correlation of domestic non-tradable output with foreign tradable output.

We follow the working paper version of Guvenen (2009) in calibrating relative risk aversion, the time discount rate, and stock market participation. Specifically, we set relative

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8The working paper version of Guvenen (2009) differs from the published paper in that it uses
Table 2: Calibration

The model is calibrated at annual frequency.

<table>
<thead>
<tr>
<th>Parameter Source</th>
<th>Parameter</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>$\gamma_p = 3$</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{np} = 10$</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Weight on traded goods</td>
<td>$a_T = 0.55$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td>Home bias in tradables</td>
<td>$a_D = 0.72$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\frac{1}{1-\rho} = 0.85$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{1-\phi} = .74$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td>Endogenous discount factor</td>
<td>$\eta = .1$</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Steady-state discount rate</td>
<td>$\omega C^{-\eta} = 0.95$</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Distribution cost</td>
<td>$\nu = 0.85$</td>
<td></td>
</tr>
<tr>
<td>Labor share</td>
<td>$\theta_L = 0.7$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td>Tradable share</td>
<td>$\theta_T = 0.35$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td>Stock market participation rate</td>
<td>$\mu = 0.3$</td>
<td>Guvenen (2009)</td>
</tr>
</tbody>
</table>

| Endowments | $\Psi^T = 0.27$ | |
| Autocorrelation of tradables | $\Psi^N = 0.45$ | |
| Autocorrelation of non-tradables | | |
| Implied moments | $\text{std}(Y^T) = 0.023$ | |
| | $\text{std}(Y^N) = 0.010$ | |
| | $\text{cor}(Y^N, Y^T = 0.64)$ | |
| | $\text{cor}(Y^T, Y^{T*}) = 0.74$ | |
| | $\text{cor}(Y^N, Y^{N*}) = 0.61$ | |
| | $\text{cor}(Y^N, Y^{T*}) = 0.46$ | |

risk aversion to $\gamma_p = 3$ for stockholders and $\gamma_{np} = 10$ for non-stockholders. The steady-state discount rate is $\omega C^{-\eta} = 0.95$.\(^9\) We calibrate the stock market participation rate to $\mu = 0.3$. While Guvenen (2009) uses a parameter value of 0.2, he also points to recent evidence that stock market participation has increased. We take this into account by choosing a slightly higher participation rate than him as we calibrate the model to more recent data.

The remaining utility parameters refer to agent’s preferences over the different types of power utility (as this paper) instead of recursive Epstein-Zin preferences.

\(^9\)We use a value of $\eta = .1$ for the curvature of the endogenous discount factor, which is reasonably small while still producing a stable model solution.
goods. Here, we follow Corsetti et al. (2008). Like them, we set the utility weight of tradables to \( \theta_T = 0.35 \), matching the share of traded goods in the US consumption basket, and the home bias in tradable goods to \( a_D = 0.72 \). Similarly, we chose the elasticity of substitution between the two traded goods to be \( \frac{1}{\Gamma - \rho} = 0.85 \) and the elasticity between the traded and non-traded good to be \( \frac{1}{\Gamma - \varphi} = .74 \), which the authors obtain by performing a method of moments estimation on a model whose goods market structure is similar to ours.

A key parameter in our model is the distribution cost parameter \( v \). As will become apparent in the next section, the distribution cost are quantitatively the most important feature of the model in reducing the correlation of consumption growth between countries. The higher \( v \), the lower the consumption correlation. While we want to restrict the magnitude of the distribution cost to be consistent with results in previous studies, we chose it to be on the high end of that spectrum. This allows us to evaluate how far the present model can go in matching the low consumption correlation in the data. When we study the quantitative importance of distribution cost for our results, we will then conduct extensive sensitivity analysis with regards to this parameter. There are several studies that estimate the distribution margin, which is defined as \( \kappa = v \frac{\rho}{\rho^*} \). Burstein et al. (2003) find that the share of the retail price accounted for by distribution services is between 40% to 50% in the US, depending on the industry. Anderson and Wincoop (2004) find that distribution cost average more than 55% among industrialized countries. Considering this evidence, we set \( \nu = 0.85 \), which implies a steady-state distribution margin of 64% in our model.

6 Results

6.1 Full Model

Table 3 summarizes the moments implied by the fully featured model. The model matches asset prices rather well. The Sharpe ratio of 0.31 is nearly identical to that in the data. While the model produces a realistic price of risk, the equity premium is only 3.06%, about half of what it is in the data. This is not surprising, however, given that there is no financial leverage in the model and hence the quantity of risk is less than in the data. This is also reflected in the fact the the equity premium is about half as volatile in the model as in the data. The risk-free rate is 0.73%, which is just slightly lower than in the data. The standard deviation of the risk-free rate is higher than in the data, with 9.94% in the model compared to 5.44% in the data.

The model-implied correlation of consumption growth is 0.75, which is somewhat larger than the value of 0.67 that is implied using non-durable consumption data and considerably larger than the value of 0.52 the we measure using total household final consumption
Table 3: Results

All moments are annual and in percent.

<table>
<thead>
<tr>
<th>Asset markets</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity premium</td>
<td>3.06</td>
<td>6.17</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.73</td>
<td>1.94</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Volatility of risk-free rate</td>
<td>9.94</td>
<td>5.44</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.31</td>
<td>0.32</td>
<td>Guvenen (2009)</td>
</tr>
<tr>
<td>Exchange rate growth volatility</td>
<td>3.52</td>
<td>9.50</td>
<td></td>
</tr>
</tbody>
</table>

| Consumption growth             |       |      |                 |
| Aggregate volatility           | 1.25  | 1.95 |                 |
| Volatility participants/non-participants | 3.30 | > 2  | Guvenen (2009)  |
| Cross country correlation      | 0.75  | ≈ 0.6|                 |

expenditure. The model hence makes substantial progress in generating less than perfect consumption co-movement between countries though it falls short of fully explaining the low consumption correlation in the data.

Finally, we find that the volatility of exchange rate growth is low. It is less than half than what we measure in the data. This finding is consistent with much of the literature, e.g. the international real business cycle model of Backus et al. (2008).

We proceed by analyzing the role of distribution cost for the model mechanism in Section 6.2 and then quantify the importance of all our main model ingredients for our results in Section 6.3.

6.2 The Importance of Distribution Cost

Table 4 shows how the model results change for varying values of the distribution cost parameter $\nu$. The comparative statics illustrate the importance of distribution cost for the model’s ability to produce a consumption correlation below unity. In fact, without distribution cost, consumption co-moves nearly perfectly between countries despite the existence of non-tradable goods and a low trade elasticity. The correlation only drops significantly for values of the distribution cost that are on the high end of the empirically observed spectrum, reaching a correlation of 0.75 in our benchmark calibration with $\nu = 0.85$.

To provide intuition for the effect of distribution cost on the correlation of consumption,
Table 4: Distribution Cost

The table shows the correlation of aggregate consumption growth between countries, the Sharpe ratio, and the volatility of the exchange rate in the benchmark model for varying degrees of distribution cost $\nu$.

<table>
<thead>
<tr>
<th>Distribution cost $\nu$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption correlation</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
<td>0.93</td>
<td>0.81</td>
<td>0.75</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.23</td>
<td>0.23</td>
<td>0.22</td>
<td>0.22</td>
<td>0.27</td>
<td>0.31</td>
</tr>
<tr>
<td>Exchange rate volatility</td>
<td>1.57</td>
<td>1.83</td>
<td>2.27</td>
<td>3.01</td>
<td>3.90</td>
<td>3.52</td>
</tr>
</tbody>
</table>

we return to our analysis from Section 2.2. In particular, we focus on how distribution cost lower the effective elasticity of substitution between the traded goods at the consumer level and hence amplify the magnitude of the income effect studied in that section.

To see how introducing distribution cost helps lower the correlation of consumption growth, consider the equivalent of Equation 3 with distribution cost,

$$\frac{\partial C_H}{\partial \tau} = \omega (1 - \kappa) (1 - a_H) \left( \frac{P_F}{P_H} \right)^{1-\omega} - (1 - a_H) \left( \frac{P_F}{P_H} \right)^{1-\omega} - \kappa a_H.$$

Similar to above, this equation shows the response of domestic demand for the home good to a fall in its price (increase in $\tau$) in a model without financial markets. The expression here however takes account of distribution cost, which are linear in the distribution margin $\kappa = \nu \frac{p_N}{p_H}$. We see that an increase in the distribution margin lowers the magnitude of the substitution effect and increases the (negative) importance of the income effect.

Next, we turn to the importance of distribution cost for the price of risk in the economy. Without the cost, the Sharpe ratio is 0.23. It increases to 0.31 for the high value of the cost in our benchmark calibration. The reason is that the utility-based consumption index becomes more volatile as the relative shares of tradables to non-tradables fluctuate more strongly. This increase in volatility due to the distribution cost is also reflected in the volatility of the exchange rate, which increases in the cost as well. It is, however, low compared to the data for the entire range of the parameter studied here.

6.3 Relative Importance of Model Ingredients for the Results

In this section, we quantify the relative importance of our main model ingredients, limited stock market participation, non-traded goods, and distribution cost, for our main results.
Table 5: The Contribution of the Model Ingredients

The table shows the correlation of aggregate consumption growth between countries, the sharpe ratio, and the volatility of exchange rate growth for six different model specifications. The two asset market specifications permit either full stock market participation ($\mu = 1$) or limited stock market participation ($\mu = 0.3$). The goods market either includes only traded goods (T), traded and non-traded without distribution cost (T/NT), or the fully featured goods market specification with traded and non-traded goods as well as distribution cost (T/NT/Dist).

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>T/NT</th>
<th>T/NT/Dist</th>
<th>T</th>
<th>T/NT</th>
<th>T/NT/Dist</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = 1.0$</td>
<td>$0.99$</td>
<td>$0.97$</td>
<td>$0.74$</td>
<td>$0.03$</td>
<td>$0.04$</td>
<td>$0.06$</td>
</tr>
<tr>
<td>$\mu = 0.3$</td>
<td>$1.00$</td>
<td>$0.99$</td>
<td>$0.75$</td>
<td>$0.18$</td>
<td>$0.24$</td>
<td>$0.31$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumption correlation</th>
<th>Sharpe ratio</th>
</tr>
</thead>
</table>
| Exchange rate volatility
| $\mu = 1.0$ | $0.64$ | $1.37$ | $2.14$ |
| $\mu = 0.3$ | $0.67$ | $1.57$ | $3.52$ |

Table 5 shows our main moments of interest, the consumption growth correlation, the Sharpe ratio, and the exchange rate volatility for six different model specifications. We study three special cases with respect to the goods market: only traded goods, traded and non-traded goods but without distribution cost, and traded and non-traded goods with distribution cost. For each of these three cases, we solve a version of the model with and without limited stock market participation.

First, we find that virtually all of the reduction in the consumption growth correlation comes from distribution cost. Irrespective of the financial market setup, we find that the consumption growth correlation is around 0.75 with distribution cost and close to unity without.

With regards to the Sharpe ratio, we find that the model without stock market participation only produces a small price of risk that does not exceed a Sharpe ratio of 0.06. However, once limited stock market participation is introduced, the price of risk does increase significantly. If all goods are tradable, the Sharpe ratio reaches 0.18 and increases significantly both with the addition of non-traded goods and by introducing distribution cost. The goods market setup matters for asset prices as both non-tradability and distribution cost raise the volatility of the utility based consumption index.

Finally, the model produces very smooth exchange rates in all versions, ranging from a standard deviation of 0.64% in the model with full stock market participation and all traded goods to a standard deviation of 3.52% in the model with limited stock market participation and non-traded goods with distribution cost.
6.4 Consumption Dynamics by Agent Type

Our analysis thus far has focused on the correlation of aggregate consumption growth between countries and we have shown that the model is capable of producing a correlation as low as 0.75. We next analyze the consumption dynamics in more detail by focusing on the different agent types.

Here, we find that the current model has an unappealing implication. In particular, consumption growth between stockholders and non-stockholders is nearly perfectly negatively correlated. The reason for this is the low persistence of output that we measure in the data and to which we calibrate our endowment processes. As agents are hit by a positive supply shock, they expect output to mean-revert quickly. Hence they expect negative future consumption growth. Non-stockholders would like to increase their precautionary savings to smooth the expected reversion of output. Stockholders, on the other-hand, are reluctant to increase their borrowing substantially. As a result, the interest rate falls and reduces the value of non-stockholders’ bond holdings (which are positive, on average). This reduction in wealth forces non-stockholders to reduce their consumption despite the positive endowment shock. While calibrating output to be highly persistent with an auto-correlation above 0.95 annually resolves this problem, we find that output is just not nearly this close to a unit root in the data.

As expected, we find that the correlation of consumption growth between foreign and domestic stockholders is unity as they have access to virtually complete financial markets. The above fact that consumption of non-stockholders is almost perfectly negatively correlated with that of stockholders then implies that the correlation of consumption growth between foreign and domestic non-stockholders is nearly unity as well. Consumption for both groups of non-stockholders hence moves together and against that of stockholders. The result that the aggregate consumption growth between countries is only 0.75 despite consumption for stockholders and non-stockholders co-moving nearly perfectly then comes from the cross-correlation of stockholders consumption in one country with that of non-stockholders in the other. This correlation is almost perfectly negative, driving down the aggregate consumption growth correlation.

It is worth pointing out that the low aggregate consumption growth correlation the model achieves still obtains if we allow all agents to participate in the stock market, as can be seen from table 5. Hence, this results does not hinge on the unappealing co-movement of consumption between the different groups of agents. That said, we regard improving the model to ameliorate its implications along this dimension as a crucial next step for future research.
7 Conclusion

We propose a general equilibrium two-county macro-finance model that features limited stock market participation, non-traded goods and distribution cost. The model makes significant progress towards rationalizing the coexistence of three stylized data facts that have been a challenge for theory thus far: i) The high equity premium, ii) relatively smooth exchange rates, and iii) the low international correlation of consumption growth.

Consistent with closed-economy models, the limited stock market participation friction produces a high and realistic price of risk. We further find that distribution cost play a central role for reducing international consumption co-movement while also amplifying risk premia. The model naturally produces a low exchange rate volatility that is even lower than in the data, irrespective of the severity of the frictions we study.

Future research will need to focus on resolving the stark implications for the consumption dynamics between agent types that are implied by the model.
References


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APPENDIX

A  Data

Our main data sources are the OECD National Accounts and the IMF International Financial Statistics. To ensure that we have long time series, we use data at the annual frequency. For most measures, we are able to obtain data from 1970 to 2012. Table 1 provides summary statistics for the data.

A.1  Production of Tradables and Non-Tradables

We use annual data covering 1970 to 2012 from the OECD on value-added by sector for the US, Canada, France, Germany, Italy, Japan, and the UK. We then categorize output to be either tradable or non-tradable, depending on its sector, as described in the main text.

The data we retrieve is measured in constant prices and PPPs fixed in the OECD base year (2005). We then divide by each country’s population (also from the OECD) to get sectoral output in per-capita terms. Finally, we detrend the data by taking logs and applying an hp-filter with smoothing parameter 6.25. Table 1 shows the resulting correlations of detrended output across sectors between the US and our set of six foreign countries.

A.2  Consumption

We use two different measures of consumption provided by the OECD, one that measures only non-durable consumption and one that measures total final household consumption.

Our measure for non-durable consumption is the sum of household final consumption expenditure for non-durables and services. These time series are in constant prices (OECD base year = 2005) and hence are additive. This measure is available starting in 1981 for Canada, 1959 for France, 1991 for Germany, 1995 for Italy, 1994 for Japan, and 1970 for the US. It is not available for the UK.

To have a longer time series of consumption, we also obtain final household consumption expenditure in constant prices of the OECD base year. This measure is available from 1970 to 2012 for all countries.

For both measures, we then divide by the country’s population, take logs, and compute the growth rate. Panel B of table 1 shows the correlation of US consumption growth with our set of foreign countries for the two measures.
A.3 Exchange Rates

We retrieve annual end-of-period nominal exchange rate data from the IMF IFS Database. Exchange rates are in terms of foreign currency to USD. To convert the nominal exchange rates to real terms, we use the deflator for household final consumption expenditure provided by the OECD. Panel C of table 1 shows the volatility of real exchange rate growth that we obtain. The data cover 1970 to 2012 for all countries.

A.4 Trade Weights

We obtain data on imports and exports between the US and our set of foreign countries from the IMF Direction of Trade Statistics database. The data are in USD terms and span 1970 to 2012 for all countries. Then, for every year, we determine a country’s trade weight with the US as the sum of imports and exports between that country and the US divided by the sum of all imports and exports between the US and our complete set of foreign countries. This procedure yields one trade weight for every country in every year. We then compute the time-series averages for every country. The resulting trade weights are reported in Panel D of table 1.

B Goods Market Clearing

The goods market clearing conditions are for domestic tradables, domestic non-tradables, foreign tradables, and foreign non-tradables (in that order) are

\[
\begin{align*}
\mu \left(c_p^H + c_p^{\ast}\right) + (1 - \mu) \left(c_{np}^H + c_{np}^{\ast}\right) &= \theta_T Y^T \\
\mu \left(c_p^N + \nu c_p^H + \nu c_p^{\ast}\right) + (1 - \mu) \left(c_{np}^N + \nu c_{np}^H + \nu c_{np}^{\ast}\right) &= (1 - \theta_T) Y^N \\
\mu \left(c_p^F + c_p^{\ast}\right) + (1 - \mu) \left(c_{np}^F + c_{np}^{\ast}\right) &= \theta_T Y^{T*} \\
\mu \left(c_p^N + \nu c_p^H + \nu c_p^{\ast}\right) + (1 - \mu) \left(c_{np}^N + \nu c_{np}^H + \nu c_{np}^{\ast}\right) &= (1 - \theta_T) Y^{N*},
\end{align*}
\]

where * denotes foreign variables.

C Portfolios Choice

This section outlines how we adapt the method in Devereux and Sutherland (2010) and Devereux and Sutherland (2011) to the case with multiple assets and a non-zero exchange rate.
C.1 Portfolio Choice Equations

In what follows, we denote the base asset, corresponding to the international bond in the main text, as asset 4. The exchange rate is denoted by $E$ and all returns are converted to units of the home consumption basket.

For every asset $m$, the home and foreign portfolio choice equations are

$$
E \left[ C_{t+1}^{\gamma} \left( R_{m,t+1} - R_{4,t+1} \right) \right] = 0
$$

$$
E \left[ C_{t+1}^{\ast - \gamma} \frac{1}{E_{t+1}} \left( R_{m,t+1} - R_{4,t+1} \right) \right] = 0.
$$

C.2 Steady-state portfolio

Expanding the portfolio choice equations to the second order accuracy and taking the difference yields

$$
E \left[ \left( \hat{C}_{t+1} - \hat{C}_{t+1}^{\ast} - \hat{E}_{t+1} / \gamma \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] = 0 + O \left( \epsilon^3 \right).
$$

The steady-state portfolio is the one that satisfies this equation as outlined in Devereux and Sutherland (2011).

C.3 First-order portfolio

Following Devereux and Sutherland (2010), we expand the third-order portfolio choice equations to the third order. For the domestic country, this is

$$
E \left[ \hat{C}^{\gamma} \left( \hat{R}_{m} - \hat{R}_{4} \right) - \gamma \left( \hat{R}_{m} - \hat{R}_{4} \right) \hat{C}^{\gamma} \hat{C}_{t+1} + \hat{C}^{\gamma} \hat{R} \hat{C}_{t+1} - \hat{C}^{\gamma} \hat{R} \hat{R}_{m,t+1} - \hat{C}^{\gamma} \hat{R} \hat{R}_{4,t+1} \right.
$$

$$
+ \frac{1}{2} \gamma^2 C^{\gamma} \hat{R} \hat{C}_{t+1} + \frac{1}{2} \gamma^{\gamma} C^{\gamma} \hat{R} \hat{R}_{m,t+1} - \frac{1}{2} \gamma^{\gamma} C^{\gamma} \hat{R} \hat{R}_{4,t+1}]
$$

$$
+ \frac{3}{6} \gamma C^{\gamma} R \hat{C}_{t+1} \hat{R}_{m,t+1} - \frac{3}{6} \gamma^{\gamma} C^{\gamma} R \hat{C}_{t+1} \hat{R}_{m,t+1} - \frac{3}{6} \gamma^{\gamma} C^{\gamma} R \hat{C}_{t+1} \hat{R}_{m,t+1} - \frac{3}{6} \gamma^{\gamma} C^{\gamma} R \hat{C}_{t+1} \hat{R}_{m,t+1} + \frac{3}{6} \gamma^{\gamma} C^{\gamma} R \hat{C}_{t+1} \hat{R}_{m,t+1} + \frac{3}{6} \gamma^{\gamma} C^{\gamma} R \hat{C}_{t+1} \hat{R}_{m,t+1}
$$

$$
- \frac{1}{6} \gamma^{\gamma} C^{\gamma} \left( \hat{R}_{m} - \hat{R}_{4} \right) C_{t+1}^{\gamma} + \frac{1}{6} \gamma^{\gamma} C^{\gamma} \hat{R} \hat{C}_{t+1} - \frac{1}{6} \gamma^{\gamma} C^{\gamma} \hat{R} \hat{R}_{m,t+1} \right] = 0 + O \left( \epsilon^4 \right)
$$

The third-order expansion of the foreign portfolio choice equation is (where $C^*$ is replaced
by \( C \) for notational convenience)

\[
E \left[ C^{-\gamma} \frac{1}{E} (\hat{R}_m - \hat{R}_4) - \gamma (R_m - R_4) \frac{1}{E} \hat{C}_{t+1} + C^{-\gamma} \frac{1}{E} \hat{E}_{t+1} + C^{-\gamma} \frac{1}{E} \hat{R}_{m,t+1} - C^{-\gamma} \frac{1}{E} \hat{R}_{4,t+1} \right]
\]

\[
- \frac{1}{2} \gamma \left( \hat{C}_{t+1} - \hat{C}_{t+1}^* \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) - \frac{3}{6} \gamma C^{-\gamma} \left( \hat{C}_{t+1}^2 + \hat{C}_{t+1} \hat{E}_{t+1} + \frac{3}{6} \gamma C^{-\gamma} R \hat{C}_{t+1}^2 \hat{R}_{m,t+1} \right)
\]

\[
+ \frac{3}{6} \gamma C^{-\gamma} \left( \hat{R}_{E_{t+1}}^2 \hat{R}_{m,t+1} - \frac{3}{6} \gamma C^{-\gamma} \hat{R}_{E_{t+1}}^2 \hat{R}_{4,t+1} + \frac{3}{6} \gamma C^{-\gamma} \hat{R}_{E_{t+1}} \hat{R}_{m,t+1} \right)
\]

\[
+ \frac{1}{6} \gamma \left( \hat{R}_{E_{t+1}}^3 \hat{R}_{m,t+1} - \frac{1}{6} \gamma \hat{R}_{E_{t+1}}^3 \hat{R}_{4,t+1} + \gamma \hat{C}_{t+1} \hat{E}_{t+1} \hat{R}_{m,t+1} \right]
\]

= \( O \left( \epsilon^4 \right) \)

Take the Difference of the portfolio choice equations to get

\[
E \left[ \left( \hat{C}_{t+1} - \hat{C}_{t+1}^* \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] = 0 + O \left( \epsilon^4 \right)
\]

The first-order portfolio choice is the one that satisfies this equation.

Next, sum the Euler Equations to get

\[
E_t \left[ \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right] = E_t \left[ -\frac{1}{2} \left( \hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) - \frac{1}{6} \left( \hat{R}_{m,t+1}^3 - \hat{R}_{4,t+1}^3 \right) \right]
\]

\[
+ \frac{\gamma}{2} \left( \hat{C}_{t+1} + \hat{C}_{t+1}^* \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right)
\]

\[
- \frac{\gamma^2}{4} \left( \hat{C}_{t+1}^2 + \hat{C}_{t+1}^* \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) + \frac{\gamma}{4} \left( \hat{C}_{t+1} + \hat{C}_{t+1}^* \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right)
\]

\[
+ \frac{1}{4} \hat{E}_{t+1} \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) - \frac{1}{4} \hat{E}_{t+1} \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right)
\]

\[
+ \frac{1}{4} \hat{E}_{t+1} \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) - \frac{1}{2} \gamma \hat{C}_{t+1} \hat{E}_{t+1} \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right)
\]

= \( O \left( \epsilon^4 \right) \)
The state space solution for \((\hat{C}_{t+1} - \hat{C}^*_t - E_{t+1}/\gamma)\) and \(\hat{r}_{x,t+1}\) can be expressed as:

\[
(\hat{C} - \hat{C}^* - E_{t+1}/\gamma) = \begin{bmatrix} \tilde{D}_0 + [\tilde{D}_2]_i [\varepsilon]^i + \tilde{D}_3 \left( [z^f]^k + [z^s]^k \right) + [\tilde{D}_4]_{ij} [\varepsilon]^i [\varepsilon]^j + [\tilde{D}_5]_{ki} [\varepsilon]^i [k]^k \end{bmatrix} + O (\varepsilon^3)
\]

\[
[\hat{r}_x]_m = \begin{bmatrix} \tilde{R}_0 + [\tilde{R}_1]_m \xi + [\tilde{R}_2]_{mi} [\varepsilon]^i + [\tilde{R}_3]_{mk} \left( [z^f]^k + [z^s]^k \right) + [\tilde{R}_4]_{mij} [\varepsilon]^i [\varepsilon]^j + [\tilde{R}_5]_{mki} [\varepsilon]^i [z^f]^k + \tilde{R}_6]_{mij} [z^f]^i [z^f]^j \end{bmatrix} + O (\varepsilon^3)
\]

Up to first-order accuracy, the expected excess return is zero and, up to second-order accuracy, it is a constant. This implies that \([\tilde{R}_3]_{mk} [z^f]^k = 0\) and the terms \([\tilde{R}_3]_{mk} [z^s]^k\) and \([\tilde{R}_6]_{mij} [z^f]^i [z^f]^j\) are constants. It also follows that

\[
[\tilde{R}_0]_m = E [\hat{r}_x]_m - [\tilde{R}_3]_{mk} [z^s]^k - [\tilde{R}_4]_{mij} [\Sigma]^{ij} - [\tilde{R}_6]_{mij} [z^f]^i [z^f]^j
\]

so

\[
[\hat{r}_x]_m = E [\hat{r}_x]_m - [\tilde{R}_4]_{mij} [\Sigma]^{ij} + [\tilde{R}_1]_m \xi + [\tilde{R}_2]_{mi} [\varepsilon]^i + [\tilde{R}_4]_{mij} [\varepsilon]^i [\varepsilon]^j + [\tilde{R}_5]_{mki} [\varepsilon]^i [z^f]^k + O (\varepsilon^3)
\]

Now recognize that \(\xi\) is endogenous and given by \(\xi = [\gamma]_{mk} [z^f]^k [\hat{r}_x]_m\). This is a second-order term, so \(\hat{r}_x\) can be replaced by its first-order parts, i.e., by \([\tilde{R}_2]_{mi} [\varepsilon]^i\). This implies that \(\xi = [\tilde{R}_2]_{mi} [\gamma]_{mk} [\varepsilon]^i [z^f]^k\).

Now, we can write (note, here the asset index that’s summed over is \(q\), the one that’s held fixed is \(m\)):

\[
(\hat{C} - \hat{C}^* - E_{t+1}/\gamma) = \begin{bmatrix} \tilde{D}_0 + [\tilde{D}_2]_i [\varepsilon]^i + \tilde{D}_3 \left( [z^f]^k + [z^s]^k \right) + [\tilde{D}_4]_{ij} [\varepsilon]^i [\varepsilon]^j + \left( [\tilde{D}_5]_{ki} + [\tilde{D}_1]_m [\gamma]_{mk} [\varepsilon]^i [k]^k \right) + \tilde{D}_6]_{ij} [z^f]^i [z^f]^j \end{bmatrix} + O (\varepsilon^3)
\]

\[
[\hat{r}_x]_m = E [\hat{r}_x]_m - [\tilde{R}_4]_{mij} [\Sigma]^{ij} + [\tilde{R}_2]_{mi} [\varepsilon]^i + [\tilde{R}_4]_{mij} [\varepsilon]^i [\varepsilon]^j + \left( [\tilde{R}_5]_{mki} + [\tilde{R}_1]_m [\tilde{R}_2]_{qi} [\gamma]_{qk} [\varepsilon]^i [z^f]^k \right) + O (\varepsilon^3)
\]
Furthermore, we use the following expressions for consumption

\[
\hat{C} = \left[ \hat{C}_2^H \right]_i [\epsilon] + \left[ \hat{C}_3^H \right]_k [z^f]^k + O (\epsilon^2)
\]

\[
\hat{C}^* = \left[ \hat{C}_2^F \right]_i [\epsilon] + \left[ \hat{C}_3^F \right]_k [z^f]^k + O (\epsilon^2),
\]

return to asset \( m \)

\[
[\hat{r}]_m = \left[ \hat{R}_2^m \right]_i [\epsilon] + \left[ \hat{R}_3^m \right]_k [z^f]^k + O (\epsilon^2),
\]

and the exchange rate

\[
\hat{E} = \left[ \hat{H}_2 \right]_i [\epsilon] + \left[ \hat{H}_3 \right]_k [z^f]^k + O (\epsilon^2).
\]

Fixing asset \( m \), we get

\[
\left[ \hat{D}_2 \right]_i \left[ \hat{R}_2 \right]_{mj} \left[ \Sigma \right]^{ij} + \left[ \hat{D}_2 \right]_i \left( \left[ \hat{R}_5 \right]_{mkj} + \left[ \hat{R}_1 \right]_m \left[ \hat{R}_2 \right]_j [\gamma]_{qk} \right) \left[ \Sigma \right]^{ij} [z^f]^k
\]

\[
+ \left( E [\hat{r}]_m - \left[ \hat{R}_4 \right]_{mj} \left[ \Sigma \right]^{ij} \right) \left[ \hat{D}_3 \right]_k [z^f]^k + \left[ \hat{R}_2 \right]_m \left( \left[ \hat{D}_5 \right]_{kj} + \left[ \hat{D}_1 \right]_m \left[ \hat{R}_2 \right]_j [\gamma]_{qk} \right) \left[ \Sigma \right]^{ij} [z^f]^k
\]

\[
+ \left[ \hat{R}_4 \right]_{mj} \left[ \hat{D}_3 \right]_k \left[ \Sigma \right]^{ij} [z^f]^k - \gamma \left[ \hat{R}_2 \right]_m \left( \left[ \hat{C}_2^H \right]_j \left[ \hat{C}_3^H \right]_k - \left[ \hat{C}_2^F \right]_j \left[ \hat{C}_3^F \right]_k \right) \left[ \Sigma \right]^{ij} [z^f]^k
\]

\[
+ \frac{1}{\gamma} \left[ \hat{R}_2 \right]_m \left[ \hat{H}_2 \right]_j \left[ \hat{H}_3 \right]_k \left[ \Sigma \right]^{ij} [z^f]^k + \frac{1}{2} \left( \left[ \hat{R}_2^m \right]_i \left[ \hat{R}_2^m \right]_j - \left[ \hat{R}_2^1 \right]_i \left[ \hat{R}_2^1 \right]_j \right) \left[ \hat{D}_3 \right]_k \left[ \Sigma \right]^{ij} [z^f]^k
\]

\[
+ \left( \left[ \hat{C}_2^F \right]_i \left[ \hat{R}_2 \right]_{mj} \left[ \hat{H}_3 \right]_k + \left[ \hat{C}_3^F \right]_k \left[ \hat{R}_2 \right]_{mj} \left[ \hat{H}_2 \right]_i \right) \left[ \Sigma \right]^{ij} [z^f]^k = 0 + O (\epsilon^4)
\]

Since this is at the steady-state portfolio, \( \left[ \hat{D}_2 \right]_i \left[ \hat{R}_2 \right]_{mj} \left[ \Sigma \right]^{ij} = 0 \) for all assets \( m \) and the above equation is homogeneous in \( [z^f]^k \) so that the following equation must be satisfied for
all \( k \) and \( m \):

\[
\begin{align*}
[\hat{D}_2]_j \left( [\hat{R}_5]_{mkj} + [\hat{R}_1]_m [\hat{R}_2]^\gamma_j [\gamma_{qk}] \right) [\Sigma]^{ij} \\
+ \left( E[\hat{r}_x]_m - [\hat{R}_4]_{mj} [\Sigma]^{ij} \right) [\hat{D}_3]_k + [\hat{R}_2]_{mi} \left( [\hat{D}_5]_{kj} + [\hat{D}_1] [\hat{R}_2]^\gamma_j [\gamma_{qk}] \right) [\Sigma]^{ij} \\
+ [\hat{R}_4]_{mj} [\hat{D}_3]_k [\Sigma]^{ij} - \gamma [\hat{R}_2]_{mi} \left( [\hat{C}_2^H]_j [\hat{C}_3^H]_k - [\hat{C}_2^F]_j [\hat{C}_3^F]_k \right) [\Sigma]^{ij} \\
+ \frac{1}{\gamma} \left( [\hat{R}_2]_{mi} [\hat{H}_2]_j [\hat{H}_3]_k [\Sigma]^{ij} + \frac{1}{2} \left( [\hat{R}_2]^m_j [\hat{R}_2]^m_j - [\hat{R}_2]^4_j [\hat{R}_2]^4_j \right) [\hat{D}_3]_k [\Sigma]^{ij} \right) \\
+ \left( [\hat{C}_2^F]_i [\hat{R}_2]_{mj} [\hat{H}_3]_k + [\hat{C}_2^F]_i [\hat{R}_2]_{mj} [\hat{H}_2]_i \right) [\Sigma]^{ij} = 0 + O(\epsilon^4)
\end{align*}
\]

Furthermore, we can express the second order of the expected excess return of asset \( m \) as

\[
E_t \left[ \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right] = E_t \left[ -\frac{1}{2} \left( \hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) + \frac{\gamma}{2} \left( \hat{C}_{t+1} + \hat{C}_{t+1}^c \right) \left( \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] + O(\epsilon^3)
\]

Evaluating this using the first-order state-space solution for consumption, returns, and the exchange rate yields

\[
E_t \left[ \hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right] = -\frac{1}{2} \left( [\hat{R}_2]^m_i [\hat{R}_2]^m_j [\Sigma]^{ij} - [\hat{R}_2]^4_i [\hat{R}_2]^4_j [\Sigma]^{ij} \right) \\
+ \frac{\gamma}{2} \left( [\hat{C}_2^H]_i + [\hat{C}_2^F]_i \right) \left( [\hat{R}_2]^m_j - [\hat{R}_2]^4_j \right) [\Sigma]^{ij} \\
+ \frac{1}{2} \left( [\hat{H}_2]_i \left( [\hat{R}_2]^m_j - [\hat{R}_2]^4_j \right) [\Sigma]^{ij} + O(\epsilon^3) \right)
\]

\[
= \frac{1}{2} \left( [\hat{R}_2]^4_i [\hat{R}_2]^4_j - [\hat{R}_2]^m_i [\hat{R}_2]^m_j \right) \\
+ \gamma [\hat{C}_2^H]_i [\hat{R}_2]_{mj} + \gamma [\hat{C}_2^F]_i [\hat{R}_2]_{mj} \\
+ \left[ \hat{H}_2]_i [\hat{R}_2]_{mj} \right) [\Sigma]^{ij} + O(\epsilon^3)
\]
Using this, we get

\[
E[\tilde{r}_m \tilde{D}_3] = E[\tilde{r}_m] \left( [\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k - \frac{1}{\gamma} [\tilde{H}_3]_k \right)
\]

\[
= \frac{1}{2} \left( [\tilde{R}_4]_i [\tilde{R}_4]_j - [\tilde{R}_2]_i [\tilde{R}_2]_j \right) \left( [\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k \right) [\Sigma]^{ij}
\]

\[
- \frac{1}{2\gamma} \left( [\tilde{C}_2^H]_i [\tilde{R}_2]_m + [\tilde{C}_2^F]_i [\tilde{R}_2]_m \right) \left( [\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k \right) [\Sigma]^{ij}
\]

\[
+ \frac{\gamma}{2} \left( [\tilde{C}_2^H]_i [\tilde{R}_2]_m + [\tilde{C}_2^F]_i [\tilde{R}_2]_m \right) \left( [\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k \right) [\Sigma]^{ij}
\]

\[
- \frac{1}{2} \left( [\tilde{C}_2^H]_i [\tilde{R}_2]_m + [\tilde{C}_2^F]_i [\tilde{R}_2]_m \right) \left( [\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k \right) [\Sigma]^{ij}
\]

\[
+ \frac{1}{2} \left( [\tilde{H}_2]_i [\tilde{R}_2]_m \left( [\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k \right) - \frac{1}{\gamma} [\tilde{H}_3]_k \right) [\Sigma]^{ij}
\]

It follows that

\[
[\tilde{D}_2]_i \left( [\tilde{R}_5]_m [\tilde{R}_2]_j [\gamma]_{mk} + [\tilde{R}_2]_i [\tilde{R}_2]_j [\gamma]_{qk} \right) [\Sigma]^{ij}
\]

\[
+ [\tilde{R}_2]_m \left( [\tilde{D}_5]_j + [\tilde{D}_1]_j \right) [\tilde{R}_2]_j [\gamma]_{mk} \right) [\Sigma]^{ij}
\]

\[
+ \left( [\tilde{C}_2^H]_i - [\tilde{C}_2^F]_i \right) \left( [\tilde{R}_2]_m \left( [\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k \right) [\Sigma]^{ij}
\]

\[
- \frac{\gamma}{2} \left( \tilde{R}_2 \right)_m \left( \tilde{C}_2^H \right)_i - \tilde{C}_2^F \right)_i \left( \left( \tilde{C}_3^H \right)_k + \tilde{C}_3^F \right)_k \right) [\Sigma]^{ij}
\]

\[
+ \frac{1}{\gamma} \left( \tilde{H}_2 \right)_i \tilde{R}_2 \right)_m \left( \tilde{H}_3 \right)_k [\Sigma]^{ij}
\]

\[
- \frac{1}{2} \left( \tilde{H}_2 \right)_i \tilde{R}_2 \right)_m \left( \tilde{H}_3 \right)_k - \tilde{H}_2 \right)_i \right)_j \tilde{R}_2 \right)_j \left( \tilde{R}_3 \right)_m \left( \tilde{R}_3 \right)_k \right) [\Sigma]^{ij}
\]

\[
+ \left( \tilde{C}_3^H \right)_i \tilde{R}_2 \right)_m \left( \tilde{H}_3 \right)_k + \tilde{C}_3^F \right)_m \tilde{H}_2 \right)_i \left( \tilde{C}_3^F \right)_k \right) [\Sigma]^{ij}
\]

\[
+ \frac{1}{2} \left( \tilde{H}_2 \right)_i \tilde{R}_2 \right)_m \left( \tilde{C}_3^H \right)_k - \tilde{C}_3^F \right)_k \right) [\Sigma]^{ij}
\]

\[
= 0
\]

Next, using the fact that \( [\tilde{D}_2]_i [\tilde{R}_2]_k \left[ \Sigma \right]^{ij} = 0 \) and that \( \tilde{R}_2 \right)_j = 0 \), this simplifies to

\[
[\tilde{D}_2]_i [\tilde{R}_5]_m \left[ \Sigma \right]^{ij}
\]

\[
+ [\tilde{R}_2]_m \left( [\tilde{D}_5]_j + [\tilde{D}_1]_j \right) \tilde{R}_2 \right)_j \left[ \gamma \right]_{mk} \right) \left[ \Sigma \right]^{ij}
\]

\[
- \frac{\gamma}{2} \left( \tilde{R}_2 \right)_m \left( \tilde{C}_2^H \right)_i - \tilde{C}_2^F \right)_i \left( \tilde{C}_3^H \right)_k + \tilde{C}_3^F \right)_k \right) \left[ \Sigma \right]^{ij}
\]

\[
+ \left( \tilde{C}_3^F \right)_i \tilde{R}_2 \right)_m \left( \tilde{H}_2 \right)_i \left( \tilde{C}_3^F \right)_k \right) \left[ \Sigma \right]^{ij}
\]

\[
+ \frac{1}{2} \left( \tilde{H}_2 \right)_i \tilde{R}_2 \right)_m \left( \tilde{C}_3^H \right)_k - \tilde{C}_3^F \right)_k \right) \left[ \Sigma \right]^{ij} = 0
\]
which can be solved for $[\gamma]_{mk}$. 

\[ \text{which can be solved for } [\gamma]_{mk}. \]