Estimation of Systemic Vascular Bed Parameters for Artificial Heart Control

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Abstract—An extended Kalman filter (EKF) estimator for the identification of systemic circulation model parameters during cardiac ejection and cardiac filling is described. The estimator is being developed for use in the control of a cardiac ventricular assist device. A lumped element circuit with a time-varying capacitor was used to represent the systemic circulation and the left ventricle. Since the hemodynamic variables that are measurable in patients with impaired cardiac function vary dramatically as the patients move through different levels of care, the estimator was designed so that it can be used with different sets of blood pressure and flow measurements. Preliminary evaluation of the performance of the estimator using data from a computer simulation and from a patient during open-heart surgery is presented. The robustness of the estimator to variations in parameter initialization is also described.

Index Terms—Artificial heart, Kalman filter, nonlinear models, parameter estimation, vascular bed parameters, ventricular assist device.

I. INTRODUCTION

VENTRICULAR assist devices (VAD) are mechanical support systems used in tandem with an impaired heart to reduce its workload. The control of a VAD involves regulation of the output of the device to meet the cardiovascular demand of the patient. Since the hemodynamics of the cardiovascular system reflect the metabolic demand of the body and the pumping capability of the heart [1], [2], incorporating hemodynamic parameters into the control of a VAD would facilitate the implementation of an effective control strategy [3]. These parameters are often difficult to measure directly, especially in the chronic setting. Thus, indirect estimation techniques will be required for long-term control. If a mathematical model representing the hemodynamic properties of the cardiovascular system is used, online parameter estimation techniques can be applied to identify the relevant parameters.

For the purpose of VAD control, a cardiovascular model should be able to simulate the left atrial pressure, cardiac output, and arterial pressure of the individual patient. The model should characterize the systemic circulation, which is the load on the natural heart. Since a limited number of variables can be measured, only a relatively simple model can be identified. Previous investigations have shown that simple models are adequate to reproduce the primary hemodynamic variables [4]–[7]. Most of the previous models used in parameter estimation incorporated lumped element approximations to the systemic circulation [6], [7], a variable compliance for the left ventricle [8]–[10], and a constant compliance to represent the pulmonary circulation [11], [12]. These models are often represented by electrical circuits, using electrical-mechanical and electrical-fluid analogs.

Procedures to estimate model parameters for an individual patient during systole from experimental data have been described by several authors [4], [5], [8]. Each of these estimation procedures was designed to utilize specific measurements. Clark et al. [4] used the aortic pressure, brachial arterial pressure, left ventricular pressure, and left ventricular volume as measurements. Deswysen [8] and Deswysen et al. [5] estimated the systemic circulation parameters with the aortic pressure and the aortic flow as measurements. Recursive methods to track parameter values over time have been developed using recursive least squares [1], [13], auto-regressive models [14], [15], and a Kalman filter [8], [9]. These estimators were also based on specific physiological measurements.

The measurements that are typically available from patients with impaired cardiovascular function change as the clinical environment changes. Extensive pressure and flow measurements can be obtained in the operating room. As the patient is relocated to the intensive care unit, then acute care, and finally long-term care facilities, however, measurement possibilities, particularly those based on invasive techniques, are reduced. When a measurement becomes unavailable, the ability to identify a particular circulatory model changes. The estimator may no longer converge for some of the parameters, or parameter accuracy may be decreased. Since the previous parameter estimation approaches were based on specific physiological measurements, these estimators have to be reformulated when the available measurements change.

There are other limitations with some of the previous estimators. Most of the estimators mentioned above [1], [4], [5], [13] identify the parameters only during the ejection phase of the cardiac cycle and may not be able to characterize the vascular bed properties in diastole because some of the system parameters are pressure dependent [16], [17]. Also, transforming the discrete estimates to express the parameters of a continuous model may lead to errors due to lack of a unique transformation between the discrete and continuous-time domains.

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This paper describes an estimation procedure based on an extended Kalman filter (EKF) that can estimate the state variables (pressures and flows) and the model parameters simultaneously. A continuous-time model is used, and estimates can be obtained using arbitrary sets of discrete measurements. For preliminary evaluation of effectiveness of the EKF, the estimator is applied to data from a computer simulation model and from an open-heart surgery patient, and effects on system identification of the availability of different sets of measurements are systematically investigated. The organization of the paper is as follows. The model of the cardiovascular system used in this study and the physiologic meanings of the model parameters and variables are reviewed in Section II. The estimation method and estimation algorithm are presented in Section III. The effects of availability of measurements on the identifiability of the individual model parameters are discussed in Section IV, and results from using the estimator with data from computer simulation and from an open-heart surgery patient are presented in Section V. Computer simulation results are discussed in Section VI, and some directions for further work are suggested.

II. CARDIOVASCULAR MODEL

The cardiovascular system model considered in this paper, shown in Fig. 1 [11], consists of the left ventricle as a time-varying capacitance, the systemic circulation as a four-element modified windkessel model [5], the pulmonary circulation and left atrium as a single capacitance, and the heart valves as diodes in series with resistors. The physiological meaning of the model parameters and variables are summarized in Tables I and II.

The system is nonlinear because of the presence of the heart valves. To simplify the estimation procedure, the diodes were assumed to be ideal. Based on the states (on or off) of the diodes, the cardiac cycle can be divided into four linear intervals [18] as indicated in Fig. 2.

1) Ejection—The aortic valve is open ($D_A$ on) and the mitral valve is closed ($D_M$ off). The blood from the ventricle is rapidly ejected into the aorta.

2) Isovolumic relaxation—The aortic and mitral valves are closed ($D_A$ and $D_M$ are off). The ventricular muscle starts to relax and ventricular pressure falls rapidly.

3) Passive filling—The aortic valve is closed ($D_A$ off) and the mitral valve is open ($D_M$ on). Blood from the venous circulation returns to the ventricle.

4) Isovolumic contraction—The aortic and mitral valves are closed ($D_A$ and $D_M$ are off). Ventricular pressure increases rapidly without a change in volume.

Parameter estimations in the ejection phase and the filling phase were performed independently so that the estimates obtained from one interval were not affected by the other. The vascular bed properties can therefore be characterized by two independent sets of parameter values. This approach...
will be more accurate than using the parameters obtained only from the ejection phase if the model parameters are pressure dependent, as has been suggested [16], [17].

III. ESTIMATION ALGORITHM

An EKF [19] for a continuous-time model with discrete measurements was chosen to estimate the vascular bed parameters. This estimator allows the parameters to be included as state variables and estimated simultaneously with the other variables. Different sets of measurements can be used in the EKF without changing the entire algorithm.

In the ejection phase the differential equations of the model can be written as

\[
\frac{d}{dt} f_A = -(R_C/L_S) \cdot f_A - (1/L_S) \cdot P_S + (1/L_S) \cdot P_A \\
\frac{d}{dt} P_S = (1/C_S) \cdot f_A - \frac{1}{R_S \cdot C_S} \cdot P_S + \frac{1}{R_S \cdot C_S} \cdot P_R \\
\frac{d}{dt} P_R = \frac{1}{R_S \cdot C_R} \cdot P_S - \frac{1}{R_S \cdot C_R} \cdot P_R.
\] (1)

Including the model parameters as states of the system, the state vector can be defined as

\[
X_E = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T
\]

where \( x_1 = f_A, x_2 = P_S, x_3 = P_R, x_4 = 1/L_S, x_5 = R_C/L_S, x_6 = 1/C_S, x_7 = 1/(R_S \cdot C_S), \) and \( x_8 = 1/(R_S \cdot C_R). \) The state equations can be written as (1), augmented by equations for the model parameters in which the time derivatives are set to zero.

A noise term representing model uncertainty, assumed to be a zero mean white Gaussian process, \( W_E(t) \sim N(0, Q_E(t)) \), was also included in the state equations. \( Q_E(t) \) describes the level of confidence in the estimates of the state variables [20]. When elements in \( Q_E(t) \) have large values, the confidence in the corresponding state estimates are lower. In this paper, \( Q_E(t) \) was assumed to be constant over time. The system dynamic equations can then be expressed as

\[
d(X_E)/dt = f_E(X_E) + W_E
\]

where

\[
f_E(X_E) = [-x_3x_1 - x_4x_2 + x_4 \cdot P_A, x_6x_1 - x_7(x_2 - x_3), x_8(x_2 - x_3), 0, 0, 0, 0, 0]^T
\]

and \( W_E = [w_{E1}, w_{E2}, w_{E3}, 0, 0, 0, 0, 0]^T. \) The measurement vector, \( Y_E(X_E) \), is

\[
Y_E(X_E) = h_E(X_E(t_k)) + v_E(t_k)
\]

where \( h_E = [x_1, x_3]^T \) if \( f_A \) and \( P_R \) are measurable, \( h_E = x_1 \) if only \( f_A \) is measurable, and \( h_E = x_3 \) if only \( P_R \) is measurable. The measurement noise term, \( v_E(t_k) \), is also assumed to be a white Gaussian process with \( v_E(t_k) \sim N(0, R_E). \)

In the passive filling phase, since there is no aortic flow, the peripheral pressure \( P_S \) is approximately equal to the aortic pressure \( P_A \). The differential equations during cardiac filling can be written as

\[
\frac{d}{dt} P_R = \frac{1}{R_S \cdot C_R} \cdot P_A - \frac{1}{R_S \cdot C_R} \cdot P_R - (1/C_R) \cdot f_R
\\
\frac{d}{dt} P_A = \frac{1}{R_S \cdot C_S} \cdot P_A + \frac{1}{R_S \cdot C_S} \cdot P_R.
\]

In this phase, the state vector is

\[
X_F = [x_1, x_2, x_3, x_4, x_5]^T
\]

where \( x_1 = P_R, x_2 = P_A, x_3 = 1/C_R, x_4 = 1/(R_S \cdot C_R), \) and \( x_5 = 1/(R_S \cdot C_S). \) The state-space system equations augmented by the unknown model parameters can be expressed in the form of (3) with

\[
f_F(X_F) = [-x_2x_1 + x_3x_2 - x_3 \cdot f_R, x_5x_1 - x_5x_2, 0, 0, 0, 0]^T
\]

and \( W_F = [w_{F1}, w_{F2}, 0, 0, 0]^T. \) \( W_F(t) \) is a noise term representing model uncertainty, again assumed to be a zero mean white Gaussian process, \( W_F(t) \sim N(0, Q_F(t)) \). The measurement vector is

\[
Y_F(X_F) = h_F(X_F(t_k)) + v_F(t_k)
\]

where \( h_F = [x_1, x_3]^T \) if \( P_R \) and \( P_A \) are measurable, \( h_F = x_1 \) if only \( P_R \) is measurable, and \( h_F = x_3 \) if only \( P_A \) is measurable. \( v_F(t_k) \) is a white Gaussian process to represent measurement noise, with \( v_F(t_k) \sim N(0, R_F). \)

In the filling phase, if the left ventricular pressure \( P_V \) is a measurable output instead of \( P_R \), the system equations are the same as in (6), but the state vector is redefined as

\[
X_V = [x_1, x_2, x_3, x_4, x_5, x_6]^T
\]

where \( x_1 = P_R, x_2 = P_A, x_3 = 1/C_R, x_4 = 1/(R_S \cdot C_R), x_5 = 1/(R_S \cdot C_S), \) and \( x_6 = R_M. \) The system dynamic equations in matrix form can be written as (3) with

\[
f_V(X_V) = [-x_4x_1 + x_5x_2 - x_3 \cdot f_R, x_5x_1 - x_5x_2, 0, 0, 0, 0]^T
\]

\[
X_V = [x_1, x_2, x_3, x_4, x_5, x_6]^T
\]
and $W(t) = [w_{1}, w_{2}, 0, 0, 0, 0]^{T}$. $W(t)$ is assumed to be a zero mean white Gaussian process, $W(t) \sim N\{0, Q(t)\}$. The measurement vector is

$$Y_{V}(X_{V}) = h_{V}(X_{V}(t_{k})) + v(t_{k})$$  \hspace{1cm} (12)

where $h_{V} = [v_{2}, x_{1} - x_{6} \cdot f_{R}]^{T}$ if $P_{V}$ and $P_{A}$ are measurable, or $h_{V} = x_{1} - x_{6} \cdot f_{R}$ if only $P_{V}$ is measurable. The measurement noise term, $v(t)$, is a Gaussian white process with $v(t) \sim N\{0, R_{V}(t)\}$.

Since $f_{A}$ and $f_{R}$ are zero in the isovolumic phases ($D_{A}$ and $D_{R}$ are off), unique solutions for the individual parameters do not exist, and only time constants $R_{S}C_{R}$ and $R_{S}C_{S}$ can be obtained by solving two differential equations using $P_{A}$ and $P_{R}$ as measurements.

Table III summarizes the system equations, available measurements, and model parameters that can be estimated for each interval of the cardiac cycle.

### IV. IDENTIFIABILITY OF THE MODEL PARAMETERS

Whether a system model can be identified depends on the measurements that are available [21]. This section determines the minimum measurement sets required for the cardiovascular system model so that studies to evaluate the performance of the estimator will incorporate adequate measurement data.

If the model parameters can be determined from the input–output transfer function coefficients, unique solutions of the parameters exist. The signals that can be measured from the system during ejection are left atrial pressure $P_{L}$, aortic pressure $P_{A}$, and aortic flow $f_{A}$. During filling, the relevant measurements are $P_{V}$ and venous return flow $f_{R}$. These signals were used to identify individual model parameters during different phases of the cardiac cycle.

During the ejection phase, $P_{L}$ was used as the system input and $P_{A}$ and $P_{R}$ were the measurable state variables. When $P_{A}$ and $f_{A}$ are both measurable, the transfer function can be expressed as

$$H_{E}(s) = \frac{f_{A}(s)}{P_{A}(s)} = \frac{[b_{2} \cdot s^{2} + b_{1} \cdot s]}{DEN(s)}$$  \hspace{1cm} (13)

where $DEN(s) = a_{3} \cdot s^{3} + a_{2} \cdot s^{2} + a_{1} \cdot s + 1$, and $a_{3} = L_{S}R_{S}C_{S}C_{R}$, $a_{2} = R_{C}R_{S}C_{S}C_{R} + L_{S}C_{S}C_{R}$, $a_{1} = R_{S}C_{R}$, and $b_{2} = R_{S}C_{S}C_{R}$, and $b_{1} = C_{S}C_{R}$. The model parameters can then be solved in terms of the

### TABLE III

**SUMMARY OF VASCULAR BED PARAMETER ESTIMATION**

<table>
<thead>
<tr>
<th>Heart Action</th>
<th>Aortic Valve</th>
<th>Mitral Valve</th>
<th>System Equations</th>
<th>Estimator (Measurement)</th>
<th>Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ejection (D on)</td>
<td>OPEN</td>
<td>CLOSED</td>
<td>$d(f_{A})/dt = (P_{A} - R_{C}f_{A} - P_{S})/L_{S}$</td>
<td>EKF</td>
<td>$R_{C}, L_{A}$</td>
</tr>
<tr>
<td></td>
<td>(D off)</td>
<td>(D off)</td>
<td>$d(P_{S})/dt = [f_{A} - (P_{S} - P_{R})]/C_{S}$</td>
<td></td>
<td>$C_{S}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$d(f_{P})/dt = (P_{S} - P_{R})/(R_{S}C_{R})$</td>
<td></td>
<td>$R_{S}, C_{R}$</td>
</tr>
<tr>
<td>Filling (D off)</td>
<td>CLOSED</td>
<td>OPEN</td>
<td>$d(P_{S})/dt = [(P_{A} - P_{R})/(R_{S}C_{R})$</td>
<td>EKF</td>
<td>$C_{R}, C_{S}$</td>
</tr>
<tr>
<td></td>
<td>(D on)</td>
<td>(D on)</td>
<td>$d(f_{P})/dt = (-P_{A} + P_{R})/(R_{S}C_{R})$</td>
<td></td>
<td>$R_{S}, R_{M}$</td>
</tr>
<tr>
<td>Iso-</td>
<td>CLOSED</td>
<td>CLOSED</td>
<td>$d(P_{S})/dt = (P_{A} - P_{R})/C_{S}$</td>
<td>Solving Linear Equations</td>
<td>$R_{S}C_{R}, R_{S}C_{S}$</td>
</tr>
<tr>
<td>Volumic (D off)</td>
<td>(D off)</td>
<td>(D off)</td>
<td>$d(f_{P})/dt = (-P_{A} + P_{R})/(R_{S}C_{S})$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE IV

**SUMMARY OF P(0) & Q USED IN THE ESTIMATOR**

<table>
<thead>
<tr>
<th>Heart Activity</th>
<th>P(0)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ejection</td>
<td>5000*1_{ns8}</td>
<td>$Q_{0}(1,1) = 500; Q_{0}(2,2) = 0.1; Q_{0}(3,3) = 1$</td>
</tr>
<tr>
<td>Filling (P_{L} was measurable)</td>
<td>5*1_{ns5}</td>
<td>$Q_{f}(1,1) = 100; Q_{f}(2,2) = 0.01$</td>
</tr>
<tr>
<td>Filling (P_{V} was measurable)</td>
<td>5*1_{ns6}</td>
<td>$Q_{v}(1,1) = 100; Q_{v}(2,2) = 0.01$</td>
</tr>
</tbody>
</table>

*Only nonzero elements in Q are listed.*
Fig. 3. Parameter estimation in the ejection phase using $P_A$ and $f_A$ as measurements.
Fig. 4. Parameter estimation in the filling phase using $f_R$ and $P_R$ as measurements.

Hence, the individual model parameters are identifiable if $P_A$ and $f_A$ are measurable during the ejection phase. If $P_A$ and $P_R$ are measurable but not $f_A$, the transfer function is

$$H_{F2}(s) = P_R(s)/P_A(s) = 1/DEN_F(s),$$

(14)

Since the individual model parameters cannot be determined from the coefficients of $DEN_F(s)$, the parameters are not identifiable with $P_A$ and $P_R$ in the ejection phase.

In the filling phase, the venous return flow, $f_R$, was used as the system input, and $P_V$, $P_R$, and $P_A$ were used as output measurements. When $P_V$ and $f_R$ are measurable, the transfer function, $P_V(s)/f_R(s)$, can be written as

$$H_{F1}(s) = -(b_2 \cdot s^2 + b_1 \cdot s + 1)/DEN_F(s)$$

(15)

where $DEN_F(s) = a_2 \cdot s^2 + a_1 \cdot s$ and $b_1 = R_S C_R$, $b_2 = R_M R_S C_R$. The model parameters can be solved from the coefficients of $H_{F1}(s)$ as

$$R_M = b_2/a_2$$
$$C_R = a_2/(b_1 - R_M a_4)$$
$$C_S = a_1 - C_R$$
$$R_S = a_2/(C_S C_R).$$

When $P_R$ and $f_R$ are measurable, the transfer function, $P_R(s)/f_R(s)$, can be expressed as

$$H_{F2}(s) = -(k \cdot s + 1)/DEN_F(s)$$

(16)

where $k = R_S C_S$. The model parameters can be solved from the coefficients of $H_{F2}(s)$ as

$$C_R = a_2/k$$
$$C_S = a_1 - C_R$$
$$R_S = a_2/(C_S C_R).$$

If only $P_A$ and $f_R$ are measurable, the transfer function, $P_A(s)/f_R(s)$, can be written as

$$H_{F3}(s) = -1/DEN_F(s).$$

(17)

A unique solution of the model parameters cannot be obtained if only the coefficients of $DEN_F(s)$ can be estimated, and the individual model parameters are not identifiable in this case.
Fig. 5(a). Parameter estimation in the ejection phase using noise-free measurements. Parameter estimates were initialized randomly.

To summarize, the model parameters can be identified with at least $P_A$ and $f_A$ as measurements in the ejection phase and with $f_R$ and either $P_V$ or $P_R$ in the filling phase. These minimum measurement sets, augmented by additional measurements, will be used in the performance evaluation studies described below.

V. PERFORMANCE EVALUATION OF THE ESTIMATOR

The estimation algorithm described in Section III was implemented in MATLAB\textsuperscript{1} and applied to data from a computer simulation [11] and to human data obtained during open-heart surgery. The use of EKF to estimate the vascular bed parameters during the ejection phase and during the filling phase was tested independently. Since $P_A$ and $f_A$ are required measurements in the ejection phase, while $f_R$ and either $P_V$ or $P_R$ are necessary in the filling phase, these measurements were chosen to test the estimator.

The estimation procedure requires knowledge of the measurement noise covariance, $R_k$, the plant noise covariance, $Q$, and the initial error covariance, $P(0)$ [19]. Inappropriate settings of these terms would result in slow convergence or divergence of the estimates [20]. In order to determine the appropriate values of these covariance matrices, the estimators for ejection and filling were both applied to simulation data during a single heart beat, using model parameters initialized to the known correct values. This test also assessed the stability of the estimators and provided an indication of estimation performance with different measurement sets.

Then, estimator performance during heart ejection when model parameters are initialized to incorrect values was considered. In this case, the estimator was run over multiple heart beats to allow time for the parameter to converge. Random noises were added to the pressure and flow measurements to describe the effects of measurement noise. Finally, the estimation procedure was applied to data obtained from a patient during open-heart surgery, in which left ventricular pressure and aortic flow measurements were available.

A. Parameter Estimation in a Single Heart Beat Using Simulation Data

To determine the appropriate values for $R_k$, $Q$, and $P(0)$, estimation using simulated data, for which the initial model parameter values were set to the actual values used in simulation, were run. The values of $R_k$, $Q$, and $P(0)$ were adjusted in this test to ensure that all of the predicted errors of the state variables were bounded by the standard deviations predicted by the estimator with these values. The estimates converged during a single cardiac cycle. $R_k$ was set to be an identity matrix for all $k$ in all cases. The appropriate values of $P(0)$ and $Q$ used in the estimation during heart ejection and filling are listed in Table IV.

Fig. 3 shows estimation results obtained during the ejection phase, using $P_A$ and $f_A$ as measurements. The first column

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\textsuperscript{1}The MathWorks Inc., Natick, MA.
Fig. 5(b). Parameter estimation in the ejection phase using noisy measurements. Parameter estimates were initialized randomly.

shows the parameter estimates during ejection, and the second column shows the predicted errors and the standard deviations of the parameters. The convergence of the parameter estimates in the filling phase using \( P_R \) and \( f_R \) as measurements is shown in Fig. 4.

Table V shows the final parameters values obtained by the estimator using different sets of measurements. The actual values used in the simulation are also listed. A blank cell indicates a parameter that cannot be identified for that condition. The percent errors between the final estimated values and the actual parameter values, defined by

\[
\text{Error}(\%) = \frac{|\theta - SV(\theta)|}{SV(\theta)} \times 100\%
\]  

where \( SV(\theta) \) is the actual parameter value in the computer simulation [11], were used as the quantitative indexes for the accuracy of the estimates. This error is indicated in parentheses in Table V. These results provide a comparison of performance with different measurement sets, since differences due to convergence problems have been minimized by starting parameters at their true values.

In the ejection phase, all parameters can be estimated by the EKF with \( P_A \) and \( f_A \) as measurements, but not as accurately as when \( P_R \) is also used. \( P_R \) is an important measurement to make the EKF converge rapidly, due to the small effect of \( C_R \) on the input impedance of the systemic circulation. In the filling phase, \( f_R \) and either \( P_V \) or \( P_R \) are necessary to identify the parameters, and the estimates are improved if \( P_A \) is also available.

B. Robustness of the Estimator to Parameter Initialization

The estimator requires initialization of the model parameters. If these initial settings are not close to the actual values, it may take longer for the estimates to converge, or the estimates may not converge at all. In clinical applications, these parameter values would be unknown, and it is necessary to describe the ability of the estimator to converge to the appropriate values. An additional set of simulations was performed in which a noise component was added to the measurement signals. These simulations mimic the actual clinical situations, with unknown initial parameter values and noisy measurements.

The estimator performance was determined during ejection using \( P_A \), \( f_A \), and \( P_R \) as measurements. The estimation used multiple cardiac cycles because the estimator required more than one cardiac cycle to converge. At the beginning of each heart ejection, the state variables, \( f_A \), \( P_S \), and \( P_R \), were reset to the values of \( f_A = 0 \), \( P_S = AP_{ED} \), and \( P_R = LAP_{ED} \), where \( AP_{ED} \) is the end-diastolic arterial pressure and \( LAP_{ED} \) is the end-diastolic left atrial pressure, while the first 3 × 3 submatrix in the covariance matrix, \( P(\theta) \), related to the state estimates, was reset to be \( 50I_{3\times3} \), where \( I \) is an identity matrix. These adjustments reduced the transient of the estimates when a new cycle was started and improved the tracking ability of the estimator. The estimator was evaluated by initializing the parameters as 0.5, 1.1, and 1.5 times the actual values. In addition, ten random selections of initial values from a normal distribution of \( N\{SV, SV^2\} \), where \( SV \) represents the actual parameter values in simulation, were used.
### TABLE V
SUMMARY OF THE PARAMETERS ESTIMATED BY EKF USING SIMULATION DATA

<table>
<thead>
<tr>
<th>Phase</th>
<th>Measurements</th>
<th>$R_C$ (Error)</th>
<th>$L_S$ (Error)</th>
<th>$C_S$ (Error)</th>
<th>$R_S$ (Error)</th>
<th>$C_R$ (Error)</th>
<th>$R_M$ (Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Simulation)</td>
<td>3.980e-2</td>
<td>1.025e-3</td>
<td>2.896</td>
<td>8.738e-1</td>
<td>4.000</td>
<td>1.e-2</td>
<td></td>
</tr>
<tr>
<td>Ejection</td>
<td>$P_A$, $f_A$, $P_R$</td>
<td>3.978e-2</td>
<td>1.031e-3</td>
<td>2.885</td>
<td>8.631e-1</td>
<td>3.98</td>
<td>-</td>
</tr>
<tr>
<td>Ejection</td>
<td>$P_A$, $f_A$</td>
<td>3.934e-2</td>
<td>1.028e-3</td>
<td>2.887</td>
<td>7.951e-1</td>
<td>3.682</td>
<td>-</td>
</tr>
<tr>
<td>Filling</td>
<td>$f_R$, $P_R$, $P_A$</td>
<td>-</td>
<td>-</td>
<td>2.902</td>
<td>8.735e-1</td>
<td>3.989</td>
<td>-</td>
</tr>
<tr>
<td>Filling</td>
<td>$f_R$, $P_R$</td>
<td>-</td>
<td>-</td>
<td>2.942</td>
<td>8.676e-1</td>
<td>3.990</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE VI
SUMMARY OF THE TEST RESULTS

<table>
<thead>
<tr>
<th>Initial 0(0)</th>
<th>$L_S$</th>
<th>$R_C$</th>
<th>$C_S$</th>
<th>$R_S$</th>
<th>$C_R$</th>
<th># cardiac cycles to converge</th>
<th>MNE of input impedance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV*</td>
<td>1.03e-3</td>
<td>3.98e-2</td>
<td>2.896</td>
<td>0.874</td>
<td>4.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.5-SV</td>
<td>1.00e-3</td>
<td>3.92e-2</td>
<td>3.035</td>
<td>0.945</td>
<td>3.679</td>
<td>129</td>
<td>5.61</td>
</tr>
<tr>
<td>1.1-SV</td>
<td>1.04e-3</td>
<td>3.95e-2</td>
<td>2.890</td>
<td>0.886</td>
<td>3.942</td>
<td>15</td>
<td>1.31</td>
</tr>
<tr>
<td>1.5-SV</td>
<td>1.12e-3</td>
<td>4.07e-2</td>
<td>2.683</td>
<td>0.847</td>
<td>4.126</td>
<td>106</td>
<td>4.18</td>
</tr>
<tr>
<td>N[SV, SV^2]</td>
<td>1.03e-3</td>
<td>3.93e-2</td>
<td>2.91</td>
<td>0.886</td>
<td>3.94</td>
<td>6</td>
<td>1.24</td>
</tr>
<tr>
<td>0.5-SV**</td>
<td>9.98e-4</td>
<td>3.91e-2</td>
<td>3.046</td>
<td>0.942</td>
<td>3.698</td>
<td>267</td>
<td>5.23</td>
</tr>
<tr>
<td>1.1-SV**</td>
<td>1.07e-3</td>
<td>4.01e-2</td>
<td>2.787</td>
<td>0.873</td>
<td>3.967</td>
<td>140</td>
<td>2.44</td>
</tr>
<tr>
<td>1.5-SV**</td>
<td>1.13e-3</td>
<td>4.09e-2</td>
<td>2.668</td>
<td>0.850</td>
<td>4.075</td>
<td>527</td>
<td>3.93</td>
</tr>
<tr>
<td>N[SV, SV^2]**</td>
<td>1.08e-3</td>
<td>4.05e-2</td>
<td>2.719</td>
<td>0.872</td>
<td>3.973</td>
<td>34</td>
<td>3.31</td>
</tr>
</tbody>
</table>
The estimator was first tested by using data from simulation as measurements. Then, random noise was added to the measurement signals
\[ z_i(t_k) = s_i(t_k) + p_i \times N\{0, 1\} \times s_i(t_k) \]
where \( z_i(t_k) \) is the \( i \)th measurement at time \( t_k \), \( s_i(t_k) \) is the \( i \)th noise-free signal at time \( t_k \) from simulation, and the \( i \)th measurement noise at time \( t_k \) is a normal distribution proportional to the signal. The values \( p_1 = 4\% \), the percentage error of the aortic flow measurement, and \( p_2 = 1\% \), the percentage of the left atrial pressure measurement, were used. These values were twice the percentage errors described by the sensors’ manufactures. The \( Q \) and \( P(0) \) obtained from the previous test, listed in Table IV, were used to start the estimator. The measurement noise covariance \( \hat{R}_k \) was set to be identity for the estimator test using noise free data. For the estimation test with measurement noises, \( R_k \) was set to
\[ R_k = \begin{bmatrix} 72.77 & 0 \\ 0 & 0.01 \end{bmatrix} \]
obtained by calculating the covariance of the error between the measurement vector and the signal vector. The estimator was stopped when all parameter estimates changed by less than 5\% during a complete cardiac cycle and the mean percentage error, defined by
\[ \left[ \sum_{k=1}^{n} \left( \theta_k - SV(\theta_k) \right) / SV(\theta_k) \times 100\% \right] / n \]
\[ k = 1, \ldots, n \]
where \( \theta_k \) is the \( k \)th parameter estimate, \( SV(\theta_k) \) is the actual value of \( \theta_k \), and \( n \) is the number of unknown parameters, was
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TABLE VII
SUMMARY THE PARAMETERS ESTIMATED BY EKF USING HUMAN DATA

<table>
<thead>
<tr>
<th>Phase</th>
<th>Measurements</th>
<th>RC</th>
<th>LS</th>
<th>CS</th>
<th>RS</th>
<th>CR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ejection</td>
<td>PV, fA</td>
<td>4.03e-2</td>
<td>9.90e-4</td>
<td>0.573</td>
<td>0.293</td>
<td>1.028</td>
</tr>
</tbody>
</table>

Fig. 8. PV and fA from a patient for the estimator test.

less than 5%. The final estimates were used as the steady-state values of the parameters. The total number of cardiac cycles to reach the steady state was used as the index of convergence speed.

The ability of the EKF to estimate the systemic circulation input impedance was also used as a measure of performance. In studies of patients, the actual cardiovascular model parameters are not known. However, the vascular bed input impedance (PA/fA), which provides information about the physical state of the vascular system and the load faced by the left ventricle [22], [23], is often used as an index of the status of health of the circulation. If the input impedance (PA/fA) generated from the EKF estimates is close to the impedance calculated from the measured PA and fA signals, the estimated values would be acceptable. The percentage of mean normalized error (MNE) of the input impedance, defined by

\[
\text{MNE} = \left(\frac{\left|Z_{\text{SV}}(j2\pi f) - Z_{\Phi}(j2\pi f)\right|}{\left|Z_{\text{SV}}(j2\pi f)\right|}\right) \times 100\%
\]  

(21)

where ZSV(j2πf) is the impedance calculated from the actual parameter values and ZΦ(j2πf) is the impedance from the estimates, was used to characterize the accuracy index of the estimates. Since the model being estimated has infinite impedance at zero frequency, the impedance comparison was started at f = 0.01 Hz.

Fig. 5(a) shows one of the ten random parameter initialization tests using noise-free data from simulation, and Fig. 5(b) shows one of the same tests using noisy measurement data. The steady-state parameter values, the number of cardiac cycles to converge, and the MNE of the impedance for all of the tests are shown in Table VI. The actual parameter values used in the simulation are also listed. The error in the impedance is small, since the simulation can be exactly described by a linear circuit. The final values of the estimates are close to the actual values. The estimates converged more quickly if the initial estimates were closer to the actual parameters. When the additive noises were applied to the measurements, the estimates approached to the actual values but took longer to converge. The input impedances calculated from the steady-state estimates are compared to impedances obtained from the actual parameter values in Figs. 6 and 7, where SV represents the actual parameter values in simulation.

C. Application to Human Data

The estimation scheme in the ejection phase was also applied to left ventricular pressure (PV) and aortic flow (fA) measurements obtained from a patient undergoing coronary bypass surgery. A 4F high-fidelity fiber-optic pressure catheter (Camino Laboratories, San Diego, CA) was advanced into the left ventricle through a small incision in the right superior pulmonary vein to measure PV. Aortic flow (fA) was measured by an electromagnetic flow probe placed around the ascending aorta. All signals were sampled by a data acquisition system (model RTS-132, Significat, Hudson, MA) at a sampling rate of 150 Hz and stored on a computer workstation (Apollo Computer Inc., model DN3550, Chelmsford, MA). Left ventricular ejection period was determined by the period between the first and second zero crossing of fA. PV was used in place of PA because PV is close to PA during the ejection phase. The measurement waveforms used for estimation are shown in Fig. 8.

The performance of the estimator is shown in Fig. 9, and the steady-state vascular bed parameters estimated by EKF are summarized in Table VII. The parameter values for RC and LS are close to those used in the computer simulations, which were selected to represent a normal cardiovascular system, and the remaining parameters were within 20% to 30% of the simulation values. These parameter estimates yield a higher systemic circulation input impedance (PV/fA) at low frequency than the impedance calculated directly from the measurements, as shown in Fig. 10. The remainder of the impedance function was of the same order of magnitude as the function obtained directly, although the former showed a minimum at 7 Hz, while the latter was uniform across frequency.

VI. DISCUSSION

An online estimator based on the EKF algorithm was developed for the estimation of vascular bed parameters. The
parameters were estimated using a continuous-time model, so that no discrete-to-continuous domain conversion was required. This estimator can be used with different available measurements without reformulating the entire algorithm.

The estimator requires at least two measurements to identify the individual parameters of the vascular bed. In the ejection phase, all parameters can be estimated by the EKF with the aortic pressure and the aortic flow as measurements. The estimator was found to converge more quickly with more accurate results when the left atrial pressure is also used. In the filling phase, the venous return flow and either left ventricular pressure or the left atrial pressure are necessary to identify
the parameters, and the results are more accurate if the aortic pressure is also available.

When the parameters are initialized close to their actual values, computer simulation showed that the estimates approached the steady state in less than 15 cardiac cycles with noise-free measurements and in less than 140 cardiac cycles with noisy measurements. To obtain rapid convergence, it would be desirable to start the estimator by using initial parameter estimates from least square estimation. If the initial estimates are not close to their actual values, the estimation can be performed over multiple cardiac cycles by resetting the state variables and the state covariance at the start of each cycle until the estimates converge. The estimator was able to converge for initial estimates of 50% to 150% of the correct values. In most clinical situations, initial estimates of these parameters could be obtained within this range. The estimator can converge within 120 cardiac cycles (about 1–2 min) using perfect measurements and within 500 cardiac cycles (about 5–10 min) or less with noisy measurements. The estimates can be used to represent the input impedance of the circulation with reasonable accuracy.

These estimates produce a higher input impedance at zero frequency than the input impedance calculated directly from the measurements. The higher input impedance may be due to bias from the use of incorrect noise statistics in the EKF [24]. Since clinical data are noisier than the simulation model results, it would be beneficial to characterize the plant noise statistics for further application to human clinical data. Another possible reason for biased estimates may be from the low sampling rate used. The EKF uses a linear approximation of the system dynamics over a small range of current state estimates. If the sampling rate is too low, the approximation range is too large and may be inaccurate. Under this condition, either increasing the sampling rate to reduce the approximation range or including second-order terms in the estimator may be helpful [25]. Finally, the very low-frequency impedance value may be affected by the pulmonary circulation. This effect would not be included in the model here, in which the pulmonary circulation was represented by a simple compliance.

It has been shown here that flow measurements are necessary to estimate the individual model parameters. Flow measurements, however, are impossible to obtain in some clinical settings and difficult under any conditions. In addition, pressure transducers are not desirable for long-term monitoring [26]. For patients being treated with a ventricular assist device (VAD), it may be possible to include the VAD’s in the cardiovascular model and use measurements from them to identify the cardiovascular model parameters. This approach could provide a clinically feasible alternative to invasive pressure and flow measurements.

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REFERENCES

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