Morphological reduction on the SLIP template

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Summary

The Spring-Loaded Inverted Pendulum (SLIP) model is commonly used to describe the dynamics of walking and running in animals and machines. More representative models have extended SLIP to capture effects of body and limb morphology, and leg number. For each deviation from this template a wide variety of new models have been introduced to better capture the details of the dynamics. In this work we use new tools for generalized template–anchor systems that formalize the relationship of the different bodies' parametrization through the *morphological reduction*, which describes the link between representative morphology (e.g. limb shape) and SLIP parameters (e.g. spring stiffness).

Introduction

SLIP is known to be a good approximation of the dynamics of animal and robot running (Blickhan 1989), predicting sagittal plane center of mass trajectories and ground reaction forces for both bipeds and polypeds. There are many proposed extensions to the SLIP model including to three dimensions (Seipel and Holmes 2005), multiple joint legs (Dadashzadeh et al. 2014, Maus et al. 2008, Saranli et al. 1998), body inertia (Ankaralı and Saranlı 2011, Maus et al. 2008, Poulakakis and Grizzle 2009), and others. While most of these extensions to the SLIP model included a comparison in the predicted dynamics of a given system, no analysis has been done on the collapse of dimension from the parameter space of real body–limb morphology to the SLIP parameters and comparing the consequences of differing morphologies.

Methods

This paper uses the generalized template–anchor relationship first detailed in (Libby et al. 2016, Appendix A), which includes mappings between not just the state spaces but also the control and parameter spaces of different system models. The original concept of templates and anchors was introduced in (Full and Koditschek 1999) and is sometimes considered to require the template dynamics be an attracting invariant submanifold of the anchor dynamics (Altendorfer et al. 2004).

The generalized relationship relaxes this requirement and allows for an analysis of: i) the parameter spaces of the models through the *morphological reduction* map, Ξ; ii) *active* and *passive* anchoring controllers, τ_Y, which drive the more complex model dynamics to those of the simpler template; iii) *exact* and *approximate* anchorings of the systems' dynamics; and iv) error characteristics for different parameter regions when the anchoring is only approximate. In this work we apply these new tools to analyze the SLIP template and its many variants.

Results

In this work, we present limited example results applying the new analysis tools from (Libby et al. 2016) to the SLIP template and related models. 3D-SLIP (Seipel and Holmes 2005) is, in the case of vertical hopping, a passive exact anchor for the SLIP template, while non-vertical hopping is passive but approximate. Multi-jointed models, e.g. (Saranli et al. 1998), use active anchoring control, and the morphological reduction, Ξ, of each multi-jointed models allows for a direct comparison between them. Asymmetric SLIP, which includes body inertia, (Poulakakis and Grizzle 2009), has dynamics that are diffeomorphic to that of (an energy stabilized variant of) SLIP when using an active anchoring controller.

Discussion

The morphological reduction and other tools of analysis enabled by the generalized template–anchor relationship provide new design insights into legged spring-mass running. Through this analysis, we can directly compare dramatically different body forms through their common template, and identify regions of the parameter space where the simpler model is a good approximation. A complete anchoring (i.e. the set of algebraic maps between state, control, and parameter spaces) enables both body design and feedback controller to be lifted from the simple model (where they are easily computed) to the more complex one (representing the desired morphology).

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