

The Effects of Advertising on Customer Retention and the Profitability of Auctions

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Abstract

A fundamental marketing problem faced by auctioneers is the promotion of their auction in order to recruit bidders. Merchants would like to attract as many bidders as possible to an auction, since the higher the number of bidders, the higher the expected winning price and profitability. This suggests that an intense advertising campaign that raises awareness for the largest possible customer base would be desirable. However, from a customer's viewpoint a large number of competing bidders decreases the perceived chance of winning, which subsequently deters the customer from participating in future auctions. In this paper we examine this tradeoff between the auctioneers' desire to expand their customer base and the bidders' desire for attractive values. We model the entry decision of consumers based on their expected gains from participating in the auction and derive the auctioneer's optimal advertising policy. Our results suggest that controlling growth by limiting advertising spending can encourage consumer retention and increase an auctioneer's long term profitability.

Keywords: Auctions; Advertising; Customer Acquisition; Customer Retention

1. Introduction

The profitability of an auction for a merchant¹ would seem to be positively related to the number of bidders. The more bidders who participate then the higher the expected winning offer. Hence, one would expect that the most profitable auctions to be those that aggressively advertise and recruit as many people as possible. However, from a customer's viewpoint a large number of competing bidders decreases the perceived chance of winning, which lowers the desire to participate in the auction. It is this tradeoff for the auctioneer between more consumers and a higher expected price versus lower consumer satisfaction and retention in future auctions that motivates our study.

We develop an analytical model to show how advertising can influence customer retention and profitability for an auction. We propose a two-period model in which an auctioneer sells one item in each time period using a second-price sealed bidding format. The auctioneer must decide upon an advertising budget which influences the number of consumers who participate. If a consumer is aware of the auction, the consumer must decide whether to participate in the auction. This framework accounts for the endogenous participation decision made by consumers, and from it we derive the optimal advertising strategy for the auctioneer that maximizes expected profits.

Our results show that too much advertising by an auctioneer can reduce a bidder's satisfaction and subsequent retention in future auctions, which lowers the auctioneer's profits. Onsale provides a real-life case study that illustrates this problem. Onsale's business model was to sell surplus computer merchandise through online auctions (Moon 1999). After one year in operation they were poised to earn a positive operating margin from auctions. However, Onsale decided to aggressively expand its customer base using advertising. A major problem was that Onsale's customer base grew faster than its supply of surplus merchandise. The initial satisfaction of consumers with the site was lessened as the chance of winning

¹ We examine this problem by focusing on a merchant that offer auctions on a regular basis, like uBid, as opposed to *listing* sites, like eBay, who extract revenue from commission. The revenue from a *merchant* auctioneer comes from selling a product that they own. Although our model could be applied to auctioneers selling through eBay.

decreased. Although there are many factors that contributed to Onsale's eventual demise², the point is that an auctioneer needs to consider the problems of promoting an auction and balance these marketing considerations against the economic ones.

Most research on auctions can be found in the economics literature. The focus has largely been on the optimal bidding strategy and the effects of auction format on the auctioneer's expected revenue. Recent advances in auction theory have incorporated new issues such as budget constraints, externalities between bidders, and competing auctioneers (refer to Klemperer 1999 for a thorough review). Although there is work in economics on optimal auction design using entry fees or posting reserve prices (e.g. Riley and Samuelson, 1981; Engelbrecht-Wiggans, 1987, 1993), no work has been done to examine the bidder's retention value to an auctioneer and the design a promotional strategy that balances customer acquisition and retention effects. To a large extent marketing related issues such as advertising, the role of channels, and consumer learning have largely been ignored in the auction literature (Chakravarti et al. 2002). Our research shows how marketing questions, such as advertising and customer retention, can influence optimal auctioneer decisions. Additionally, our paper contributes to the marketing literature by showing how competition amongst customers can impact the long-term profitability of a firm. Most research in marketing focuses upon a posted-price retail setting, where customers act as price takers, and not price setters as in an auction context.

The paper is organized as follows. In §2 we review research on auction design in economics and marketing that pertains to our problem. §3 presents our assumptions about auctioneer and bidder behavior. We derive the analytical results of our model in §4. §5 relaxes some of the distributional assumptions we make in §3 for analytical tractability and conduct several simulation studies to understand the robustness of our results. We conclude the paper in §6 by discussing the managerial implications of the model and areas for further research.

² We argue that too much growth was a factor in their demise, but there were many others, such as awareness, competition, and complications from their merger with Egghead that all contributed to Onsale's failure. Recently, Onsale has restarted operations after Egghead's bankruptcy.

2. Literature Review

There are two streams of work that are relevant in our research. The first is the recent stream of work in economics concerning online auctions. Although our model can be applied in either an online or offline format, online auctioneers such as Onsale or uBid provide strong similarities with the assumptions that we make about auctioneer and bidder behavior. For example, online auctions tend to use email and banner ads frequently to draw traffic to their auction, which corroborates our focus on advertising. Also, online auctions tend to occur more frequently which suggests consumer retention could be important.

Online auctions provide many changes to traditional auctions which have forced economists to revisit auction theory (Pinker, Seidmann and Vakrat, 2001). One problem is generating consumer awareness of an auction. Unlike traditional auctions, online auctions don't have a physical presence in the market that may naturally generate awareness. In order to gather enough bidders the auctioneer has to advertise and/or lengthen the duration of its auctions. There is great variation in the length of online auctions. Some may last only a few minutes while others may last for weeks. On average, the length of auctions on the Internet is usually about a week, which is much longer than traditional auctions (Lucking-Reiley, 1999a). As a result awareness of online auctions may be higher than for traditional auctions. Compared to the traditional auctions, online auctions provide more geographic and temporal convenience which makes it possible to advertise to large populations of buyers (Lucking-Reiley, 1999a; Bajari and Hortacsu, 2002).

The most common online auction format is a second-price sealed-bid auctions with ascending bids. These auctions are favored both by researchers and auctioneers because of their simplicity (Lucking-Reiley, 1999a). However, due to the unique problems on online auctions, such as sniping, efforts have to be made to preserve the attributes of the second-price sealed bid auction. For example, to avoid all bidders making bids at the last minute, which would yield a first-price sealed format, most English auction sites add an extension period after the closing time if there is still enough activity. Alternatively, eBay explicitly asks

bidders to submit their maximum willingness to pay and uses a “proxy” to automatically increase the bids up to that amount. Before the auction ends, no one can observe each bidder’s bid, but only observe the amount that is required to win at that point. This type of format yields behavior identical to a second-price sealed-bid auctions.

Sequential auction theory has long focused on the price path of the auction. Weber (1983) argues that on average, prices should not drift upwards or downwards for either first-price and second-price sealed auctions. The reason is that the effect of reduced competition in later auctions cancels out the reduced quantity of goods left for bidding. However, empirically what has been observed in the traditional auction is that the price tends to decline as sequential auctions proceed. This “declining price anomaly” has sparked much research. McAfee and Vincent (1993) argue that risk-aversion while Englebrecht-Wiggans (1991) present a distributional argument to explain why demand for early objects exceeds that of later ones. These arguments are based on the typical assumption of “limited consumption capacity” for the sequential auction literature (Weber 1983). That is each bidder desires only one item and the winner in the first round drops out in the second round. Therefore, bidders have to make tradeoffs between the surpluses of winning the first round of auction and the second.

In our model we do not require the winner of the first auction to drop out, nor do we assume the rest of the bidders remain in the second round. Although the “limited consumption capacity” assumption may be reasonable for durable product categories, it seems too restrictive for products found on online auctions in which consumers have the freedom to come back and bid for items in the same product category again or may seek outside options before the next round. As a result the two auctions become effectively isolated³; and the only connection between the two auctions is the number of customers. The first-round

³ In a controlled experiment, Zeithammer (2002) finds that when the number of bidders increases (as many as eight), the deflation of the first bid result from customers’ forward looking behavior will diminish. At an Online auction the number of customers tends to be large, which means that shading the first bid for the sake of the surplus from the second auction is unlikely to become an equilibrium bidding strategy.

bidders know that the surplus from bidding again in the next period is minimal because new customers may be recruited by the seller in the second period. Hence they may not want to shade their bids as described in the traditional sequential auction literature.

In addition, the nature of the online auctions casts doubt on the “reduced competition” and the “reduced auction objects”. At online auctions it is not the case that later objects receive lower demand if the auctioneer is a profit-maximizer when recruiting new customers to join the bidding process. This contrasts to traditional auctions where the quantity is fixed. Therefore, we feel it is more appropriate to view online auctions as sequential auctions with infinite rounds.

A second stream of research that is related to our problem is work on consumer satisfaction and retention. One would expect that more satisfied consumers are more likely to buy again. The relationship between customer satisfaction and retention has generated some debate (Gale 1997, Bolton 1998). Recent empirical findings by Bolton (1998) indicate that satisfaction explains a large amount of variation in the duration of service provider-customer relationship and customer acquisition and retention should not be regarded as two independent processes. When making customer acquisition decisions, firms need to forecast the duration of a customer relationship accurately (Thomas, 2001). Bolton and Lemon (1999) argue that the usage of service is determined by the consumer’s perceived “fairness” of the exchange between the prices they pay and the service they get. In addition, consumers update this “fairness” of the exchange using price and usage. Increasing customer satisfaction and retention can be used as a defensive strategy by firms because it will develop a positive reputation for the firm and makes the firm less vulnerable in competition.

3. Model

We consider an auctioneer who has one product to sell in each period of a two-period, sequential auction. The customers’ participation and bidding decisions are illustrated in Figure 1. In order to recruit customers to visit and bid at the auction, the auctioneer buys advertisements to build traffic (e.g., banner

advertisements or e-mail marketing campaigns). The auctioneer must decide how much—if any—advertising to do in each period in order to maximize expected profits. A consumer becomes aware of the auction either due to an advertisement in that period or previously bidding at the auction.

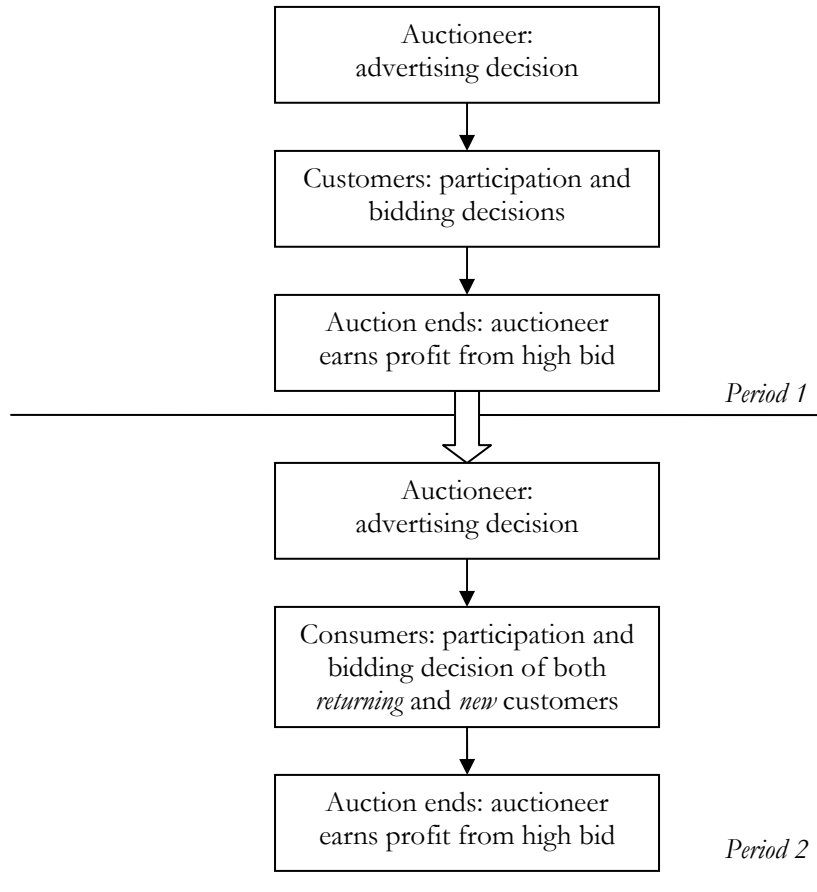


Figure 1. The Model Process

Consumers must make a decision about whether to participate in the auction based upon their expectation of the competition they expect from other bidders, their valuation of the product, and the cost of participating in the bidding process. They may bid in either or both periods, but at the end of the first period the

consumer uses their experience from the first auction as a signal of competitiveness for the second-period auction when deciding whether to participate in the second round.⁴

Assumption 1: Auction setup. We assume the auctioneer uses a second-price sealed bid, independent private value (IPV) auctions. The second-price sealed bid auction is commonly observed in practice and well established in the literature. Additionally, we assume that the auctioneer owns the product, is risk neutral, and has no reservation value for the products he is selling. The auctioneer can only provide one product for sale in each period, does not have a budget constraint for advertising, and does not discount future profits. Finally, we assume that customers are risk neutral and symmetric.

Assumption 2: Consumer's auction participation costs. We assume that consumers incur a cost when bidding at an auction (e.g. Levin and Smith, 1994, Samuelson, 1985). For instance the consumer may have to expend time and effort to become familiar with the product or auction rules. This cost may include both the opportunity cost of the time spent in participating in the auction, as well as a cognitive effort expended (Lucking-Reiley, 2002). For convenience we assume that each bidder incurs a cost c when a bid is made and 0 otherwise.

Assumption 3: Customer valuations. We assume that customer valuations for the product are stochastic identically across consumers and time. Specifically, customers' values (defined as v_i for the i th customer) for the product in each period are independent and follow a uniform distribution on the interval $[0, \bar{v}]$. (The uniform distribution is made to yield closed form solutions in §4, but is relaxed in §5.) \bar{v} is the upper bound of the value distribution and is assumed to be known. It can also be thought of as the price at which the customers can purchase the product without bidding in the retail market. Even though the value distributions for the two periods are assumed to be stochastically identical, the realization of the draws for the

⁴ The model could be formulated so that the two periods are structurally symmetric, that is, in the first period, besides the invited customers, there are some customers who knew about the auction site but were not informed by the advertisements. These customers could be thought of as returning customers from “period zero”, or arriving do to some non-purchased advertisements. Incorporating these “free customers” into the model does not alter our main results concerning retention. Hence, for the simplicity we assume all customers are solicited through advertising.

same customer in the two periods can be different. Hence, the two auctions are linked only through the set of bidders, which is determined by the advertising decisions of the auctioneer and the endogenized participation decisions of the customers. Finally, these imperfect substitutes (in different conditions) arrive randomly at the auction (Engelbrecht-Wiggans 1994).

Assumption 4: Customers' knowledge about their competitors. Unlike previous auction research, we do not assume consumers know how many bidders they are competing against. In practice consumers must infer the level of competition from signals such as past experience, word of mouth, or advertising. Since our focus is upon advertising, hence we concentrate upon the signal generated by the auctioneer's advertising intensity. We assume that the number of bidders (n) follows a Poisson distribution whose rate is determined by the advertising intensity (x) of the auctioneer, which we assume is observed by consumers, $n \sim \text{Poisson}(x)$. This distributional assumption is supported by recent empirical evidence found at online auctions (Vakrat and Seimann, 2000; Pinker, Seidmann and Vakrat, 2001).

Assumption 5: Customers' auction participation decisions are endogenous. Since bidding incurs a cost by consumers who participate, it is natural to assume that consumers follow a rule that guides their participation decisions. This rule should endogenously depend upon their chance of winning. This point was also supported by a field experiment conducted by Lucking-Reily (1999b). If bidder i participates and wins then his utility is the surplus (excess value above second highest bid) less the participation cost, and if the bidder loses then his utility is decreased by the participation cost:

$$U_i = \begin{cases} (v_i - \tilde{x})^+ - c & s \leq v_i \\ -c & s > v_i \end{cases} \quad (1)$$

where $(x)^+$ refers to the positive part of the x , s defines the participation threshold,

$\tilde{x} = \max\{v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n\}$, and n refers to the number of potential bidders in the period. If a bidder

is the only one that bids, then he wins the auction paying zero and $U_i = v_i - c$. Notice our threshold rule is

different from the conventional treatment of entry cost in the auction literature. Our model does not require

bidders to pay a fee to know their value for the product. We believe our assumption more closely resembles online auctions.

A consumer's expected utility of bidding given the number of bidders is:

$$E[U_i | v_i, v_j, N = n] = v_i F(s)^{n-1} + (n-1) \int_s^{v_i} (v_i - z) F(z)^{n-2} f(z) dz - c \quad (2)$$

We also refer the reader to Menezes and Monteiro (2000) for a similar derivation. After integrating by parts, (2) becomes

$$s[F(s)]^{n-1} + \int_s^{v_i} [F(x)]^{n-1} dx - c \quad (3)$$

where F denotes the cumulative distribution of the bids. The first part of equation (2) represents the case when all other bidders' values are less than the threshold, i.e., a consumer with the threshold value will win at the auction and is the only bidder. The second part captures the more general case, the expected gain from bidding when a customer is facing other opponents that also have values higher than the threshold s .

The notion of participation threshold ties closely to the optimal auction design literature (Riley and Samuelson, 1981, Samuelson, 1985, Menezes and Monterio 2000). It is defined as the point at which a consumer is indifferent between bidding or not. Note that a consumer can only win the auctioned product with a bid equal to this threshold if she is the only one bidding. Therefore, the equation that defines the participation threshold is:

$$sF(s)^{n-1} - c = 0. \quad (4)$$

After averaging over all possible numbers of customers, the resulting participation threshold becomes

$$s = \frac{cF(s)}{e^{x(F(s)-1)}}. \quad (5)$$

Under the uniform $[0, \bar{v}]$ assumption, s further simplifies to

$$s = \bar{v} - \frac{\bar{v} \ln(\bar{v}/c)}{x} \quad (6)$$

Notice, s increases in the highest willingness to pay \bar{v} ⁵, the bidding cost c , and the arrival rate x . Therefore, the participation threshold depends on the advertising effort (x) but not directly on n , which is random, as customers will not be able to know it with certainty *a priori*.

Assumption 6: Effect of Advertising on Consumer Arrival Rate. We assume that the auctioneer’s advertising intensity influences the arrival rate. In particular, we use $x = \frac{A}{k}$ to model the relationship between the advertising spending (A) and the arrival rate (x) for the following analysis, where k is the average unit cost of recruiting a customer⁶. Therefore, optimizing the auctioneer’s expected two-period profit over advertising level A can be viewed equivalently as a maximization problem over by choosing the optimal arrival rate x .

4. Optimal Customer Recruiting Strategy

In this section, we solve the decision problem and find the optimal level of advertising for the online auctioneer for five different cases. First, we consider a myopic auctioneer who naively optimizes advertising in each period separately. This case serves as a benchmark for our other cases. In the next two cases we consider a monopolist auctioneer who faces exogenous and endogenous retention, respectively. In the last two cases we consider an auctioneer operating in a duopoly both for exogenous and endogenous retention, to under the effects of competition between auctions. Each of these cases builds upon the other, hence we present each in sequence. The proofs of our propositions and results are given in the Appendix.

⁵ In order for $s \in [0, \bar{v}]$, this requires that $x > \ln(\bar{v} / c)$. In the following analysis, we can see that this condition is met in equilibrium.

⁶ The insights of the model do not depend on the functional form between arrival rate (x) and advertising spending (A). A positive one-to-one mapping between x and A maintains the main results of the model.

Proposition: *The symmetric subgame perfect equilibrium in the sequential auction defined above is that only customers with values for the object greater than s_t will bid in period t , and their bids equal their true values.*

In general the expected profit for the auctioneer adopting the second-price sealed bid auction format can be written as⁷ :

$$E\Pi = \sum_{t=1}^2 \int_{s_t}^{\bar{v}} \sum_{n_t=2}^{\infty} n_t (n_t - 1) (1 - F(v)) v F(v)^{n_t-2} f(v) \frac{1}{n_t!} x_t^{n_t} e^{-x_t} dv - kx_t \quad (7)$$

where n_t and x_t are the number of bidders and arrival rate in period for these bidders, respectively. The auctioneer's decision problem is to choose a non-negative advertising spending level to maximize the total expected profit function, which is:

$$\max_{A_1, A_2} E\Pi \quad s.t. \quad A_1, A_2 > 0 \quad (8)$$

We solve the problem backwards to yield:

$$\max E\Pi = \max_{x_1} E\Pi_1 \left(\max_{x_2} E\Pi_2(x_2 | x_1) \right) \quad (9)$$

In the second period, we have both new customers who are recruited by advertising in that period and some returning customers from the previous period. Other consumers from the first period will drop out from the bidding pool simply because they do not want the product anymore, or decide to seek outside options. We denote the proportion of customers from period 1 who are aware in period 2 as α , $\alpha \in [0,1]$.

Case I: Myopic Auctioneer

This case examines a myopic seller, who recognizes existence of the participation threshold, but treats the two-period auctions as if they were run independently and takes no customer retention into

⁷ Note that when $n < 2$, then no value will be transferred from the auctioneer to the customers. Hence, the firm's expected profit averages over all the possible numbers of bidders starting from two.

account. In this case, the optimization problem reduces to two identical decisions in each period. (This is equivalent to the solution of a one period problem.) We obtain the following result:

Result 1. *The total expected profit function for the auctioneer is concave and has a unique optimal advertising level.*

The optimal advertising level that maximizes the expected total profit in this case is

$$x_1^* = x_2^* = \sqrt{\frac{2\bar{v} - 2c - c \ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{k}}. \quad (10)$$

Throughout the following analysis, we use q to denote the optimal advertising in the myopic case to further simplify the notation. For future reference, the total expected profit for the myopic case is $2\bar{v} - 2c - 2c \ln(\bar{v}/c) - 4kq$, and the corresponding total optimal advertising spending is $2kq$.

Case II: Monopoly Auctioneer Recognizing Retention

A customer may decide not to bid in the first period, but may return and participate in the second period if his value is high enough (this follows from assumption 3). The pool of eligible bidders is defined as those consumers who are informed about the auction in both periods. Following the properties of the Poisson distribution, the total number of customer arrivals in the second period follows a Poisson distribution with rate $\lambda_2 = x_2 + \alpha x_1$. Customers are aware of this distribution and would rationally expect new customers to join the bidding and vice versa. The expected total profits and the optimal solutions to this problem are given in Table 1. Comparing this case with the myopic auctioneer we state two results of interest.

Result 2. *The expected profit is strictly higher than that of the myopic case when a monopoly auctioneer takes into account retention which is strictly positive.*

Figure 2 shows the difference in expected profit when retention is properly taken into account versus the myopic case. Accounting for retention changes the advertising strategies for both periods. The optimal advertising level is higher in the first period and lower in the second period compared with the myopic

solution. The profit gain is the result of saving from unnecessary adverting spending. This is consistent with previous empirical findings in marketing research on customer retention (e.g. Thomas, 2001).

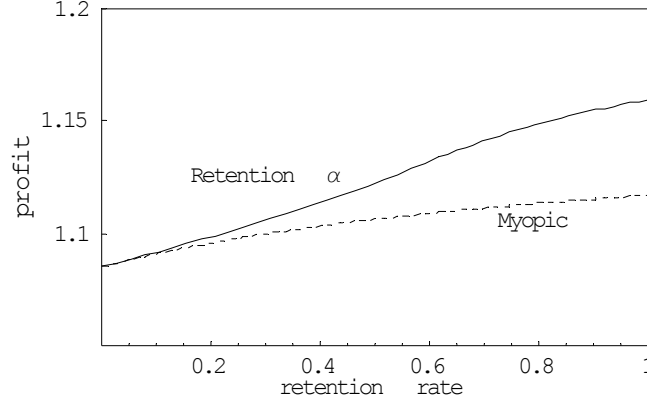


Figure 2: Comparison of profits with retention by an optimizing and myopic auctioneer ($k=0.005$, $\bar{v}=1$, $\epsilon=0.1$)

Result 3. For the monopoly case: (a) Not accounting for retention will result in overspending in total advertising.

(b) When $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ the optimal spending on customer acquisition in the second period decreases in retention, whereas

the first period optimal spending increases in retention. When $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$, as α increases, the total optimal advertising spending should be decreased, compared to the myopic case.

The intuition for this result is that since the retention rate is strictly positive, spending more in the first period results in more consumers the second period, of which α percent of first period customers will stay in the second period. However, in the second period due to the higher retained customer base, the customer's perception of competition increases. Hence, the spending should be reduced accordingly, even more so when the retention rate is high.

Case III: The Duopoly Competition with Retention

Suppose there are two auctioneers in the market, who are competing for bidders in each period. For simplicity, we consider the case in which the auctioned goods at the two competing auction sites are identical, and the two auction firms are symmetric. In reality, it is possible that consumers are aware of both auction sites, for example, from being exposed to advertisements of both firms. However, they will bid at only one auction at a time since they have only unitary quantity demand for the product.

To model this situation, we introduce a parameter γ that measures the overlap in the number of the bidders between the two auctions, which is defined as $\gamma = \frac{D}{x+y}$, where D is the Poisson rate of customers who are aware of both auction sites, and x and y respectively denote the rates of aware customers at the each of the two auction sites. This parameter $\gamma \in [0,1)$ measures the degree to which advertising efforts overlap at the two auctions and essentially captures the competition intensity in the market⁸. To simplify our model we assume that γ is constant across periods. We also use a random splitting rule (e.g. Lippman and McCardle, 1997) for assigning the customers who were aware of the both firms, e.g., each firm gets half of the overlapping customer pool.

In this case, the number of bidders for either of the auctions in the second period is

$$\lambda_2 = (1 - \frac{\gamma}{2})(\alpha x_1 + x_2) - \frac{\gamma}{2}(\alpha y_1 + y_2), \quad (11)$$

where x_2 and y_2 are the arrival rates of the two symmetric auctioneers in the second period. Notice that λ_2 consists of two parts: the customers from the previous period and the newly recruited customers. In each part, the rival auctioneer is contributing to the arrival rate, through the overlapping, informed customers in both periods. The duopoly competition model yields the following notable finding:

⁸ When $\gamma = 1$ the model reduces to a case where two auctioneers are competing for the same customers.

Result 4. For the case of exogenous retention: (a) total advertising spending increases in γ and (b) increasing γ reduces profit.

As long as the two auctioneers are competing for the same consumers, i.e., $\gamma > 0$, the industry's total advertising spending will be strictly greater than that of the monopoly case. This result holds for both periods and when comparing the duopoly model to either the myopic model or the monopoly model with retention. An extreme case occurs when γ goes to 1 which drives auctioneer advertising spending to infinity. In general, the expected profit decreases in the intensity of competition, γ (shown in Figure 3). This occurs because as competition becomes more intense the firm's spending on advertising becomes less efficient. Note that the duopoly framework reduces to the monopoly case when $\gamma = 0$.

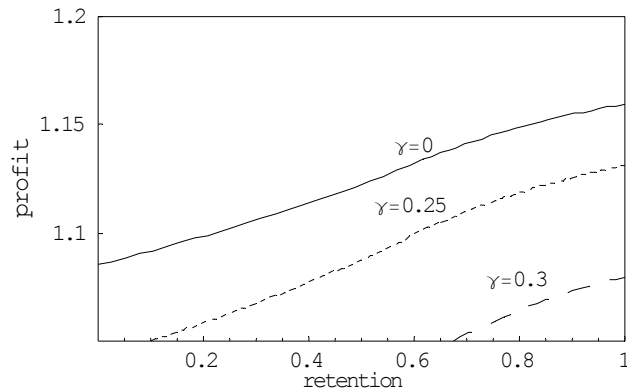


Figure 3: Competition Effects ($\kappa=0.005$, $\bar{v}=1$, $c=0.1$)

Case IV: Monopoly Auctioneer with Endogenous Retention

In the above model we have assumed the retention rate, α , to be exogenous. In this section we relax this assumption by modeling retention as the proportion of consumers who perceive their value to be too low given the perceived advertising intensity in the first round of auction. In the second period, the potential number of customers who *return* are those whose values are above their threshold in the first period,

$(\frac{\bar{v} - s_1}{\bar{v}})x_1$. Together with the newly recruited bidders, the arrival rate becomes $\lambda_2 = x_2 + \frac{(\bar{v} - s_1)}{\bar{v}}x_1$.

Following Equation (4), the threshold for participation in the second period simplifies to

$$s_2 = \bar{v} - \frac{\bar{v} \ln(\bar{v}/c)}{x_2 + \ln(\bar{v}/c)}. \quad (12)$$

We solve the monopoly auctioneer's decision problem with retention as defined above and find the optimal advertising levels to be:

$$x_1^* = q \text{ and } x_2^* = q - \ln\left(\frac{\bar{v}}{c}\right) \quad (13)$$

Similar to the exogenous case, if the auctioneer is myopic and does not recognize retention, he will overspend in the second period, which leads to lower expected profits. The extent of overspending depends on the logarithm of the ratio of the upper bound on value over the bidding cost.

Case V: Duopoly Competition with Endogenous Retention

When retention is considered endogenously in the symmetric duopoly competition, the arrival rate, which consists of both returning customers and new advertising efforts in the second period, becomes

$$\lambda_2 = \left(1 - \frac{\gamma}{2}\right) \frac{\bar{v} - s_1}{\bar{v}} x_1 - \frac{\gamma}{2} \frac{\bar{v} - s_1}{\bar{v}} y_1 + \left(1 - \frac{\gamma}{2}\right) x_2 - \frac{\gamma}{2} y_2, \quad (14)$$

The symmetric equilibrium for the first and the second period are respectively

$$x_1^* = y_1^* = \frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)} q \text{ and } x_2^* = y_2^* = \frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)} q - \ln \frac{\bar{v}}{c} \quad (15)$$

As in the exogenous case, we find that increased spending by auctioneers increases the competition intensity, and summarize our findings with the following two results:

Result 5: *When retention is endogenized, the previous findings in the exogenous cases still hold.*

Comparing the models with endogenous retention to the model where retention is assumed exogenous, we make the following observations: First, the optimal ad spending levels in the first period of the exogenous retention model (A1 in Figure 4) are always higher than those of the endogenous case for the monopoly auctioneer. This is true for the duopoly case when α is large. Exogenous models (both monopoly and duopoly) suggest auctioneers invest less in customer acquisition in the second period (A2 in Figure 4) for most retention values except those on the lower end.

Second, the expected profit from the endogenous case exceeds those of the myopic model.

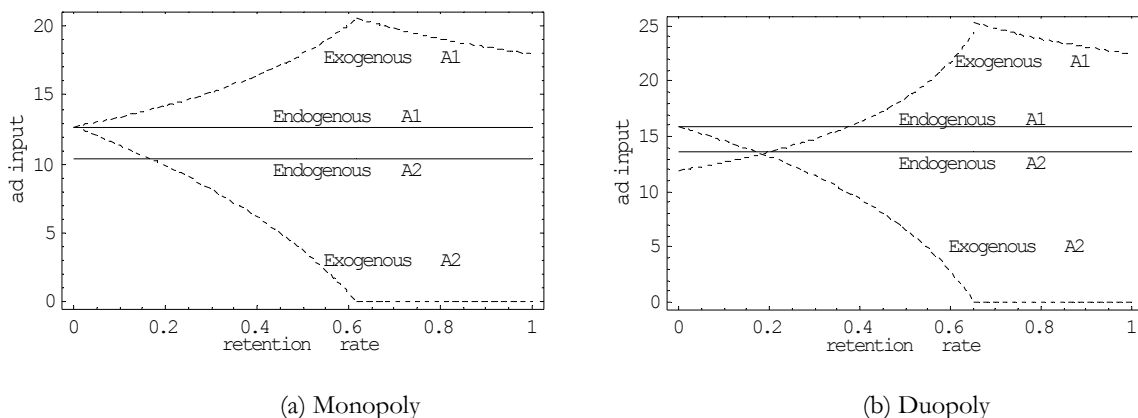


Figure 4: Advertising Strategies when Retention is Endogenous and Exogenous ($k=0.005$, $\bar{v}=1$, $\epsilon=0.1$, $\gamma=0.25$).

Result 6. *When customers' valuations are i.i.d. Uniform $[0, \bar{v}]$, for all the monopoly cases considered the total expected profit increases in retention α and the upper bound of the distribution \bar{v} ; and decreases in bidding cost c and the advertising cost k .*

5. A Numerical Study

Like most other papers in the auction literature, the closed-form results obtained in the last section relied upon the assumption that the consumers' values are uniformly distributed on $[0, \bar{v}]$. In this section,

we examine how these results change if we consider other distributions. Specifically, we consider the Pareto and Weibull distributions since they can accommodate a variety of distribution shapes.

First, we consider a Pareto distribution, which has a CDF of $F(v) = 1 - \theta / v^\theta, (v > 1, \theta > 0)$. This distribution can capture a potential value distribution commonly observed in practice: most customers are bargain hunters, namely, a large amount of the density lies in the lower quantiles; but some bidders may have very high values, which is represented by the long tail of the distribution. As the shape parameter (θ) increases, the variance decreases and the tail becomes thinner (see Figure 5a).

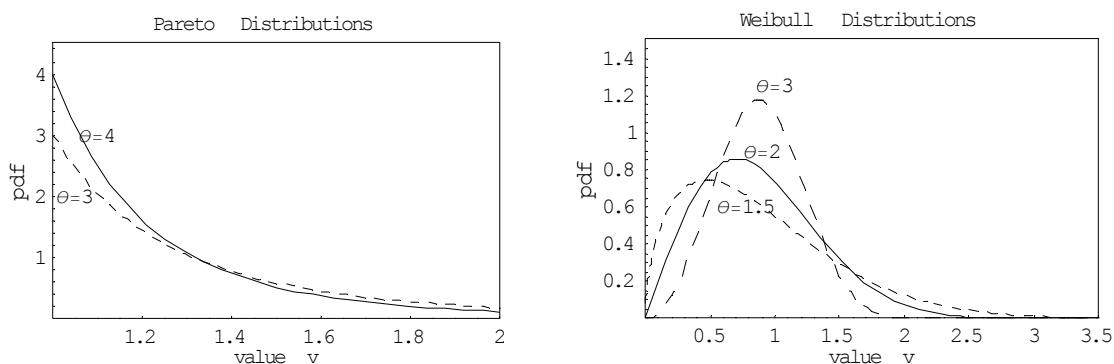


Figure 5: (a) Pareto Distributions ($\theta=3$ and 4), and (b) Weibull Distributions ($\theta=1.5, 2,$ and 3)

We outline our numerical analysis as follows: First we solve for the participation threshold with Poisson arrivals as modeled in the Uniform distribution case. Given the complexity of the probability distribution function, the threshold has to be solved numerically. After we have the threshold solutions, we numerically integrate the expected profit function over its defined range, with the number of bidders following a Poisson distribution as we assumed previously. In the Pareto distribution case this integration range is from the threshold (which is greater than 1) to infinity.

We illustrate our findings in Figure 6. First, similar to the results for the uniform distribution, the expected profit based on the Pareto distribution (in a single period) is also uni-modal, and defines a unique optimal advertising spending strategy. Second, when consumers' participation is endogenous, the expected

profit is lower than that of a benchmark model where the bidding threshold is considered to be zero. This is due to the endogenously determined bidding threshold (i.e., the competition effect among customers). When the threshold is strictly positive, some of the profit is lost due to consumers' perceived competitiveness at the auction. We also find that if the firm ignores the endogeneity of participation, it would spend too much on advertising which results in less profit (i.e., optimal ad spending under EP' is higher than under EP). Third, the bidding threshold is non-decreasing in advertising input, which also conforms to the results we had for the Uniform distribution. Comparing the two cases with different shape parameters, both the bidding threshold and the expected profit are lower for the case when $\theta=4$ than $\theta=3$. The intuition is that as θ becomes large, more mass is placed over lower values, which reduces expected profits. Additionally, the tail of the distribution becomes thinner and thus the threshold tapers off more quickly.

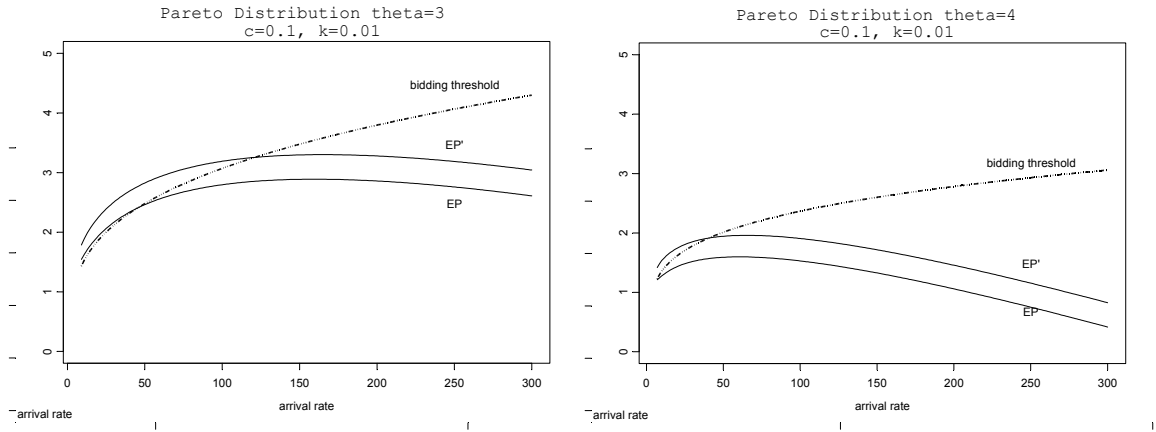


Figure 6: Comparison of Expected Profit with Endogenous Participation (EP) and a benchmark model (EP')

Next, we consider the bell-shaped Weibull distributions. The c.d.f. of the Weibull distribution with a scale parameter θ is $F(v) = 1 - e^{-v^\theta}$, where $\theta > 0, v > 0$. The mean and variance are:

$$E(v) = \Gamma(1 + 1/\theta)$$

$$Var(v) = \Gamma(1 + 2/\theta) - \Gamma(1 + 1/\theta)^2.$$

The Weibull distribution has flexible shapes and by varying its shape parameter θ from 1.5 to 3, the mode of the distribution gradually shifts from the left to the right (see Figure 5b).

The results for the Weibull distributions are similar to those of the Pareto distributions (see Figure 7). They suggest that as the shape parameter increases, the expected profit decreases, as does the participation threshold and the expected revenue. The optimal advertising input level decreases as well. These results are largely due to the tail behavior of the Weibull distribution. As θ increases, the tail becomes thinner and hence it is less likely to get a large value by spending more on advertising.

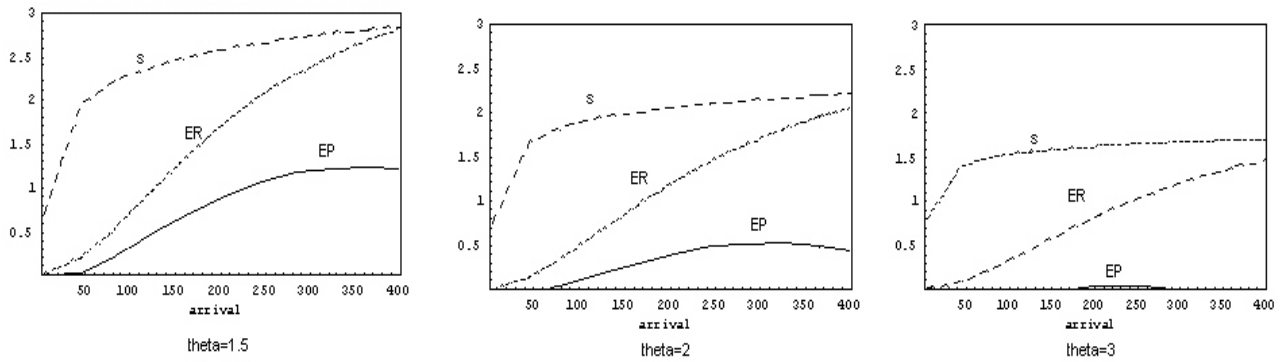


Figure 7: Expected Profit (EP), Expected Revenue (ER) and bidding threshold (s) for Weibull distributions, $(c=0.1, k=0.04, \bar{v} = \infty, \gamma = 0)$

To summarize, our numerical study suggests that the shape of the value distributions is an important consideration when the firm makes its advertising decisions. However, qualitatively regardless of the shape parameter, our basic results from the uniform distribution still hold. Namely, expected profit functions are concave and have a unique maximum in advertising spending. Although we only considered a single period in this numerical study, we expect these results to hold when the simulation is extended to the second period with the retention linking the two periods.

Effects of Upper Bound of the Value Distributions

In the above simulation study, we take the whole distribution range, namely, for both Pareto and Weibull distributions, the customers' value can go to positive infinity. However, it is often quite likely that customers' values for the auctioned product is bounded from above at some price level, for example, a price of a similar product that offered by a retail store, or a suggested maximum bidding price posted at the auction. To incorporate this case, we study the truncated Pareto and Weibull distributions with an arbitrary truncation point. We found that for the Pareto and Weibull distributions when the upper bound is finite instead of infinite then even the expected revenue has a unique mode. This result suggests that even if the cost of advertising is negligible to the online auction firms, it is not optimal to advertise as much as possible. As the upper bound of the value distribution decreases, namely, the outside option becomes attractive, the optimal level of advertising needs to be reduced accordingly. Table 2 and Figure 8 demonstrate these findings for a Pareto ($\theta=3$) and Weibull ($\theta=1.5$) distribution. The results are similar for the other distributions we studied in this section.

Upper Bound \bar{v}	Truncation Degree $1-F(\bar{v})$	Participation Threshold	Optimal Arrival Rate	Optimal Revenue
10	0.0010	5.1835	641	5.8380
8	0.0020	4.2229	331	4.6254
6	0.0046	3.2413	141	3.4184
4	0.0156	2.2295	42	2.2201

Table 2a: Optimal Expected Revenue of Pareto Distribution at Various Right Truncations ($\theta=3$, $c=0.1$, $k=0.01$).

Upper Bound \bar{v}	Truncation Degree $1-F(\bar{v})$	Participation Threshold	Optimal Arrival Rate	Optimal Revenue
3.5	0.0014	2.1741	77	2.0728
3	0.0055	2.1452	72	1.8820
2.5	0.0192	1.6808	26	1.4270
2	0.0591	1.1762	10	1.0033

Table 2b: Optimal Expected Revenue of Weibull Distribution at Various Right Truncations ($\theta=1.5$, $c=0.1$, $k=0.01$).

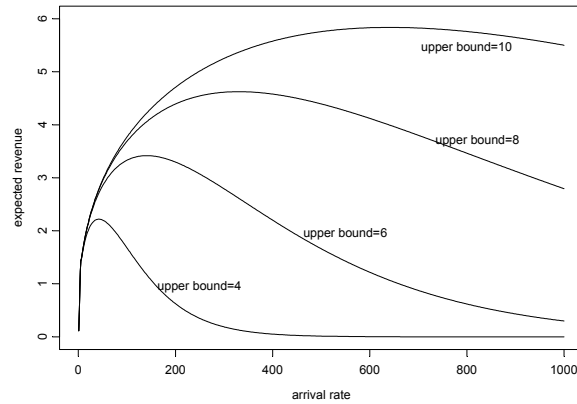


Figure 8a: Expected Revenue at Various Right Truncations (Pareto $\theta=3$, $c=0.1$, $k=0.01$).

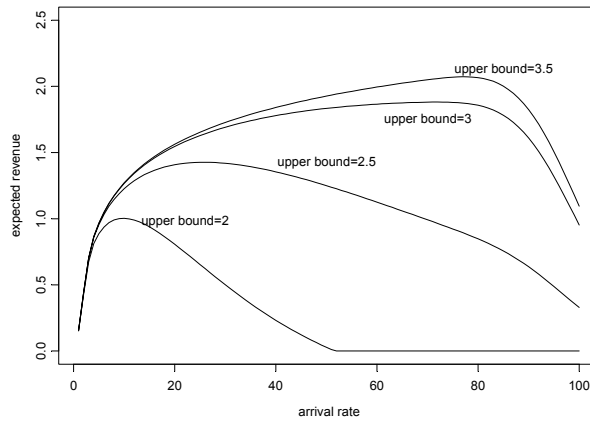


Figure 8b: Expected Revenue at Various Right Truncations (Weibull $\theta=1.5$, $c=0.1$, $k=0.01$).

6. Discussion and Conclusions

We argue that recognizing the connection between customer acquisition and retention at an auction is important for auctioneers. In this research, we explicitly model the role of customer acquisition and retention on the profitability of online auctions when customer participation decisions are endogenized. We show that an online auctioneer needs to optimize their advertising strategy to achieve higher profits. Besides reducing unnecessary operating expenses, controlling advertising spending can increase a consumer's

perceived chance of winning which can increase retention and subsequently the auctioneer's profits. Our findings contrast with past research (Levin and Smith, 1994; Menezes and Monteiro, 2000) which found that revealing knowledge about the exact number of competitors does not impact the auctioneer's expected revenue, even with endogenous participation in a private value auction with risk-neutral bidders. Our results show that advertising could serve as a signal of the potential number of bidders which can directly influence an auctioneer's revenue.

The link between a successful auction and the advertising policy of the auctioneer has not been considered in the academic literature. Yet the most successful auctions, eBay and uBid, which together account for more than 95% of online auctions, cite high customer satisfaction and loyalty as important contributors to their success (Nielsen/NetRatings and Harris Interactive 2001). The senior vice president of marketing at uBid credits their success and growth to the incredible value that auctions provide consumers. This value creates consumer loyalty, which provides the foundation for the future of the business (Press Release Network, 2001).

We also point out another conclusion from our research, namely that the auction business may be less scalable than those of conventional retailers. If the number of customers participating at an auction is increased, without a compatible increase in the number of products being sold, then customer retention is hurt and an auctioneer's profitability can decline. A possible explanation of Onsale's demise is that excessive growth (that can partially be attributed to higher advertising spending) was not balanced by an increase in product variety, which resulted in more but less happy consumers. According to Nielsen/NetRatings, total consumer spending on online auctions in May 2001 was \$556 million, an increase of 149 percent from the preceding year and 65 percent from the preceding month. Additionally, advertising spending by auction houses ranks second behind online department stores in the retail category (AdRelevance, 1999). Given the explosive growth in this category, this suggests that the continued growth of online auctions depends upon both an increase in demand and supply.

This research is not without limitations. We have deliberately made many assumptions to keep the model simple and tractable, which has assumed away many interesting aspects of the problem. First, we only consider the merchant auctioneer whose supply of auctioned goods is fixed. Although this is likely to be true for many merchant sites that actually own the products they sell or merchants who sell surplus merchandise whose quantity is limited, it is also possible that auctioneers could adapt the assortment and variety of their merchandise based upon their user base. Second, we ignored many elements of the auction policy that employ be used to recruit and retain customers, for instance, bidding duration, quantity of auctioned goods, and the seller's reservation prices and so on. Prior research indicates that these factors affect the final winning prices and the participation behavior of bidders (Engelbrecht-Wiggans 1987). These will inevitably affect consumers' perception on the value of bidding as well. Third, we did not consider constraints on market size or a firm's advertising budget. Although we would conjecture that incorporating these constraints should strengthen our major findings. Finally, there are many ways that the model can be enriched, for instance by incorporating heterogeneous bidding costs among customers, auctioneer's targetability development over the time, and the influence of communication among consumers. We hope our efforts will encourage others to consider the contribution of marketing variables to the successful design of auctions.

References

- Bajari, Patrick and A. Hortacsu (2002), "Cyberspace Auctions and pricing Issues: A Review of Empirical Findings," in Derek Jones, ed., the *New Economy Handbook*, forthcoming.
- Bolton, Ruth (1998) "A Dynamic Model of the Duration of the Customer's Relationship with a Continuous Service Provider: The Role of Satisfaction," *Marketing Science*, 17 (1), 45-65.
- Bolton, Ruth and Katherine Lemon, (1999), "A Dynamic Model of Customer Usage of Services: Usage as an Antecedent and Consequence of Satisfaction," *Journal of Marketing Research*, 36 (May), 171-186.
- Chakravarti, Dipankar, E. Greenleaf, A. Sinha, A. Cheema, J. Cox, D. Friedman, T-H, Ho, R.M. Isaac, A. Mitchell, A. Rapoport, M. Rothkopf, J. Srivastava, R. Zwick (2002) "Auctions: Research Opportunities In Marketing," *Marketing Letters*, in press.
- Engelbrecht-Wiggans, Richard (1987), "On Optimal Reservation Prices in Auctions," *Management Science*, 33 (6), 763-770.
- Engelbrecht-Wiggans, Richard (1993), "Optimal Auctions Revisited," *Games and Economic Behavior* 5, 227- 239.
- Engelbrecht-Wiggans, Richard (1994), "Sequential auctions of Stochastically Equivalent Objects," *Economic Letters*, 44, 87-90.
- Gale (1997) "Satisfaction is Not Enough," *Marketing News*, October 27, p.18.
- Klemperer, Paul (1999), "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys*, 13 (3), 227-286.
- Levin, D. and J. Smith (1994) "Equilibrium in Auctions with Entry," *American Economic Review*, 84, 585-99.
- Lippman, Steven A. and K. F. McCardle (1997), "The Competitive Newsboy," *Operations Research*, 45, (1), 54-65.
- Lucking-Reiley, David (1999a), "Auctions on the Internet; What's Being Auctioned and How?" *The Journal of Industrial Economics*, XLVIII (3), 227-251.
- Lucking-Reiley, David (2002), "Experimental Evidence on the Endogenous Entry of Bidders in Internet Auctions." *Economic Inquiry*, 40 (44), 611-619.
- Matthews, Steven (1987), "Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View," *Econometrica*, 55 (3), 633-646.
- McAfee, R. and D. Vincent (1993), "The Price Decline Anomaly," *Journal of Economic Theory*, 60, 191-212.
- Menezes, Flavio and P. K. Monteiro (2000), "Auctions with Endogenous Participation," *Review of Economic Design*, 5 (1), 71-89.
- Milgrom, Paul (1989), "Auctions and Bidding: A Primer." *Journal of Economic Perspectives*, 3 (3), 3-22.
- Milgrom, Paul and R. Weber (1982), "A Theory of Auctions and Competitive Bidding." *Econometrica*, 50 (5), 1089-1122.
- Moon, Youngme (1999), "Onsale, Inc.," Case 9-599-091, Harvard Business School Publishing, Boston, MA.

- Pinker E., A. Seidmann and Y. Vakrat (2001). "The Design of Online Auctions: Business Issues and Current Research", Working Paper CIS01-05, Simon School, University of Rochester.
- Riley, John and W. Samuelson (1981), "Optimal Auctions." *The American Economic Review*, 71 (3), 381-392.
- Thomas, Jacquelyn (2001), "A Methodology for Linking Customer Acquisition to Customer Retention," *Journal of Marketing Research*, 38 (May), 262-268.
- Samuelson, W. (1985) "Competitive Bidding with Entry Costs," *Economics Letters*, 17, 53-57.
- Vakrat, Y. and A. Seidmann (2000), "Implications of the Bidders' Arrival Process on the Design of Online Auctions," In Proceedings of the 33d Annual Hawaii International Conference on System Sciences (HICSS-33), (Jan. 2000, Maui, Hawaii).
- Wang, R. (1993) "Auctions versus Posted-Price Selling," *American Economic Review*, 83 (4), 838-851.
- Weber, R. J. (1983), "Multi-object Auction", in: *Auctions, Bidding and Contracting: Uses and Theory*, eds., R. Engelbrecht-Wiggans, M. Shubik and R. M. Strak (New York University Press), 165-194.
- Zeithammer, R. (2002) "Sequential Auctions for Substitutes: A Theory of Bargain-Hunting on eBay," working paper, MIT.

Appendix

1. Notation used in the paper

v_i	customer i 's value for the auctioned product
\bar{v}	upper bound of the value distributions
n_t	number of customers in the t -th period
x_t	arrival rate of customers generated by the advertising in the t -th period of firm 1
y_t	arrival rate of customers generated by the advertising in the t -th period of firm 2
λ_t	arrival rate of customers of the t -th period
s_t	bidding threshold of the t -th period
α	retention parameter $\alpha \in [0, 1]$
A	advertising spending
k	unit cost of advertising
c	cost of bidding
γ	competition intensity $\gamma \in [0, 1]$
q	optimal arrival rate for the myopic model
F	CDF of the customer's values for the product
θ	shape parameters for Pareto and Weibull distributions in simulation study

2. Proof of Proposition

We argue that the assumption that there will be one less bidder in the second auction by the traditional sequential auction literature is too restrictive. In the current model, no winning customers are required to exit the auction, current customers can drop out of the auction freely (with probability α in the model), and the auction firm has the capability to bring new customers to the second auction. As a result, the two auctions are effectively separated, and customers will not shade their bids in the first period because of the possible gain in the second period.

Menezes and Monteiro (2000) proved that in a fixed- n private-value second-price auction with endogenous participation, the optimal bidding strategy for bidders is to bid their true value if the value is greater than the threshold. Our proof is similar to that except that the number of bidders follows Poisson distributions with different rate in the two auctions. This again depends on the fact that the two auctions are isolated when the seller is able to replenish the demand for the good.

3. Proof of Results 1-6.

For the following proofs, we assume $\bar{v} > c \gg k > 0$.

Result 0: $q > 0$ and $q - \ln \frac{\bar{v}}{c} > 0$, where $q \equiv \sqrt{\frac{2\bar{v} - 2c - c \ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{k}}$.

Proof: Note that the numerator inside the square root is decreasing in c :

$\frac{d}{dc} 2v - 2c - c \ln(v/c)(2 + \ln(v/c)) < 0$. This implies that when c goes to \bar{v} , this part reaches its minimal,

which is 0. Since k is also positive, we have $q > 0$. Similarly we can show that $q - \ln \frac{\bar{v}}{c} > 0$.

Result 1[on Concavity of the profit function and Uniqueness of the Optimal Solution]: *The total expected profit function is concave and has a unique optimum. Furthermore, in each period, the expected profit function for the auctioneer is concave and has a unique optimal advertising level.*

Proof: From (5), we have

$$E\Pi_2 = \lambda_2^2 \int_{s_2}^{\bar{v}} \frac{\exp((\bar{v}/v - 1)x)(\bar{v} - v)v}{\bar{v}^2} dv - kx_2 \quad (A1)$$

$$\text{with } \lambda_2 = x_2 \text{ in this case hence, } s_2 = \bar{v} - \frac{\bar{v} \ln(\bar{v}/c)}{x_2}. \quad (A2)$$

After simplifying, myopic sellers total expected profit then becomes

$$E\Pi = 2 \left(\frac{(\bar{v} - c)(x - 2) - kx^2 + c \ln(\bar{v}/c)(2 - x + \ln(\bar{v}/c))}{x} \right). \quad (A3)$$

Assuming the expected profit function is differentiable with respect to n , the second order derivative with respect to x_2 is

$$\frac{\partial^2 E\Pi_2}{\partial x_2^2} = \frac{4(c - \bar{v}) + 2c \ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{x_2^3} \leq 0 \quad (\text{A4})$$

So $E\Pi_2$ is concave. By solving the first order condition

$$\frac{\partial E\Pi_2}{\partial x_2} = -\frac{2c - 2\bar{v} + kx_2^2 + c \ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{x_2^2} = 0, \quad (\text{A5})$$

we obtain

$$x_1^* = x_2^* = \sqrt{\frac{2\bar{v} - 2c - c \ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{k}}. \quad (\text{A6})$$

The proof for the concavity and the uniqueness of the solution for the other models in the paper is similar and hence is omitted here.

Result 2 [on Total Expected Profit]: *When monopoly auctioneer takes it into account retention when it is strictly positive ($\alpha > 0$), the expected profit is strictly higher than that of the myopic case.*

Proof: Let Π^{case1} and Π^{case2} denote the total expected profit for the auctioneer of Case 1 (monopoly, myopic) and of Case 2 (monopoly, exogenous retention), respectively.

$$\Pi^{ex} - \Pi^m = \begin{cases} 2kq - 2kq\sqrt{1-\alpha} > 0 & \text{when } \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ -2kq\sqrt{1+\frac{1}{\alpha}} + 4kq > 0 & \text{when } \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \quad (\text{A7})$$

Result 3 [on Advertising Spending]: *In the monopoly case, in terms of advertising spending, (a). Not accounting for retention will result in overspending in total advertising. (b). When $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ the optimal spending on customer*

acquisition in the second period decreases in retention, whereas the first period optimal spending increases in retention. When

$\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$, retention decreases optimal advertising spending.

Proof: Part (a).

The total advertising for Case 2 is $kq(1 + \sqrt{1-\alpha})$ when $\alpha \in [0, \frac{\sqrt{5}-1}{2})$; and

$kq\sqrt{1 + \frac{1}{\alpha}}$ when $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$. The total spending in both cases will be less than that of the myopic case, $2kq$.

Moreover, the myopic auctioneer will spend less than optimal in the first period and more than optimal in the second period. Let A_t^{case1} , A_t^{case2} , and denote the advertising spending in time period t , for the Case 1 and Case 2, respectively.

$$A_2^{case2} - A_2^{case1} = \begin{cases} -kq\alpha\sqrt{\frac{1}{1-\alpha}} < 0 & \text{when } \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ -kq < 0 & \text{when } \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \quad (A8)$$

$$A_1^{case2} - A_1^{case1} = \begin{cases} kq\left(\sqrt{\frac{1}{1-\alpha}} - 1\right) > 0 & \text{when } \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ kq\left(\sqrt{1 + \frac{1}{\alpha}} - 1\right) > 0 & \text{when } \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \quad (A9)$$

Part (b),

$$\frac{\partial A_2^{case2}}{\partial \alpha} = -\sqrt{\frac{1}{1-\alpha}} - \frac{1}{2} \left(\frac{1}{1-\alpha}\right)^{3/2} < 0 \text{ when } \alpha \in [0, \frac{\sqrt{5}-1}{2})$$

$$\frac{\partial A_1^{case2}}{\partial \alpha} = \begin{cases} \frac{1}{2} \left(\frac{1}{1-\alpha}\right)^{3/2} > 0 & \text{when } \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ -\frac{1}{2\alpha^2 \sqrt{1+\frac{1}{\alpha}}} < 0 & \text{when } \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \quad (A10)$$

Result 4 [on Competition between Auctioneers]: For the exogenous retention case, (a). the total advertising spending is increasing in γ ; and (b). γ reduces profit.

Proof: Part (a).

First we show that when $\alpha \in [0, M)$, γ increases in the total advertising spending.

$$\frac{dA^{case3}}{d\gamma} = \frac{3-\gamma}{2\sqrt{4-2\gamma(1-\gamma)^2}} + (1-\alpha) \frac{(2-\gamma)^2 - 2(1-\gamma)^2}{2(2-\alpha(2-\gamma) - 2\gamma)^2 \sqrt{\frac{(1-\gamma)(2-\gamma)}{2-\alpha(2-\gamma) - 2\gamma}}} \equiv h_1(\alpha, \gamma) \quad (A11)$$

We rely upon a graphic argument to prove our claim that $h_1(\gamma, \alpha)$ is positive. Notice that $h_1(\gamma, \alpha)$ is a decreasing function of α , namely it reaches its minimum when $\alpha = M$. Figure A-1 depicts the curve $h_1(\gamma, \alpha | \alpha = M)$, and shows that $h_1(\gamma, \alpha)$ is strictly positive.

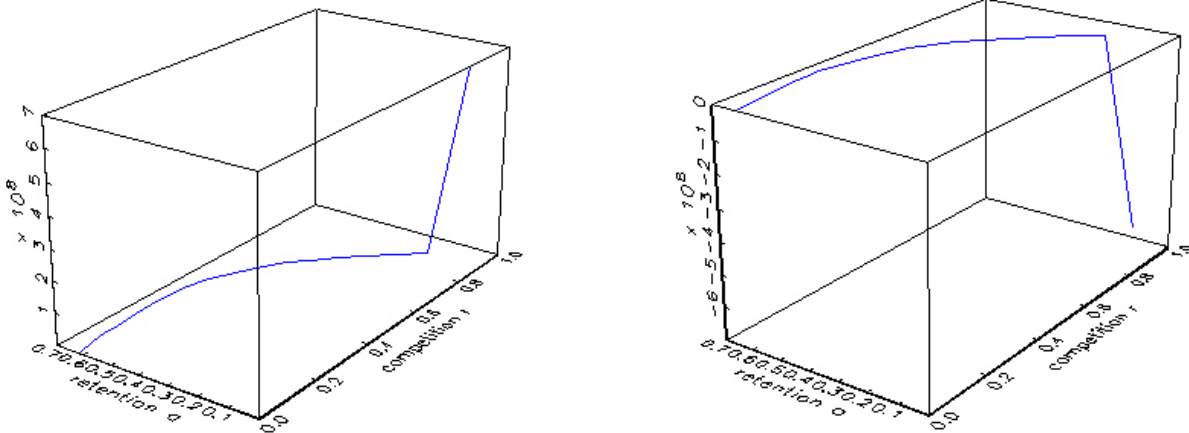


Figure A-1: Result 4 (a)

Figure A-2: Result 4 (b)

And when $\alpha \in [M, 1]$,

$$\frac{dA_{\alpha \in [M, 1]}^{case 3}}{d\gamma} = \frac{1}{\sqrt{2}(1-\gamma)} \sqrt{\frac{2(1+\alpha)(2-\gamma)}{\alpha}} > 0 \quad (A12)$$

Part (b).

Again we use graphical method to show that $\frac{dE\Pi_{\alpha \in [0, M]}^{case 3}}{d\gamma} < 0$.

$$\begin{aligned} \frac{dE\Pi_{\alpha \in [0, M]}^{case 3}}{d\gamma} &= \\ &= \frac{-3\alpha(2-\gamma)^2 + 2(1-\gamma)(5-3\gamma)}{2\sqrt{\frac{2}{2-\gamma} - \frac{\alpha}{1-\gamma}}(2-\gamma)^2(-1+\gamma)3} + (\alpha-1) \frac{(2-\gamma)^2 - 2(1-\gamma)^2}{2(2-\alpha(2-\gamma) - 2\gamma)^2 \sqrt{\frac{(1-\gamma)(2-\gamma)}{2-\alpha(2-\gamma) - 2\gamma}}} + \frac{8+3(-3+\gamma)\gamma}{2\sqrt{4-2\gamma(-2+\gamma)(1-\gamma)^2}} \\ &\equiv h_2(\gamma, \alpha) \end{aligned}$$

Note that $h_2(\gamma, \alpha)$ is an increasing function of α , and hence, reaches its maximum when $\alpha = M$. We plot

$h_2(\gamma, \alpha | \alpha = M)$. If $h_2(\gamma, \alpha | \alpha = M) < 0$, then we have proved that competition intensity decreases profit. Figure (A-2) shows that, indeed, the claim is true.

$$\frac{dE\Pi_{\alpha \in [M, 1]}^{case 3}}{d\gamma} = kq \frac{\sqrt{\frac{1+\alpha}{4\alpha-2\alpha\gamma}}(8-9\gamma+3\gamma^2)}{2(\gamma-2)(1-\gamma)^2} < 0 \quad (A13)$$

This is because $8-9\gamma+3\gamma^2 > 6-9\gamma+3\gamma^2 = (\gamma-2)(3\gamma-3) > 0$.

Result 5 [on Endogenous Retention]: *When retention is endogenized, the previous findings in the exogenous cases still hold.*

Proof: Let Π_t^{en} be the total profit of the monopoly auctioneer who considers retention effect, which is endogenously determined. And Let A_t^{en} be the optimal advertising spending in period t for this auctioneer.

$$\text{First,} \quad \Pi^{case4} - \Pi^{case1} = k \ln \frac{\bar{v}}{c} > 0 \quad (\text{A14})$$

This is consistent with Result 2. And,

$$A^{case4} - A^{case1} = 2kq - k \ln \frac{\bar{v}}{c} - 2kq < 0$$

Consistent with the first part of the Result 3, Part (a). Specifically,

$$\begin{aligned} A_2^{case4} - A_2^{case1} &= -k \ln \frac{\bar{v}}{c} < 0, \\ A_1^{case4} - A_1^{case1} &= 0 \end{aligned} \quad (\text{A15})$$

Result 4 holds for the endogenous case as well, as the following are true:

$$\frac{dA_1^{case5}}{d\gamma} = \frac{dA_2^{case5}}{d\gamma} \frac{3-\gamma}{2\sqrt{4-2\gamma}(1-r)^2} > 0 \quad (\text{A16})$$

and

$$\frac{dE\Pi^{case5}}{d\gamma} = 2kq \frac{\sqrt{4-2\gamma}(8-9\gamma+3\gamma^2)}{4(\gamma-2)(1-\gamma)^2} < 0 \quad (\text{A17})$$

Result 6 [on Other Parameters of the Model]: *When customers' values are i.i.d. distributed Uniform $[0, \bar{v}]$, for all the monopoly cases considered, the total expected profit increases in retention α , the upper bound of the distribution \bar{v} ; and decreases in bidding cost c and the advertising cost k .*

Proof:

1. The retention effect (α): For case II, it is straightforward to see that when α increases, the expected profit increase on the whole range of α . For case III, when $\alpha \in [0, M)$, it is also easy to see that the expected profit increases in α . And

$$\frac{\partial E\Pi_{\alpha \in [M,1]}^{case 3}}{\partial \alpha} = \frac{kq(4-3\gamma)}{2\alpha(1+\alpha)(1-\gamma)} \sqrt{\frac{1+\alpha}{4\alpha-2\alpha\gamma}} > 0 \quad (A18)$$

2. The bidding effect (c): In general, the first-order derivative of the expected profit functions of the five models follow the following form:

$$\frac{\partial E\Pi^{case 1}}{\partial c} = \ln \frac{v}{c} \left(-2 + \frac{4}{2} \frac{\ln \frac{v}{c}}{q} \right). \text{ Because } \ln \frac{v}{c} < q \text{ from the Result 0 above, we have } \frac{\partial E\Pi^{case 1}}{\partial c} < 0$$

$$\frac{\partial E\Pi_{\alpha \in [0, (\sqrt{5}-1)/2]}^{case 2}}{\partial c} = \ln \frac{v}{c} \left(-2 + \frac{2(1+\sqrt{1-\alpha})}{2} \frac{\ln \frac{v}{c}}{q} \right) < 0 \quad (A19)$$

$$\frac{\partial E\Pi_{\alpha \in [(\sqrt{5}-1)/2, 1]}^{case 2}}{\partial c} = \ln \frac{v}{c} \left(-2 + \frac{2(\sqrt{1+\frac{1}{\alpha}})}{2} \frac{\ln \frac{v}{c}}{q} \right) < 0$$

$$\frac{\partial E\Pi^{case 4}}{\partial c} = \ln \frac{v}{c} \left(-2 - \frac{k}{c} + \frac{4}{2} \frac{\ln \frac{v}{c}}{q} \right) < 0 \quad (A20)$$

3. The advertising cost effect (k): by examining the expected profit functions of each model, we note that there is a term $-gkq$ in each of the cases, where g denotes the positive coefficient of the $-kq$ term. We also

know that in general, $\frac{d(-kq)}{dk} = -\frac{1}{2}q < 0$. Hence, the claim is easily proved for the cases 1-3.

And
$$\frac{dE\Pi^{case 4}}{dk} = \ln \frac{\bar{v}}{c} - 2q < 0 \quad (A21)$$

$$\frac{dE\Pi^{case 5}}{dk} = \ln \frac{\bar{v}}{c} - \frac{4-3\gamma}{(1-\gamma)\sqrt{4-2\gamma}} q. \quad (A22)$$

Because
$$\frac{4-3\gamma}{(1-\gamma)\sqrt{4-2\gamma}} > \frac{4-4\gamma}{(1-\gamma)\sqrt{4-2\gamma}} = \frac{4}{\sqrt{4-2\gamma}} > 1, \frac{dE\Pi^{case5}}{dk} < 0. \quad (A23)$$

4. The upper bound effect (\bar{v}): Again, let g denote the positive coefficient of the $-kq$ term. In general,

$$\frac{dE\Pi}{d\bar{v}} = 2 - \frac{2c}{\bar{v}} + \frac{g(c + c \ln(\bar{v}/c) - \bar{v})}{\bar{v}q} = b_3(\bar{v}, c). \quad \text{This function is decreasing function of } c. \quad \text{The minimum}$$

is obtained when $c = \bar{v}$, and $b_3(\bar{v}, c) = 0$. Hence, the upper bound parameter has positive effect on the expected profit.

Optimal Ad Spending in the First Period A_1^*	Optimal Ad Spending in the Second Period A_2^*	Total Optimal Expected Profit $E\Pi^*$
Case I. Myopic		
kq	kq	$2\bar{v} - 2c - 2c \ln\left(\frac{\bar{v}}{c}\right) - 4kq$
Case II. Monopoly, Exogenous Retention		
$kq\sqrt{\frac{1}{1-\alpha}}$ if $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ $kq\sqrt{1+\frac{1}{\alpha}}$ if $\alpha \in [\frac{\sqrt{5}-1}{2}, 1)$	$kq\left(1-\alpha\sqrt{\frac{1}{1-\alpha}}\right)$ if $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ 0 if $\alpha \in [\frac{\sqrt{5}-1}{2}, 1)$	$2\bar{v} - 2c - 2c \ln\frac{\bar{v}}{c} - 2kq(1+\sqrt{1-\alpha})$ $2\bar{v} - 2c - 2c \ln\frac{\bar{v}}{c} - 2kq\left(\sqrt{1+\frac{1}{\alpha}}\right)$
Case III. Competition, Exogenous Retention		
$kq\sqrt{\frac{(1-\gamma)(2-\gamma)}{2+\alpha(\gamma-2)-2\gamma}}$ if $\alpha \in [0, M)$ $\frac{kq}{2(1-\gamma)}\sqrt{\frac{2(1+\alpha)(2-\gamma)}{\alpha}}$	$kq\left(\frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)} - \alpha\sqrt{\frac{(1-\gamma)(2-\gamma)}{2+\alpha(\gamma-2)-2\gamma}}\right)$ if $\alpha \in [0, M)$ 0 otherwise	$2\bar{v} - 2c - 2c \ln\frac{\bar{v}}{c} - kq$ $\left(\sqrt{\frac{2+\alpha(\gamma-2)-2\gamma}{(1-\gamma)^3(2-\gamma)}} + (1-\alpha)\sqrt{\frac{(1-\gamma)(2-\gamma)}{2+\alpha(\gamma-2)-2\gamma}} + \frac{4-3\gamma}{(1-\gamma)\sqrt{4-2\gamma}}\right)$ $2\bar{v} - 2c - 2c \ln\frac{\bar{v}}{c} - \frac{kq(4-3\gamma)}{1-\gamma}\sqrt{\frac{1+\alpha}{4\alpha-2\alpha\gamma}}$
Case IV: Monopoly, Endogenous Retention		
kq	$k\left(q - \ln\frac{\bar{v}}{c}\right)$	$2\bar{v} - 2c - 2c \ln\frac{\bar{v}}{c} + k \ln\frac{\bar{v}}{c} - 4kq$
Case V: Competition, Endogenous Retention		
$\frac{kq\sqrt{2(2-\gamma)}}{2(1-\gamma)}$	$k\left(\frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)}q - \ln\frac{\bar{v}}{c}\right)$	$2\bar{v} - 2c - 2c \ln\frac{\bar{v}}{c} + k \ln\frac{\bar{v}}{c} - \frac{2kq(4-3\gamma)}{(1-\gamma)\sqrt{4-2\gamma}}$

Note: $q \equiv \sqrt{\frac{2\bar{v} - 2c - c \ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{k}}$. $M = \frac{\gamma - 2 + \sqrt{20 + \gamma(-68 + \gamma(97 + 16(-4 + \gamma)\gamma))}}{4(1-\gamma)^3}$.

Table 1: Summary of Results