Customer Acquisition at Online Auctions:
Why More Bidders Can Decrease Profitability

Xin Wang and Alan L. Montgomery

January 2012

Xin Wang (e-mail: xinwang@brandeis.edu) is an Assistant Professor at the International Business School, Brandeis University, Waltham, MA 02454. Alan L. Montgomery (e-mail: alan.montgomery@cmu.edu) is an Associate Professor at the Tepper School of Business, Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA 15213. The authors thank the National Science Foundation for partially supporting this research with Grant No. 0118767. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Customer Acquisition at Online Auctions:

Why More Bidders Can Decrease Profitability

Abstract

A fundamental marketing problem faced by auctioneers is deciding upon an appropriate promotional strategy in order to acquire customers. Auctioneers need to attract as many bidders as possible since the more bidders should increase the winning price and result in higher profits. This suggests that an intense advertising campaign that raises awareness to the largest possible customer base would be desirable. However, more bidders mean decreased chances of winning and ex post may result in less satisfied customers and lower customer retention rates. To examine this trade-off, we endogenize the entry decision of customers according to their expected gains from participation, which in turn determines the auctioneer's optimal advertising policy. Our analyses of auctions in monopoly and duopoly settings suggest that controlling growth by limiting advertising spending can increase an auctioneer's long-term profitability, and can be necessary even within a single period. We show that depending on the customer valuation distribution, an auctioneer might need to refrain from advertising even when the cost of advertising is minimal.

Keywords: Auctions; Advertising, Customer Acquisition; Customer Retention

JEL Classification: D44, M37

1. Introduction

The revenue generated by an auction is positively related to the number of bidders, since more bidders should drive the winning offer higher. It would seem that the most profitable auctions should be those that aggressively advertise and recruit as many people as possible. On the other hand if we consider consumer behavior we can argue the opposite effect: increased numbers of competing bidders will reduce a bidder's chance of winning the auction which makes auctions with large number of bidders less appealing. Understanding this trade-off between more customers with a higher expected winning offer versus smaller auctions with more satisfied customers is the focus of this study.

Using an analytical model we show how customer acquisition tools such as advertising can affect the customer retention rate and subsequently the profitability of an auction. We propose a two-period model in which the auctioneer sells one item in each time period using a second-price, sealed-bid format. The auctioneer must decide on an advertising budget, which influences the number of customers who participate. Once the customer is aware of the auction, subsequent participation is based upon their perceived value of auction. (For simplicity we refer to bidders as customers, regardless of whether they actually purchase.) Using this framework we derive the optimal advertising strategy for an auctioneer that maximizes his expected profits based upon different types of customer participation decisions.

These results are important for many types of popular online auctions which have become popular since the late 1990s and are now an important base of e-commerce activity. Businesses such as HP, Dell, uBid, and Onsale have adopted separate auction channels. To attract bidders, these online auctioneers have relied on advertising, such as affiliate banner networks, comparison shopping sites, paid and organic search engines, e-mail, and televised promotions. For example, uBid spends millions of dollars on advertising, including television ads, that aims to generate traffic to the website.

To illustrate the importance of advertising to acquire customers at an auction site consider the auction site Onsale. Onsale sold surplus computer merchandise through online auctions. After a year in operation, it was poised to earn positive operating margins from its auctions, but it also chose to aggressively expand its customer base through advertising (Moon 1999). By increasing its customer base of bidders faster than its supply of surplus merchandise, Onsale failed to take advantage of the initial satisfaction of its customers, who perceived that their chances of winning had decreased. Although many factors contributed to Onsale's eventual demise, this case offers a clear example of an auctioneer that inappropriately balanced its promotional needs with its economic considerations.

Our study provides auctioneers insight into how to achieve this balance. First, extant research indicates that retention effectively increases customer value (e.g., Gupta and Lehmann 2005), but our research also reveals that retention can harm auctions if it is not managed in accordance with customer acquisition. The two processes are not independent, and both affect the auction's profit. That is, online auctions usually offer similar products sequentially, so early investments in advertising determine the size of the later customer base through retention, which represents a "carry-over" benefit. Yet too much carry-over is problematic, because customers rationally predict bidding competitiveness and opt out if they consider the auction too competitive.

Second, the retention effect on customer acquisition is not continuous or monotonic, even within a single period. Different retention levels have opposite effects on advertising, and different types of retention stipulate entirely different advertising policies. When the retention rate is high, the auctioneer should refrain from any advertising and instead increase its initial advertising spending, allowing the retention effect to increase the customer base. This result holds in both monopoly and duopoly settings. Moreover, we show that different reasons motivate customers' return decisions can have drastically different implications for advertising policy.

Third, even ignoring the cost of advertising, it is not optimal to increase advertising blindly to increase auction profit. The auctioneer needs a good understanding of the customer valuation distribution and outside competition, especially for fixed-price offers. Although the negative financial impact is greater when an auctioneer underinvests than overinvests in customer acquisition, deviating from optimal levels either way can lead to substantial financial loss, the extent of which depends on the underlying customer valuation distribution and outside fixed-price offer. Because of the competition between bidders and their endogenous participation, auction models are not as scalable as fixed prices. Thus, auctioneers and auction houses should pay special attention to expanding the customer base appropriately to achieve optimal profits.

In addition, our approach contributes to extant literature in two ways. First, most auction research in economics and business literature focuses on optimal bidding strategies and auction designs (for a review, see Klemperer 1999). In marketing, more recent advances model bidding behavior (Park and Bradlow 2005) and dynamics (Bradlow and Park 2007), demand structures (Yao and Mela 2008), seller competition (Chan, Kadiyali and Park 2007), multi-item auctions (Popkowski Leszczyc and Häubl 2010; Subramaniam and Venkatesh 2009), and reference price effects (Kamins, Dreze, and Folkes 2004; Dholakia and Simonson 2005). Yet little attention has been placed on ways to manage the customer base and relationships, and even less research examines the bidder's retention value. Thus, no proven promotional strategy for auctions balances customer acquisition and retention effects. Marketing-related issues, such as advertising, the role of channels, and consumer learning, have largely been ignored in auction literature (Chakravarti et al. 2002).

Second, marketing research notes the effects of advertising and retention on profit, though not in an auction setting, where customers must compete to get an item. The extent to which customers' retention rate and advertising policy might change in light of such competitiveness has not been studied. If customers are collectively price setters instead of takers, lifetime value measures must go beyond transaction-based metrics, because most auction participants neither win nor pay for the product, but they still generate revenue together.

The paper is organized as follows. In §2 we review research on auction design from economics and marketing that pertains to our study. We then present the model and its assumptions in §3, then derive the analytical results of the model in §4. Distributional assumptions are relaxed in the following section, and several simulation studies are conducted to confirm the robustness of our results and offer more quantitative insights. We conclude in §5 by discussing the managerial implications of our model and areas for further research.

2. Literature Review

This study draws from two streams of extant research: auctions in economics and management and customer profitability in marketing. Auction theory— including online auctions which are conducted through the Internet— is extensive. Our model can be applied to either online or offline formats, but our assumptions about auctioneer and bidder behavior are similar to those of online auctioneers like uBid and Dell Auctions.

Online auctions differ from traditional, offline auctions in many ways, which has forced economists to revisit auction theory (Pinker, Seidmann and Vakrat 2001). First, online auctions lack any physical presence in the market, so to attract enough bidders the auctioneer stretches the duration of its auctions. Second, online auctions tend to be much longer than the average for traditional auctions (Lucking-Reiley 1999a). There is great variation in the length of Internet auctions ranging from a few minutes to several weeks, but most average about a week. As a result awareness of Internet auctions may be higher than for traditional auctions. Third online auctions tend to be more frequent, which amplifies the need for customer retention. Finally, online auctions focus more on advertising using such mediums as televised ads, search keywords, and banner ads to draw traffic, which is consistent with our focus on advertising.

The most common online auction format is a second-price auction with ascending bids. Such formats are favored by both researchers and auctioneers because of their simplicity (Lucking-Reily, 1999a), though they present some unique challenges online. For example, to avoid a situation in which all bidders make bids at the last minute, which would yield a first-price sealed format, auction sites such as uBid or Amazon auctions add an extension period after the closing time if there is enough activity. Other sites explicitly ask bidders to submit their maximum willingness to pay and use proxies to increase the bids automatically up to that amount. Before the auction ends, no one can observe any other bidder's bid, just the amount required to win at that point. We follow the convention of theoretical work and consider a second-price sealed-bid framework.

Within the framework of second-price sealed-bid auctions, economists have examined price paths to understand the bidding dynamics of sequential auctions. One argument is that the price of the auctions should decline, because bidders in early auctions are willing to pay an extra risk premium associated with future risky prices (McAfee and Vincent 1993), or winner's dropping out reduces competition for the subsequent auctions (Engelbrecht-Wiggans 1994). Another school of thought posits that the price of sequential auctions in fact should go up due to the uncertainty of future supply (Jeitschk 1999), or the complementarily between the two auctioned goods (Sørensen 2006). Weber (1983) argues that on average, prices do not drift upward or downward for either first-price and second-price sealed auctions, because the effects of reduced competition and fewer goods available cancel out each other. Jank and Shmueli (2010) examine actual online auctions data and find that empirically the prices across sequential online auctions do not exhibit significant differences.

We consider online sequential auctions such that the bidders in earlier auctions can freely choose to participate or not participate in later auctions. The winner of the first auction does not need to drop out, nor are other bidders required to remain in the second round (unlike the forward-looking theory, where the all losing bidders have to stay throughout the two period, and the winning bidder has to exit).

Although the limited consumption capacity assumption may be reasonable for durable product categories, it seems too restrictive for online auctions, for which customers may return to bid for items in the same product category or seek outside options before the next round. Moreover, the auctioned goods sold are stochastically equivalent, i.e. *ex ante* they are identical to bidders. Hence, the values of bidders are not fixed across auctions, instead they are drawn from the same valuation distribution (e.g. Englebrecht-Wiggans 1994, Sørensen 2006). Therefore, we argue that assuming that bidder values are independent across auctions is more realistic in depicting consumer behavior at online auctions like uBid. Thus we model two sequential auctions effectively are isolated[1]; the only connection between them is the number of customers. The first-round bidders know that the surplus from bidding again in the next period is minimal, because the seller will recruit new customers if necessary in the second period. Thus they may prefer not to shade their bids. In addition, online auctions might not involve reduced competition or fewer auction objects, because objects offered later do not provoke less demand, assuming the auctioneer maximizes profit when recruiting new bidders.

In a multiple-auction setting, Wang, Montgomery, and Srinivasan (2008) show bidders' participation costs can have important implications for auctioneers pricing format designs, such as auctions, buy-it now auctions and fixed price offers. We follow their definition of customer participation costs as those including direct time costs, opportunity costs associated with waiting for the auction to close, transaction costs associated with bidding, and the customer's cognitive effort expended in the bidding process. Their empirical study reveals that consumers make conscientious efforts to reduce or avoid bidding costs and select product offerings based on time-to-end, reserve price, and availability of posted price. Haruvy and Leszczyc (2010) use controlled experiments to demonstrate that the wide price

[1] In a controlled experiment, Zeithammer (2002) finds that when the number of bidders increases (as many as eight), any deflation of the first bid due to customers' forward-looking behavior diminishes. In an Internet auction, more customers tend to participate, so shading the first bid to gain surplus from the second auction is unlikely to be an equilibrium bidding strategy.

dispersions at online auctions and bidder strong inertia in auction choices can be attributed to bidders' search costs. However, no research has studied how pricing decisions affect auctioneers' advertising policy or how a stipulated advertising policy might affect customers' return decisions.

Another related problem in auction design is understanding endogenous auction participation. Menezes and Monteiro (2000) consider a problem in which agents decide whether or not to participate in an auction based only upon their own valuation and participation cost but without knowing how many other participants are bidding against them. They point out that the auctioneer's profits may decrease when the number of potential participants increases. This is an intriguing result for our study since it suggests that more auction participants are not always better. However, their results do not elaborate about why and how the auctioneer's profits will decline. We believe this is an important area for study if managers are to decide about how to manage the size of the customer base, and these are questions that we pursue in our research.

As noted previously we also draw on a second stream of research related to customer value, which also involves customer acquisition, retention, and profitability strategies. Gupta et al. (2006) review conceptual frameworks and models in this field, and empirical findings (e.g., Bolton 1998) indicate that customer satisfaction explains much of the variation in the duration of a service provider–customer relationship. In this sense, customer acquisition and retention are interdependent, so when they make customer acquisition decisions, firms should forecast the duration of the customer relationship (Thomas 2001). Bolton and Lemon (1999) also argue that service usage is determined by a consumer's perception of the fairness of the exchange, which is largely a function of the price paid versus the service received. In addition, consumers update fairness perceptions over time. By increasing customer satisfaction and retention firms can develop positive reputations and thus can defend themselves better against attacks by competitors.

Reinartz, Thomas, and Kumar (2005) propose a model that helps firms balance their customer acquisition and retention efforts in a traditional fixed price selling context. They advocate an integrated framework with connections among acquisition, retention, and customer profitability. Using estimates of their empirical model, they also show that underspending on acquisition is worse than overspending in terms of returns on investment, but an optimal acquisition and retention solution is generally flat (i.e., a large deviation from the optimal strategy leads to relatively small profit decreases). However, such findings have not been examined in an auction setting, where customers collectively determine the price and customer equity has a different meaning. We therefore investigate how an auction firm's acquisition and retention expenditures affect profitability.

3. Model

We assume an auctioneer has one product to sell in each period of a two-period, sequential auction. To recruit customers to visit and bid, the auctioneer buys advertisements to increase the number of bidders (e.g., banner or TV advertisements, or online search words). The auctioneer must decide how much—if any— advertising to buy in each period to maximize total expected profits. Furthermore, the auctioneer hosts second-price, sealed-bid, independent private value auctions. We assume that the auctioneer owns the product, is risk neutral, and has no reservation value for the products being sold. The auctioneer can only provide one product for sale in each period, does not have a budget constraint for advertising, and does not discount future profits.

These assumptions provide a stylized representation of online auctioneers like uBid and Dell Auctions. However, eBay—which is currently the most popular online auction—does not fit our framework well. eBay serves as an auction intermediary which profits by facilitating auctions and does not own the products. It profits through increased listings and not just completed auctions. Hence it advertises to draw both auctioneers and bidders. As a consequence its profit function is more complex and does not fit well with our modeling assumptions.

A customer becomes aware of the auction after seeing an advertisement in that period or previously bidding. Customers must make a decision about whether to participate in the auction, based upon their expectation of competition from other bidders, their product valuation, and the efforts associated with participating in the bidding process. Customers may bid in either or both periods. At the end of the first period, they use their experience as a signal of the competitiveness of the second-period auction and decide if to bid again. In addition, we assume that customers are risk neutral and symmetric.

We assume that customers incur a cost when bidding, such as the time and effort required to become familiar with the product or auction rules, the opportunity cost of time spent participating in the auction, or cognitive efforts. Thus, in line with previous research, we assert that a bidder incurs a cost c if she makes a bid and otherwise does not incur at cost. Bidding cost could be specific to each individual bidder, but such an approach would not provide closed-form results. Green and Laffont (1984) show that with heterogeneous bidding cost that there exists a unique bidding equilibrium such that the customer will bid her true valuation if and only if her expected utility of bidding is greater than the cost of bidding. Wang, Montgomery, and Srinivasan (2008) offer a thorough simulation analysis of such asymmetric bidding costs in the context of endogenous participation.

Customer valuations for the product are stochastically identical across customers and time. Specifically, customers' values (defined as v_i for the ith customer) for the product in each period are independent and follow a uniform distribution in the interval $[0, \bar{v}]$[2]. (The uniform distribution assumption is made to yield closed-form solutions, but will be relaxed in Section 5.) We assume that $\bar{v}$,

[2] The zero-lower bound improves the tractability of the analyses. We can show that everything else being equal, when the lower bound of the valuation is positive, for example, due to the seller's reserve, our main result will be stronger (the advertising spending should be even further reduced. In other words, the zero-lower bound condition considered here yields more conservative result. Proof is available upon request.

the upper bound of the value distribution, is known; it represents the price at which customers can purchase the product through an outside vendor without bidding. The value distributions for the two periods are assumed to be stochastically identical, but the realization of the draws for the same customer in the two periods can be different. The two auctions thus are linked only by the set of bidders, which is determined by the advertising decisions of the auctioneer and customer retention behavior. Finally, auctioned products in each period are treated as imperfect substitutes (Engelbrecht-Wiggans 1994).

Unlike previous research, we do not require customers to know exactly how many other bidders are in the auction; the auctioneer does not have the privilege of such knowledge either (McAfee and McMillan 1987). In practice, customers might infer the level of competition from observable signals, such as past experience, word of mouth, or exposed advertising. Such signals, or advertising intensity (x), govern the Poisson distribution of customer arrivals: $n \sim \mathrm{Poisson}(x)$. This distributional assumption is supported by empirical evidence from online auctions (Vakrat and Seimann, 2000; Pinker, Seidmann, and Vakrat, 2001).

Because of their participation costs, customers must decide whether to participate in an auction (see also Lucking-Reily 1999b). If bidder i participates and wins, then his or her utility is the surplus (excess value above the second highest bid) less the participation cost; if the bidder loses, then her or his utility decreases by the amount of the participation cost:

$$U_i = \begin{cases} (v_i - z)^+ - c & s \le v_i \\ -c & s > v_i \end{cases}, \tag{1}$$

where $(x)^+$ refers to the positive part of the x, s defines the participation threshold, and $z = \max\{v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N\}$. Our threshold rule differs from the conventional treatment of entry cost in auction literature, because we do not require bidders to pay a fee to reveal their value for the product (e.g., Levin and Smith 1994; Samuelson 1985). Customers know their valuation for the product but also understand that if they participate in bidding, they face internalized participation costs. We believe this scenario resembles real-world online auctions. Therefore, a customer's expected utility of bidding conditioned upon the number of bidders is:

$$E\left[U_i \mid v_i, v_j, N = n\right] = v_i F(s)^{n-1} + (n-1)\int_s^{v_i} (v_i - z) F(z)^{n-2} f(z) dz - c \,. \tag{2}$$

Menezes and Monteiro (2000) offer a similar derivation. After integrating by parts, (2) becomes

$$s\left[F(s)\right]^{n-1} + \int_s^{v_i} \left[F(x)\right]^{n-1} dx - c \,, \tag{3}$$

where s is the participation threshold, n is the number of bidders, and F denotes the cumulative distribution of the bids. In the first part of (2), all other bidders' values are less than the threshold, and the customer with the threshold value wins at the auction as the only bidder. The second part captures the more general case or the expected gain from bidding when a customer faces other opponents who also have values higher than the threshold s.

The notion of a participation threshold ties closely to optimal auction designs (Riley and Samuelson 1981; Samuelson 1985; Menezes and Monterio 2000) and represents the point at which a customer is indifferent between bidding or not. A customer can only win the auctioned product with a bid equal to this threshold if he or she is the only one bidding. Therefore, the equation that defines the participation threshold is:

$$sF(s)^{n-1} - c = 0. \tag{4}$$

After averaging over all possible numbers of customers, the participation threshold becomes

$$s = \frac{cF(s)}{e^{x(F(s)-1)}} \,. \tag{5}$$

Under the uniform [0, $\bar{v}$] assumption, s further simplifies to

$$s = \bar{v} - \frac{\bar{v}\ln(\bar{v}/c)}{x} \,. \tag{6}$$

Notice that s increases with the highest willingness to pay $\bar{v}$,[3] the bidding cost c, and advertising intensity x. All else being equal, a higher bidding threshold means fewer customers.

Advertising intensity is modeled as $x = \frac{A}{k}$, where A is advertising spending, and k is the average unit cost of recruiting a customer. The main results do not change on the basis of the functional forms of x and A. Any positive one-to-one mapping between x and A would maintain the insights of the model; therefore, optimizing the auctioneer's expected two-period profit over the advertising level A is equivalent to a maximization problem for choosing the optimal x. In Section 5, we generalize the model by considering a case with no advertising costs.

4. Optimal Customer Acquisition Strategy

Even though prior research has shed some light on bidders' endogenous participation decisions, none has examined the implications of such decisions on seller's profits and customer acquisition strategies. In this section, we tackle this problem and find the optimal level of advertising for the online auctioneer with different retention assumptions. We first analyze the monopoly case, and then proceed to the case of a duopoly competition.

Proposition: *The symmetric subgame perfect equilibrium in the sequential auction defined above is that only customers with values for the object greater than s_t will bid in period t, and their bids equal their true values.*

In general the expected profit for an auctioneer adopting a second-price, sealed-bid auction format can be written as:[4]

---

[6] In order for $s \in [0, \bar{v}]$, this requires that $x > \ln(\bar{v}/c)$. In the following analysis, we can see that this condition is met in equilibrium.

[4] When $n < 2$, no value transfers from the auctioneer to customers. The firm's expected profit averages over all possible numbers of bidders, starting with two.

$$E\Pi = \sum_{t=1}^{2} \int_{s}^{\bar{v}} \sum_{n_t=2}^{\infty} n_t (n_t - 1)(1 - F(v)) v F(v)^{n_t - 2} f(v) \frac{1}{n_t!} x_t^{\;n_t} e^{-x_t} dv - k x_t \;. \tag{7}$$

The auctioneer's decision problem is to choose a non-negative advertising spending level to maximize the total expected profit function, or

$$\max_{A_1, A_2} E\Pi(A_1, A_2) \;\; s.t. \;\; A_1, A_2 > 0 \,. \tag{8}$$

We solve the problem backwards to find:

$$\max E\Pi = \max_{x_1} E\Pi_1 \left(\max_{x_2} E\Pi_2 (x_2 \mid x_1) \right). \tag{9}$$

The second period includes both new customers recruited by advertising in that period and some returning customers from the previous period. Other customers from the first period drop out from the bidding pool because they do not want the product anymore or decide to seek outside options. We denote the proportion of customers from period 1 who return in period 2 as $\alpha \in [0,1]$. Therefore, advertising decisions are linked through retention, and both elements influence the customer's perceptions of competitiveness through the bidding threshold.

Case 1: Myopic Auctioneer

As a benchmark, a myopic (or naïve) auctioneer recognizes existence of the participation threshold but treats two-period auctions as if they were independent and takes no customer retention into account. The optimization problem thus reduces to two identical decisions in each period (equivalent to the solution of a one-period problem). We obtain the following result:

Result 1: *The total expected profit function is concave and has a unique optimum. Furthermore, in each period, the expected profit function for the auctioneer is concave and has a unique optimal advertising level.*

The optimal advertising level that maximizes the expected total profit of the myopic auctioneer is

$$x_1^* = x_2^* = \sqrt{\frac{2\overline{v} - 2c - c\ln(\overline{v}/c)(2 + \ln(\overline{v}/c))}{k}}. \tag{10}$$

To simplify the notation, we use q to denote the optimal advertising level in the myopic case, as in Equation (10). The corresponding total optimal advertising spending is $2kq$. As we show in Table 1, the total optimal expected profit for the myopic case (Case 1) thus is $2\overline{v} - 2c - 2c\ln(\overline{v}/c) - 4kq$.

Case 2: Monopoly Auctioneer that Recognizes Retention

A customer may decide not to bid in the first period but then participate in the second period if her value is high enough. Each customer hence has a chance of coming back, and the decision to return to a second auction is completely random (as opposed to following a decision rule, as in Case 3). Following the properties of the Poisson distribution, the customer arrival in the second period follows a Poisson distribution with rate $\lambda_2 = x_2 + \alpha x_1$. In reality, returning bidders reasonably anticipate new bidders, and vice versa. The expected total profits and the optimal solutions to this problem, as summarized in Case 2 in Table 1, suggest interesting results in comparison with Case 1.

Result 2. *Accounting for customer retention when it is positive yields higher seller's profit than not accounting for it. Disregarding positive customer retention, an auctioneer will underspend on advertising in the first period and overspend in the second, which leads to suboptimal profits.*

This increase in the auctioneer's profit is not simply from cost savings. Notice that when customer retention is positive that optimal ad spending is actually higher in the first period and then lower in the second than that in the myopic case (constant kq). The profit gain results from the proper allocation of advertising dollars according to the customer retention rate. Notice that when the retention rate is high, the auctioneer runs a higher risk of overinvesting in advertising in the second period. The reason is that more customers carry over from the previous period. Therefore, advertising spending should be reduced.

In the first period, if the auctioneer does not acknowledge the retention effect, it underinvests, because it fails to realize that advertising dollars spent in the first period offer benefits in both periods.

This result is consistent with previous empirical findings in marketing that suggest customer acquisition and retention should not be viewed as independent. As Thomas (2001) shows, failing to account for the interactions of customer acquisition with retention results in large bias in estimates of the net present value of customers and customer relationships, prompting erroneous customer relationship management decisions and negative financial impacts. Similarly, if the auctioneer disregards retention, it underspends in the first period and overspends in the second, which leads to suboptimal profit.

Another important finding is that within each period, the optimal advertising spending is a step function of customer retention, which reflects the non-negative advertising spending boundary. The level of customer retention could have qualitatively different impacts on optimal advertising levels, even in a single period.

Result 3: *When retention is exogenous, a monopoly auctioneer (a) in the first period, should increase the advertising level as α increases when* $\alpha \in [0, \frac{\sqrt{5}-1}{2})$*, but reduce advertising as α increases when* $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$. *(b) In the second period, this auctioneer should decrease advertising when* $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ *and stop advertising when* $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$.

The intuition for this result is that positive retention motivates the auctioneer to invest more in acquiring new customers in the first period when the retention rate is low to medium, or $\alpha \in [0, \frac{\sqrt{5}-1}{2})$. If the customer base is too large, it is counterproductive in both periods, because it not only increases customers' perceived competition but also implies unnecessary spending. Therefore, adverting should be reduced in the second period and in the first if retention is high, or $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$. The auctioneer should eliminate advertising altogether in the latter period if retention is high.

In this case, we also see that when customer retention is randomly determined, higher advertising intensity in the first period increases the bidding threshold for the second period ($s_2 = \bar{v} - \frac{\bar{v}\ln(\bar{v}/c)}{x_2 + \alpha x_1}$), which can indirectly reduce total profits. Therefore, auctioneers must carefully balance the marginal returns of advertising and costs over time, as we discuss further in Section 5.1.

Case 3: Monopoly Auctioneer with Endogenous Retention

For Case 2, we assumed the retention rate α was exogenous. However, prior marketing research identifies poor price perceptions as a major reason that customers drop a service provider (Keaveney 1995). Thomas, Blattberg, and Fox (2004) explicitly model customer retention as a function of the firm's pricing strategy. We therefore consider a case in which customer retention depends on the price expectation of the auction; those with an expected positive surplus (according to the customer's own valuation and the auction's expected price) from the first auction return for a second period. In other words, customers with valuations equal to or greater than the bidding threshold are potential retained customers, and the remainder are lapsed customers. Effectively, the retention rate becomes $\alpha = (\frac{\bar{v} - s_1}{\bar{v}})$.

By including newly recruited bidders, the arrival rate for the second period becomes $\lambda_2 = x_2 + \frac{(\bar{v} - s_1)}{\bar{v}} x_1$.

Following Equation (6), the threshold for participation then simplifies to

$$s_2 = \bar{v} - \frac{\bar{v}\ln(\bar{v}/c)}{x_2 + \ln(\bar{v}/c)}. \tag{11}$$

We solve the auctioneer's decision problem with retention, as defined previously, and the optimal advertising levels then turns out to be constant and quite simple:

$$x_1^* = q \text{ and } x_2^* = q - \ln(\frac{\bar{v}}{c}). \tag{12}$$

The solutions are constant in both periods, because the retention rate $\alpha = (\frac{\bar{v} - s_1}{\bar{v}})$ is a constant. After simplification $\lambda_2 = x_2 + \frac{(\bar{v} - s_1)}{\bar{v}} x_1$ becomes $x_2 + \ln(\bar{v}/c)$, which does not depend on x_1, and the first-period optimal advertising level equals that of the monopoly case. In other words, if the auctioneer does not recognize endogenous retention when it exists, it may perform well in the first period, because the optimal advertising level is the same as that in the myopic case. However, the auctioneer then will overspend in the second period and earn suboptimal profits. The extent of overspending depends on the logarithm of the ratio of the upper bound of value over the bidding cost. When bidders' participation cost is small, and the upper bound of the valuation is high, overspending can be quite large.

Figure 1, Panel a, illustrates these findings in a monopoly setting. Comparing the advertising decisions in the exogenous and endogenous cases (i.e., first-period advertising is the same as in the myopic case), we easily confirm the importance of knowing the rate of returning customers and how they make that decision. These factors have direct implications for the auctioneer's customer acquisition strategies and profitability.

Case 4: Duopoly Competition with Exogenous Retention

Suppose there are two auctioneers in the market, competing for bidders in each period. For simplicity, we consider the case in which the auctioned goods are identical on both sites, and the two auctioneers are symmetric. In reality, customers may be aware of both auction sites, perhaps because they have been exposed to advertisements by both. However, they can bid at only one auction at each time, because they have unitary quantity demand.

To model this situation, we introduce a parameter γ that measures competition intensity[5] between the two firms, $\gamma = \frac{D}{x+y}$ and $\gamma \in [0,1)$, where D is the Poisson rate of all potential customers, and x and y denote the rates of aware customers on each of the auction sites. To simplify the analysis, we assume that γ is constant across periods and use a random splitting rule (Lippman and McCardle, 1997) to assign customers aware of the both firms. That is, each firm gets half the overlapping customer pool.

In this case, the number of bidders for either auction in the second period is

$$\lambda_2 = (1 - \frac{\gamma}{2})(\alpha x_1 + x_2) - \frac{\gamma}{2}(\alpha y_1 + y_2), \tag{13}$$

where x_2 and y_2 are the arrival rates of the two symmetric auctioneers in the second period. From (13), one can see that λ_2 consists of two parts: customers from the previous period and newly recruited customers. In each part, the rival auctioneer also contributes to the arrival rate, through overlapping, informed customers in both periods.

The optimal advertising levels appear in Table 1, Case 4, and are graphically illustrated in Figure 1, Panel b. Similar to the monopoly case with exogenous retention (Result 3 in the Appendix), it is easy to prove that in the first period, advertising could increase or decrease with retention, depending on the retention rate α. The case for non-monotonicity is parallel that for the monopoly case.

Case 5: Duopoly Competition with Endogenous Retention

When retention is endogenous in symmetric duopoly competition, the arrival rate, which consists of both returning customers and new advertising efforts in the second period, becomes

---

5 When $\gamma = 1$, the model reduces to a case in which two auctioneers compete for the same customers.

$$\lambda_2 = (1-\frac{\gamma}{2})\frac{\bar{v}-s_1}{\bar{v}}x_1 - \frac{\gamma}{2}\frac{\bar{v}-s_1}{\bar{v}}y_1 + (1-\frac{\gamma}{2})x_2 - \frac{\gamma}{2}y_2, \tag{14}$$

and the symmetric equilibrium for the first and the second period are

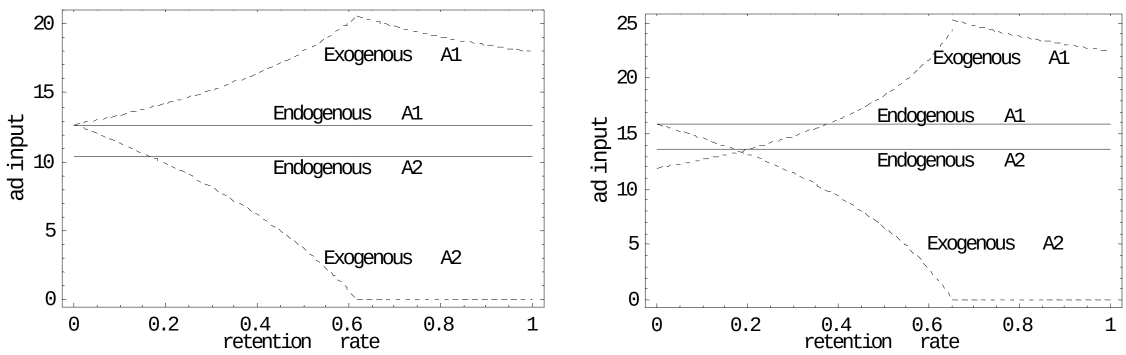
$$x_1^* = y_1^* = \frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)}q \text{ and } x_2^* = y_2^* = \frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)}q - \ln\frac{\bar{v}}{c}. \tag{15}$$

As in the exogenous case, increased spending by auctioneers increases competitive intensity, which leads to this statement:

Result 4. *For the duopoly competition cases the total advertising spending increases with competition intensity γ, and an increasing γ reduces profits. The effects of retention on advertising are similar to those in the monopoly case (see Result 3).*

Comparing the exogenous and endogenous retention models in the duopoly setting, we again note that it is vital for the auctioneer to know how customers decide to come back and thus how the customer bases in different periods are linked. As Figure 1, Panel b, shows, the two models have very different implications for advertising policy; if they get confused, the auctioneer will over- or underinvest in advertising in both periods and suffer reduced profit.

If we compare Figure 1, Panels a and b, with all else being equal, the optimal advertising in the duopoly case is higher, which is due to the loss of efficiency from competition. As long as the two auctioneers compete for the same customers, $\gamma > 0$, the industry's total advertising spending is strictly greater than that of the monopoly case. This result holds for both periods. An extreme case occurs when γ approaches 1, which drives the auctioneer's advertising spending to infinity. In general, expected profit decreases with competitive intensity, because as competition becomes more intense, the firm's spending on advertising grows less efficient. The duopoly framework and results reduce to the monopoly case when $\gamma = 0$.



(a) Monopoly (b) Duopoly

Figure 1: Advertising Strategies when Retention Is Endogenous and Exogenous ($k = 0.005$, $\bar{v} = 1$, $c = 0.1$, $\gamma = 0.25$).

In addition, we have the following comparative statics results:

Result 5. *When customers' valuations are i.i.d. uniformly distributed on [0, $\bar{v}$], for all cases considered, the total expected profit increases with retention α and the upper bound of the distribution $\bar{v}$; it decreases with bidding cost c and advertising cost k. The optimal advertising level also increases with $\bar{v}$ and decreases in c and k.*

Therefore, retention has a positive effect on profitability overall. Nevertheless, as our analyses show, the effect of retention on advertising spending depends on the magnitude of and heuristic for retention; it is not deterministic, even in a single period (see Result 3 in the Appendix). Customer participation costs lower the auctioneer's profit, because as a sunk cost, it lowers the customer's expected utility from bidding, and because the bidding costs increase the bidding threshold. All else being equal, the auctioneer must spend more to recruit enough customers who are eligible to bid.

5. Numerical Studies

5.1. Effect of Customer Acquisition on Profitability

Our analysis shows that increasing customer acquisition spending has several conflicting effects: it increases (1) the auction's revenue and (2) total advertising costs which reduces profit, and (3) it increases the bidding threshold (*s*) which indirectly decrease revenue. These effects jointly determine the expected profit and the optimal acquisition spending. The magnitude of the net effect depends on the model parameters and distribution assumptions.[6]

In Table 2 we illustrate the relative magnitude of three effects: 1) assuming that customer valuations follow a uniform distribution [0, 1], 2) the cost of bidding (*c*) is 0.1, and 3) the cost of acquiring one customer (*k*) is 0.001. Increasing the number of customers increases the auction's expected revenue, though at a decreasing rate. It also increases the bidding threshold. The rate at which the number of customers increases the bidding threshold exceeds the rate at which its increases the revenue. When customers engage in endogenous participation, a larger customer base drives up the bidding threshold and exerts a negative impact on the expected revenue (*s*-effect). The auctioneer also incurs customer acquisition costs, which diminishes revenue even more (*k*-effect). The *s*-effect is more substantial than the *k*-effect, and when c>>k, the former is always the main driver that decreases profits.

Similar to Reinartz, Thomas, and Kumar (2005), we find that misallocation of acquisition spending is asymmetric: Underinvesting is worse than overinvesting, and the expected profit function has a relatively flat maximum. For the uniform [0, 1] distribution, the profit decreases by 0.57% and 5.55% for underinvestments (i.e., deviations from the optimal customer arrivals) by 10 and 20 customers, and by 0.538% and 1.5% for similar overinvestments. Disregarding the effect of endogenous participation, the auctioneer overestimates its expected profit by 48.55% and overspends on acquisition by 67%.

With Table 2, an auctioneer could calculate the marginal effect of customers in auctions and use it as a metric for customer value. In a traditional retail setting, customer value or profitability typically is

[6] The main results do not change qualitatively when we use different valuation assumptions. These results are available on request.

defined according to the purchases made (e.g., profit margins of past purchases). However, for auctions, only the winner makes a purchase, and the losing customers, though they do not purchase, still have value because they help set the price. At the aggregate level, the auctioneer can calculate, using a value distribution, how much more revenue or profit it could gain by recruiting more customers. If the auctioneer also has customer-specific bidding data, it can arrive at individual-specific customer profitability estimates for losing customers. A good customer profitability measure is thus a foundation for optimizing customer relationships.

Arrivals	Bidding Threshold (s)	ER	EP	ER'	EP'	*s*-Effect	*k*-Effect
4	0.4244	0.4674	0.4634	0.5275	0.5235	-0.0601	-0.0040
6	0.6162	0.5349	0.5289	0.6700	0.6640	-0.1351	-0.0060
8	0.7122	0.5686	0.5606	0.7504	0.7424	-0.1818	-0.0080
10	0.7697	0.5888	0.5788	0.8001	0.7901	-0.2113	-0.0100
20	0.8849	0.6293	0.6093	0.9000	0.8800	-0.2707	-0.0200
30	0.9232	0.6428	0.6128	0.9334	0.9034	-0.2906	-0.0300
40	0.9424	0.6495	0.6095	0.9501	0.9101	-0.3006	-0.0400
50	0.9539	0.6536	0.6036	0.9603	0.9103	-0.3067	-0.0500
60	0.9616	0.6563	0.5963	0.9668	0.9068	-0.3105	-0.0600
70	0.9671	0.6582	0.5882	0.9716	0.9016	-0.3134	-0.0700
80	0.9712	0.6596	0.5796	0.9752	0.8952	-0.3156	-0.0800

Table 2: Effect of Customer Acquisition on Expected Auction Profit for Various Distributions
Notes: ER = expected revenue; EP = expected profit. ER' and EP' = expected revenue and profit, respectively, without acknowledging endogenous participation (or bidding threshold effect). The *s*-effect is the revenue reduction due to an increasing bidding threshold, measured by (ER – ER'). The *k*-effect is the revenue reduction due to the total cost of advertising, measured by (EP' – ER'), which is the same as (EP – ER).

5.2 Valuation Distributions

Our closed-form results require the simplicity of uniformly distributed customer valuations. In this section, we consider other types of distribution, such as Pareto and Weibull, to understand the sensitivity to this assumption. We first consider a Pareto distribution, which has a cumulative density function (CDF) of $F(v) = 1 - \theta / v^{\theta}, (v > 1,\ \theta > 0)$. This distribution can capture a potential value distribution commonly observed in practice: Most customers are bargain hunters, and much of the density lies in the lower quantiles. However, some bidders have very high values, as represented by the long tail of the distribution. As the shape parameter (θ) increases, the variance decreases, and the tail becomes thinner (Figure 2, Panel a).

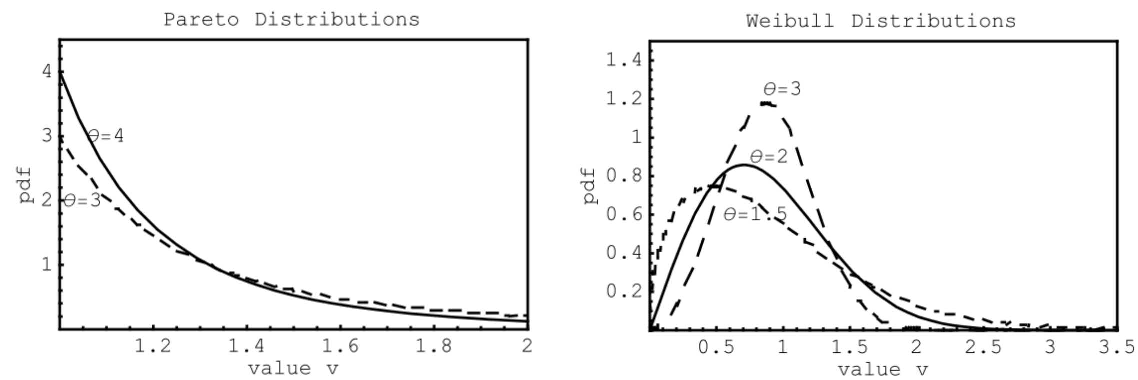


Figure 2: (a) Pareto Distributions (θ = 3 and 4), and (b) Weibull Distributions (θ = 1.5, 2, and 3)

In our numerical analysis, we solve first for the participation threshold with Poisson arrivals, as modeled in the uniform distribution case. The complexity of the probability distribution function requires that we solve for the threshold numerically. After obtaining the threshold solutions, we numerically integrate the expected profit function over its defined range, and the number of bidders follows a Poisson distribution. In the Pareto distribution case, this integration range is from the threshold (greater than 1) to infinity.

We illustrate our findings in Figure 3. First, similar to the result for the uniform distribution, the expected profit based on the Pareto distribution (in a single period) is uni-modal and defines a unique optimal advertising spending strategy. Second, when customers' participation is endogenous, the expected profit is lower than that of a benchmark model, where the bidding threshold is considered to be zero. This is due to the endogenously determined bidding threshold (i.e., competition effect among customers). When the threshold is strictly positive, some profit gets lost because customers perceive competition in the auction. If the firm ignores the endogeneity of participation, it spends too much on advertising, resulting in less profit (i.e., optimal ad spending under EP' is higher than that under EP). Third, the bidding threshold is not decreasing with advertising input, which conforms with the results we found for the uniform distribution. If we compare the two cases with different shape parameters, we find that both the bidding threshold and the expected profit are lower when $\theta = 4$ rather than 3. As θ grows larger, more mass moves over to lower values, which reduces expected profits. The tail of the distribution also is thinner, and thus, the threshold tapers off more quickly.

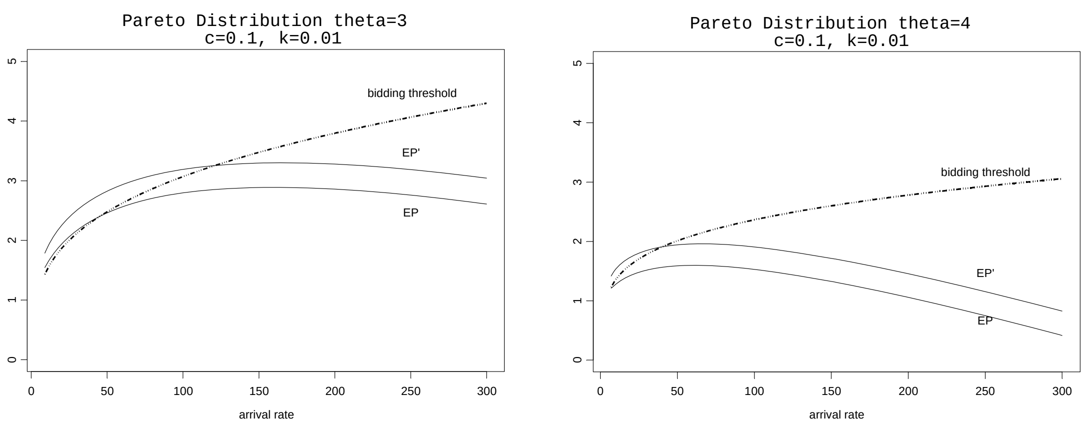


Figure 3: Comparison of Expected Profit with Endogenous Participation (EP) and a Benchmark Model (EP')

Next, we consider bell-shaped Weibull distributions. The CDF of the Weibull distribution with a scale parameter θ is $F(v) = 1 - e^{-v^{\theta}}$, where $\theta > 0, v > 0$. The mean and variance are:

$$E(v) = \Gamma(1 + 1/\theta)$$
$$Var(v) = \Gamma(1 + 2/\theta) - \Gamma(1 + 1/\theta)^2.$$

The Weibull distribution has flexible shapes; if we vary its shape parameter θ as in Figure 2, Panel b, from 1.5 to 3, the mode of the distribution gradually shifts from the left to the right. Thus the Weibull distribution results are similar to those of the Pareto distributions (Figure 4). As the shape parameter increases, expected profit decreases, as does the participation threshold and expected revenues. The optimal advertising input level decreases as well. These results are largely due to the tail behavior of the Weibull distribution. As θ increases, the tail becomes thinner, such that it is less likely to attain a large value by spending more on advertising.

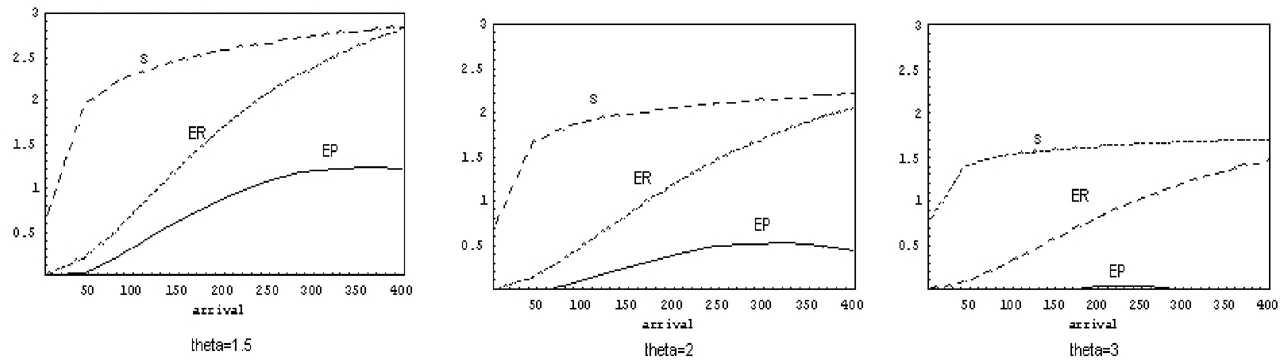


Figure 4: Expected Profit (EP), Expected Revenue (ER), and Bidding Threshold (s) for Weibull Distributions (c = 0.1, k =0.04, $\bar{v} = \infty, \gamma = 0$)

In summary, our numerical study suggests that the shape of the value distributions is an important consideration when a firm makes advertising decisions. However, regardless of the shape parameter, our basic results from the uniform distribution still hold: The expected profit functions are concave and have a unique maximum in advertising spending. Although we only considered a single period in this numerical study, the result should hold when the simulation is extended to the second period and retention links the two periods.

5.3. Effects of Upper Bound of the Value Distributions

In the simulation study, we took the whole distribution range, such that for both Pareto and Weibull distributions, customers' value could go to positive infinity. However, customers' values for an auctioned product often are bounded by some price level, such as the price for a similar product offered by a retail store or a fixed price posted by the auction. To incorporate this case, we study truncated Pareto and Weibull distributions with an arbitrary upper bound. When the upper bound is finite, even the expected revenue has a unique mode. If customers' valuations are not distributed uniformly but rather follow a distribution with a tail, there are fewer customers with large valuations (though they exist, and possibly with extremely high values). In other words, for the same amount of advertising input, the chance of recruiting a high valuation customer is smaller, compared with the uniform case. This effect is exacerbated by the fixed upper bound, because expected revenue can start to decline. Even if the cost of advertising is zero, it is not optimal to advertise as much as possible. As the upper bound of the value distribution decreases, the outside option becomes more attractive, and the optimal level of advertising should be reduced accordingly.

Upper Bound $\bar{v}$	Truncation Degree $1 - F(\bar{v})$	Participation Threshold	Optimal Arrival Rate	Optimal Revenue
10	0.0010	5.1835	641	5.8380
8	0.0020	4.2229	331	4.6254
6	0.0046	3.2413	141	3.4184
4	0.0156	2.2295	42	2.2201
3.5	0.0014	2.6743	260	2.3574
3	0.0055	2.1831	88	1.9747
2.5	0.0192	1.6901	31	1.5220
2	0.0591	1.1920	12	1.0707

Table 3: Optimal Expected Revenue at Various Right Truncations (top: Pareto $\theta = 3$, bottom: Weibull $\theta = 1.5$; c = 0.1).

In Table 3 and Figure 5, we demonstrate these findings for a Pareto ($\theta = 3$) and Weibull ($\theta = 1.5$) distribution. In Figure 5 the curvature of the expected revenue function, similar to the uniform [0,1] distribution in Section 5.1, reveals that underspending is worse than overspending. The curve is steeper on the left side of the optimal value than on the right. However, the expected profit (or revenue) function does not always have a flat maximum; rather, financial losses can be quite substantial even with just a small deviation from the optimal advertising level when outside offers are more competitive. This finding conflicts with non-auction empirical findings, such as Reinartz, Thomas, and Kumar's (2005), which again highlights the importance of understanding the customer valuation distribution and perceptions about competition in an auction setting.

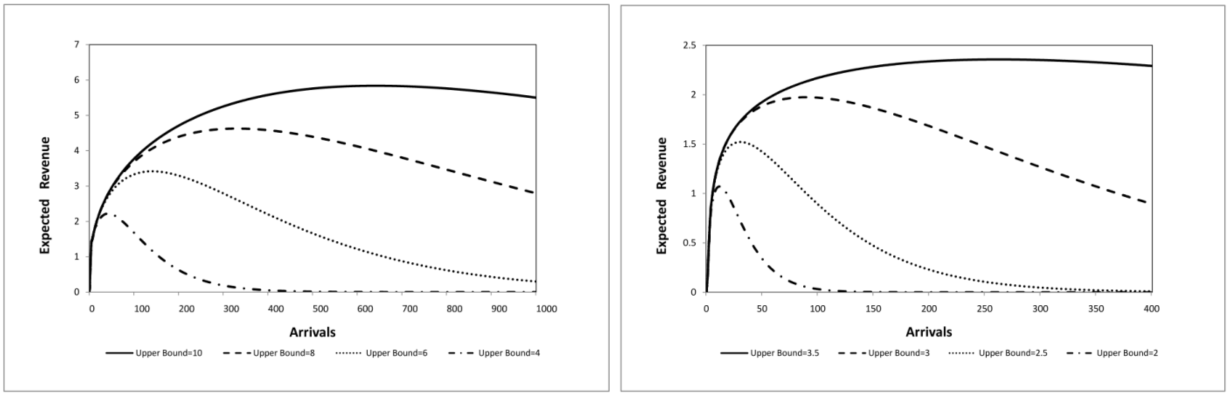


Figure 5: Expected Revenue at Various Right Truncations (left: Pareto $\theta = 3$, right: Weibull $\theta = 1.5$; c = 0.1).

6. Discussion and Conclusions

Auctioneers rely on a large customer bases and healthy customer relationships to generate their profits. However, the novelty of online auctions has started to wane, which makes it more difficult to attract customers (Holahan 2008). Consumers want not only a good value but also convenience which we assert influences their participation costs, such as the time and effort spent to win the auction. The participation cost incurred in the bidding process affects their subsequent retention decision. For auction businesses, this shift creates a new challenge to actively manage customer satisfaction within an auction setting.

An important distinction between auctions and traditional fixed-price retail environments is that in the former, customers compete against one another to obtain the product. With the increasing availability of price information and fixed-price offers, this feature of auctions is likely to become more salient to both bidders and auctioneers, especially in e-commerce marketplaces. Solutions to such a challenge include improving the customer shopping experience and overall satisfaction with the product and service offered. Moreover, auction sites should be mindful of the need to balance the growing number of customers with an increased quantity and variety of products sold. To optimize customer relationships and its customer base, the business also must understand its competition from the fixed-price world, customers' valuation distributions for the product sold, and how customers make participation and return decisions. Market research, surveys, and bidding data can help answer these questions.

Taking into account customers' participation decisions, we model the interactions between customer acquisition and retention, as well as their effects on auction profit. Our research thus makes several major contributions to auction and marketing literature. First, acquisition spending is a double-edged sword: It increases auction revenue, but it also increases customers' perceptions of competition and thus reduces auction revenue. For some distributions, the second effect is so acute that even with no advertising costs, the auctioneer should still limit the size of its customer base.

Second, customer acquisition spending has an influence through the rate of retention. Even though retention has a positive overall effect on optimal profits, its effect on acquisition is not continuous or monotonic. Rather, it depends on how customers make return decisions (exogenous or endogenous) and the overall rate of retention. Considering the interaction between advertising and retention can help auctioneers reduce unnecessary operating expenses, better manage customer perceptions of their chance of winning, and thus improve their own profits.

Third, similar to a fixed-price setting, we find that the misallocation of acquisition spending is asymmetric; underspending is worse than overspending. However, the profit function is not necessarily flat near the maximum. The profit losses from deviating from optimal spending levels can be substantial, but they depend on the underlying customer valuation distribution.

Fourth, in auctions, only the winner pays for the product, even though everyone expends some effort to participate. If a customer value assessment is based solely on a traditional transaction-based metric, only the winner has positive value. Yet the auction price, and thus revenue, is determined collectively by all participants. By analyzing the customer acquisition problem, we provide a framework for gauging the value of losing customers through the marginal revenue they generate. Auctioneers should take this chance to move beyond a transaction-based customer value metric, though more research is needed to extend our understanding of customer value (Gupta et al. 2006), especially for e-commerce that entails non–transaction-based information and activities (e.g., user-generated content, social media) that still constitute valuable assets for businesses. Moreover, with the increased integration of social networking and e-commerce, businesses should recognize and take into account the social effects in recruiting and retaining customers.

This research also suffers some limitations. To keep the analysis transparent and tractable, we assumed away some interesting aspects of online auctions. For example, we only consider merchant auctioneers with fixed supplies of auctioned goods. Although this scenario is likely accurate for many merchant sites that own the products they sell or those that sell surplus merchandise of limited quantity, it is also possible that auctioneers adapt the assortment and variety of their merchandise to their user base. We did not consider the listing auction sites such as eBay, and ignored other elements of the online auction design variables that might attract and retain customers too, such as bidding duration, quantity of auctioned goods, and the seller's reservation prices. Prior research has indicated that all these elements affect final winning prices and participation behavior (e.g., Engelbrecht-Wiggans 1987); they likely affect customers' perceptions of the value of bidding as well. Finally, our model could be enriched in several ways, such as by incorporating heterogeneous bidding costs across customers, the auctioneer's targetability development over time, and the influence of communication among customers. We hope our efforts encourage others to consider the potential contribution of marketing variables to defining successful designs for auctions, as well as to calculating proper measurements of customer equity in auctions.

References

Bajari, Patrick and A. Hortacsu (2002), "Cyberspace Auctions and Pricing Issues: A Review of Empirical Findings," in Derek Jones, ed., the *New Economy Handbook*, Elsevier.

Bolton, Ruth (1998) "A Dynamic Model of the Duration of the Customer's Relationship with a Continuous Service Provider: The Role of Satisfaction," *Marketing Science*, 17 (1), 45-65.

Bolton, Ruth and Katherine Lemon, (1999), "A Dynamic Model of Customer Usage of Services: Usage as an Antecedent and Consequence of Satisfaction," *Journal of Marketing Research*, 36 (May), 171-186.

Bradlow, Eric T. and Young-Hoon Park (2007), "Bayesian Estimation of Bid Sequences in Internet Auctions Using a Generalized Record Breaking Model," *Marketing Science*, 26 (2), 218-229.

Chakravarti, Dipankar, E. Greenleaf, A. Sinha, A. Cheema, J. Cox, D. Friedman, T-H, Ho, R.M. Isaac, A. Mitchell, A. Rapoport, M. Rothkopf, J. Srivastava, R. Zwick (2002) "Auctions: Research Opportunities In Marketing," *Marketing Letters*, 13 (3) 281-296.

Chan, Tat Y., Vrinda Kadiyali, and Young-Hoon Park (2007), "Willingness to Pay and Competition in Online Auctions," *Journal of Marketing Research*, 44 (2), 324-333.

Dholakia Utpal M. and Itamar Simonson (2005), "The Effect of Explicit Reference Points on Consumer Choice and Online Bidding Behavior," *Marketing Science*, 24 (2), 206-217.

Engelbrecht-Wiggans, Richard (1987), "On Optimal Reservation Prices in Auctions," *Management Science*, 33 (6), 763-770.

Engelbrecht-Wiggans, Richard (1994), "Sequential Auctions of Stochastically Equivalent Objects," *Economic Letters*, 44, 87-90.

Gupta, Sunil, Dominique Hanssens, Bruce Hardie, Wiliam Kahn, V. Kumar, Nathaniel Lin, Nalini Ravishanker, and S. Sriram (2006), "Modeling Customer Lifetime Value," *Journal of Service Research*, 9(2), 139-155.

Gupta, Sunil and Donald Lehmann (2005), *Managing Customers as Investments: The Strategic Value of Customers in the Long Run*. Upper Saddle River, NJ: Pearson Prentice Hall.

Haruvy, Ernan, and Peter T. L. Popkowski (2010), "Search and Choice in Online Consumer Auctions," *Marketing Science*, 29 (6), 1152-1164.

Holahan, Catherine (2008), "Auctions on eBay: A Dying Breed" *Bloomberg Business Week*, June 3, 2008. http://www.businessweek.com/technology/content/jun2008/tc2008062_112762.htm

Jank, Wolfgang and Galit Shmueli (2010), *Modeling Online Auctions (Statistics in Practice)*, 1st Edition, Hoboken, NY: Wiley.

Jeitschk, Thomas D. (1999), "Equilibrium Price Paths in Sequential Auctions with Stochastic Supply," *Economics Letters*, 64, 67-72.

Kamins, Michael, Xavier Drèze, and Valerie Folkes (2004) "Effects of Seller-supplied Prices on Buyers' Product Evaluations: Reference Prices in an Internet Auction Context," *Journal of Consumer Research*, 30 (4), 622-628.

Keaveney, Susan M. (1995), “Customer Switching Behavior in Service Industries: An Exploratory Study,” *Journal of Marketing*, 59 (2), 11-S2.

Klemperer, Paul (1999), “Auction Theory: A Guide to the Literature,” *Journal of Economic Surveys*, 13 (3), 227-286.

Levin, D. and J. Smith (1994) “Equilibrium in Auctions with Entry,” *American Economic Review*, 84, 585-99.

Lippman, Steven A. and K. F. McCardle (1997), “The Competitive Newsboy,” *Operations Research*, 45, (1), 54-65.

Lucking-Reiley, David (1999a), “Auctions on the Internet; What’s Being Auctioned and How?” *Journal of Industrial Economics*, XLVIII (3), 227-251.

Lucking-Reiley, David (1999b), “Experimental Evidence on the Endogenous Entry of Bidders in Internet Auctions.” Working paper, Vanderbilt University.

McAfee, R. Preston and J. McMillan (1987), “Auctions with a Stochastic Number of Bidders,” *Journal of Economic Theory.* 43, 1-19.

McAfee, R. and D. Vincent (1993), “The Price Decline Anomaly,” *Journal of Economic Theory*, 60, 191-212.

Menezes, Flavio and P. K. Monteiro (2000), “Auctions with Endogenous Participation,” *Review of Economic Design*, 5 (1), 71-89.

Moon, Youngme (1999), “Onsale, Inc.”, Harvard Business Publishing, Case 9-599-091.

Park, Young-Hoon and Eric T. Bradlow (2005), “An Integrated Model for Bidding Behavior in Internet Auctions: Whether, Who, When, and How Much,” *Journal of Marketing Research*, 42 (4), 470-482.

Pinker, E., A. Seidmann, and Y. Vakrat (2001), “The Design of Online Auctions: Business Issues and Current Research.” Working Paper CIS01-05, Simon School, University of Rochester.

Popkowski Leszczyc, Peter T.L. and Gerald Häubl (2010), “To Bundle or Not to Bundle: Determinants of the Profitability of Multi-Item Auctions,” *Journal of Marketing*, 74(4), 110-24.

Reinartz, Werner, Jacquelyn Thomas, and V Kumar (2005), “Balancing Acquisition and Retention Resources to Maximize Customer Profitability,” *Journal of Marketing*, 69 (January) 63-79.

Riley, John and W. Samuelson (1981), “Optimal Auctions.” *American Economic Review*, 71 (3), 381-392.

Samuelson, W. (1985) “Competitive Bidding with Entry Costs,” *Economics Letters*, 17, 53-57.

Sørensen, Søren Tang (2006), “Sequential Auctions for Stochastically Equivalent Complementary Objects,” Economics Letters, 91, 337-342.

Subramaniam, Ramanathan, and R. Venkatesh (2009), “Optimal Bundling Strategies in Multi-object Auctions of Complements or Substitutes,” *Marketing Science*, 28 (2), 264-273.

Thomas, Jacquelyn (2001), “A Methodology for Linking Customer Acquisition to Customer Retention,” *Journal of Marketing Research*, 38 (May), 262-268.

Thomas, Jacquelyn, Robert Blattberg, and Edward Fox (2004), "Recapturing Lost Customers," *Journal of Marketing Research*, 41 (February), 31-45.

Vakrat, Y. and A. Seidmann (2000), "Implications of the Bidders' Arrival Process on the Design of Online Auctions," In *Proceedings of the 33d Annual Hawaii International Conference on System Sciences* (HICSS-33), January, Maui, Hawaii.

Wang, Xin, Alan L. Montgomery and Kannan Srinivasan (2008) "When Auction Meets Fixed-Price: A Theoretical and Empirical Examination of Buy-it-Now Auctions," *Quantitative Marketing and Economics* 6 (4), 339-370.

Weber, R. J. (1983), "Multi-object Auction," in: *Auctions, Bidding and Contracting: Uses and Theory*, eds. R. Engelbrecht-Wiggans, M. Shubik, and R. M. Strak. New York: New York University Press, 165-194.

Yao, Song and Carl F. Mela (2008), "Online Auction Demand," *Marketing Science*, 27(5), 861-885.

Zeithammer, R. (2002) "Sequential Auctions for Substitutes: A Theory of Bargain-Hunting on eBay." Working paper, MIT.

Appendix

1. Notation used in the paper

v_i : customer *i*'s value for the auctioned product.

$\bar{v}$: upper bound of the value distributions.

x_t : arrival rate of customers generated by the advertising in the *t-th* period of firm 1.

y_t : arrival rate of customers generated by the advertising in the *t-th* period of firm 2.

λ_t : arrival rate of customers of the *t-th* period.

s_t : bidding threshold of the *t-th* period.

α : retention parameter $\alpha \in [0, 1]$.

A: advertising spending.

k: unit cost of advertising.

c: cost of bidding.

γ : competition intensity $\gamma \in [0, 1)$.

q: optimal arrival rate for the myopic model.

F: CDF of the customer's values for the product.

θ : shape parameters for Pareto and Weibull distributions in simulation study.

2. Proofs

No winning customers are required to exit the auction, current customers can drop out of the auction freely, and the auction firm has the capability to bring new customers to the second auction. In addition, bidders' values for the two auctions are independent (i.e. bidders values are redrawn from the valuation distribution in the second auction). As a result, the two auctions are effectively separated, and customers do not shade their bids in the first period because of the possible gain in the second period, or bid more aggressively in the first period for fear of risky future prices. Following Sørensen (2006) one can show that in such sequential auctions, the optimal (weakly dominant symmetric) bidding strategy is to bid one's valuation.

Previous literature has proved that in a second-price, sealed-bid auction with endogenous participation, the optimal bidding strategy for bidders is to bid their true value if the value is greater than the threshold (Menezes and Monteiro 2000; Wang, Montgomery and Srinivasan 2008). Our proof is similar, except that the number of bidders follows Poisson distributions with different rates in the two auctions. This proof thus depends on the isolation of the two auctions when the seller can replenish demand for the good.

For the following proofs, we assume $\bar{v} > c >> k > 0$. First, we show that $q > 0$,

where $q \equiv \sqrt{\dfrac{2\bar{v} - 2c - c\ln(\bar{v}/c)(2 + \ln(\bar{v}/c))}{k}}$.

Proof: Note that the numerator inside the square root is decreasing in c:

$\dfrac{d}{dc} 2\bar{v} - 2c - c\ln(\bar{v}/c)(2 + \ln(\bar{v}/c)) < 0$. Therefore, when c goes to $\bar{v}$, this part reaches its minimum,

which is 0. Because k is also positive, $q > 0$.

Result 1: *The total expected profit function is concave and has a unique optimum. Furthermore, in each period, the expected profit function for the auctioneer is concave and has a unique optimal advertising level.*

Proof: From Equation (2) and the uniform distribution assumption, the auctioneer's second-period expected profit simplifies to

$$E\Pi_2 = \lambda_2{}^2 \int_{s_2}^{\bar{v}} \frac{\exp((\bar{v}/v-1)\lambda_2)(\bar{v}-v)v}{\bar{v}^2} dv - kx_2 , \tag{A1}$$

with $\lambda_2 = x_2$, so

$$s_2 = \bar{v} - \frac{\bar{v}\ln(\bar{v}/c)}{x_2} . \tag{A2}$$

After simplifying, the second-period expected profit becomes

$$E\Pi_2 = 2\left(\frac{(\bar{v}-c)(x_2-2) - kx_2{}^2 + c\ln(\bar{v}/c)(2-x_2+\ln(\bar{v}/c))}{x_2}\right). \tag{A3}$$

The second-order derivative with respect to x_2 is

$$\frac{\partial^2 E\Pi_2}{\partial x_2{}^2} = \frac{4(c-\bar{v}) + 2c\ln(\bar{v}/c)(2+\ln(\bar{v}/c))}{x_2^3} \le 0 , \tag{A4}$$

so $E\Pi_2$ is concave. The proof for the concavity and the uniqueness of the solution for the first period and other models is similar and is omitted here. By solving the first-order condition,

$$\frac{\partial E\Pi_2}{\partial x_2} = -\frac{2c - 2\bar{v} + kx_2^2 + c\ln(\bar{v}/c)(2+\ln(\bar{v}/c))}{x^2} = 0 , \tag{A5}$$

we obtain

$$x_1^* = x_2^* = \sqrt{\frac{2\bar{v} - 2c - c\ln(\bar{v}/c)(2+\ln(\bar{v}/c))}{k}} . \tag{A6}$$

Result 2. *Accounting for customer retention when it is positive yields higher seller's profit than not accounting for it. If it disregards positive customer retention, an auctioneer will underspend on advertising in the first period and overspend in the second, which leads to suboptimal profits.*

Proof: Let Π^{case1} and Π^{case2} denote the total expected profit for the auctioneer in Case 1 (monopoly, myopic) and Case 2 (monopoly, exogenous retention), respectively. Then,

$$\Pi^{case2} - \Pi^{case1} = \begin{cases} 2kq - 2kq\sqrt{1-\alpha} > 0 & when\ \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ -2kq\sqrt{1+\frac{1}{\alpha}} + 4kq > 0 & when\ \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \tag{A7}$$

The total advertising for Case 2 is $kq(1+\sqrt{1-\alpha})$ when $\alpha \in [0, \frac{\sqrt{5}-1}{2})$; it is $kq\sqrt{1+\frac{1}{a}}$ when $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$. The total spending in both cases is less than that of the myopic case, $2kq$.

Moreover, the myopic auctioneer spends less than is optimal in the first period and more than is optimal in the second period. Let A_t^{case1} and A_t^{case2} denote advertising spending at time *t* for Cases 1 and 2, respectively.

$$A_2^{case2} - A_2^{case1} = \begin{cases} -kq\alpha\sqrt{\frac{1}{1-\alpha}} < 0 & when\ \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ -kq < 0 & when\ \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \tag{A8}$$

$$A_1^{case2} - A_1^{case1} = \begin{cases} kq\left(\sqrt{\frac{1}{1-\alpha}} - 1\right) > 0 & when\ \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ kq\left(\sqrt{1+\frac{1}{\alpha}} - 1\right) > 0 & when\ \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \tag{A9}$$

Result 3: *When retention is exogenous, the monopoly auctioneer (a) in the first period should increase the advertising level as α increases when* $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ *and reduce advertising as α increases when* $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$. *(b) In the second period, the auctioneer should decrease advertising when* $\alpha \in [0, \frac{\sqrt{5}-1}{2})$ *and stop advertising when* $\alpha \in [\frac{\sqrt{5}-1}{2}, 1]$.

Proof:

$$\frac{\partial A_2^{case2}}{\partial \alpha} = -\sqrt{\frac{1}{1-\alpha}} - \frac{1}{2}(\frac{1}{1-\alpha})^{3/2} < 0 \quad when\ \alpha \in [0, \frac{\sqrt{5}-1}{2})$$

$$\frac{\partial A_1^{case2}}{\partial \alpha} = \begin{cases} \frac{1}{2}(\frac{1}{1-\alpha})^{3/2} > 0 & when\ \alpha \in [0, \frac{\sqrt{5}-1}{2}) \\ -\frac{1}{2\alpha^2\sqrt{1+\frac{1}{\alpha}}} < 0 & when\ \alpha \in [\frac{\sqrt{5}-1}{2}, 1] \end{cases} \tag{A10}$$

Result 4: *For the duopoly competition cases (a) total advertising spending increases in competition intensity* γ *and (b) increasing* γ *reduces profit. (c) The effects of retention on advertising are similar to those in the monopoly case (see Result 3).*

Proof: For the duopoly with exogenous retention (Case 4), Part (a).

we first must show that when $\alpha \in [0, M)$, γ increases with total advertising spending:

$$\frac{dA^{case4}}{d\gamma} = \frac{3-\gamma}{2\sqrt{4-2\gamma}(1-\gamma)^2} + (1-\alpha)\frac{(2-\gamma)^2 - 2(1-\gamma)^2}{2(2-\alpha(2-\gamma)-2\gamma)^2\sqrt{\frac{(1-\gamma)(2-\gamma)}{2-\alpha(2-\gamma)-2\gamma}}} \equiv h_1(\alpha,\gamma) \tag{A11}$$

Notice that $h_1(\gamma,\alpha)$ is a decreasing function of α and reaches its minimum when $\alpha = M$. When $\alpha \in [M, 1]$,

$$\frac{dA^{case\,4}_{\alpha\in[M,1]}}{d\gamma} = \frac{1}{\sqrt{2}(1-\gamma)}\sqrt{\frac{2(1+\alpha)(2-\gamma)}{\alpha}} > 0\,. \tag{A12}$$

Part (b).

$$\frac{dE\Pi^{case4}{}_{\alpha\in[0,M)}}{d\gamma} =$$

$$\frac{-3\alpha(2-\gamma)^2+2(1-\gamma)(5-3\gamma)}{2\sqrt{\frac{2}{2-\gamma}-\frac{\alpha}{1-\gamma}}(2-\gamma)^2(-1+\gamma)3} + (\alpha-1)\frac{(2-\gamma)^2-2(1-\gamma)^2}{2(2-\alpha(2-\gamma)-2\gamma)^2\sqrt{\frac{(1-\gamma)(2-\gamma)}{2-\alpha(2-\gamma)-2\gamma}}} + \frac{8+3(-3+\gamma)\gamma}{2\sqrt{4-2\gamma}(-2+\gamma)(1-\gamma)^2}$$

$$\equiv h_2(\gamma,\alpha)$$

Note that $h_2(\gamma,\alpha)$ is an increasing function of α , so it reaches its maximum when $\alpha = M$,

$$\frac{dE\Pi^{case\,4}{}_{\alpha\in[M,1]}}{d\gamma} = kq\frac{\sqrt{\frac{1+\alpha}{4\alpha-2\alpha\gamma}}(8-9\gamma+3\gamma^2)}{2(\gamma-2)(1-\gamma)^2} < 0\,, \tag{A13}$$

because $8-9\gamma+3\gamma^2 > 6-9\gamma+3\gamma^2 = (\gamma-2)(3\gamma-3) > 0$.

Part (c).

$$\frac{\partial A_2^{case\,4}}{\partial\alpha} = -\frac{kq(2-\gamma)}{2\sqrt{2}\alpha^2\sqrt{\frac{(1+\alpha)(2-\gamma)}{\alpha}}(\gamma-1)} < 0,\ when\ \alpha\in[0,M)$$

$$\frac{\partial A_1^{case\,4}}{\partial\alpha} = \begin{cases} \dfrac{kq(2-\gamma)\sqrt{\dfrac{(\gamma-2)(\gamma-1)}{2+\alpha(\gamma-2)-2\gamma}}}{4+2\alpha(\gamma-2)-4\gamma} > 0 & when\ \alpha\in[0,M) \\[3ex] -\dfrac{kq(4+\alpha(\gamma-2)-4\gamma)\sqrt{\dfrac{(\gamma-2)(\gamma-1)}{2+\alpha(\gamma-2)-2\gamma}}}{4+2\alpha(\gamma-2)-4\gamma} < 0 & when\ \alpha\in[M,1] \end{cases}$$

For the duopoly with endogenous retention (Case 5), :

Part (a)

$$\frac{dA_1^{case\,5}}{d\gamma} = \frac{dA_2^{case\,5}}{d\gamma} = \frac{3-\gamma}{2\sqrt{4-2\gamma}(1-r)^2} > 0\text{, and} \tag{A14}$$

Part (b)

and $$\frac{dE\Pi^{case5}}{d\gamma} = 2kq\frac{\sqrt{4-2\gamma}(8-9\gamma+3\gamma^2)}{4(\gamma-2)(1-\gamma)^2} < 0 \quad \text{(A15)}$$

Result 5. *When customers' valuations are i.i.d. uniform distributed on [0, $\bar{v}$], for all cases considered, the total expected profit increases with retention α , and the upper bound of the distribution $\bar{v}$; it decreases with bidding cost c and the advertising cost k. The optimal advertising level increases with $\bar{v}$ and decreases with c and k.*

Proof: First, for the retention effect (α), in Case 2, it is straightforward that when α increases, the expected profit increases over the whole range of α . For Case 4, when $\alpha \in [0, M)$, the expected profit again increases with α . Furthermore,

$$\frac{\partial E\Pi^{case4}_{\alpha\in[M,1]}}{\partial\alpha} = \frac{kq(4-3\gamma)}{2\alpha(1+\alpha)(1-\gamma)}\sqrt{\frac{1+\alpha}{4\alpha-2\alpha\gamma}} > 0 \quad \text{(A16)}$$

Second, regarding the bidding effect (*c*), in general, the first-order derivatives of the expected profit functions of the five models are $\frac{\partial E\Pi^{case1}}{\partial c} = \ln\frac{v}{c}(-2+\frac{4}{2}\frac{\ln\frac{v}{c}}{q})$. Because $\ln\frac{v}{c} < q$ (from the preceding results), we know $\frac{\partial E\Pi^{case1}}{\partial c} < 0$, and

$$\begin{aligned}
\frac{\partial E\Pi^{case2}_{\alpha\in[0,(\sqrt{5}-1)/2)}}{\partial c} &= \ln\frac{v}{c}(-2+\frac{2(1+\sqrt{1-\alpha})}{2}\frac{\ln\frac{v}{c}}{q}) < 0 \\
\frac{\partial E\Pi^{case2}_{\alpha\in[(\sqrt{5}-1)/2,1]}}{\partial c} &= \ln\frac{v}{c}(-2+\frac{2(\sqrt{1+\frac{1}{\alpha}})}{2}\frac{\ln\frac{v}{c}}{q}) < 0
\end{aligned} \quad \text{(A17)}$$

$$\frac{\partial E\Pi^{case\,3}}{\partial c} = \ln\frac{v}{c}(-2 - \frac{k}{c} + \frac{4}{2}\frac{\ln\frac{v}{c}}{q}) < 0 \tag{A18}$$

Third, for the advertising cost effect (k), by examining the expected profit functions of each model, we note that there is a term $-gkq$ in each of the cases, where g denotes the positive coefficient of the $-kq$ term. We also know that in general, $\frac{d(-kq)}{dk} = -\frac{1}{2}q < 0$. The claim thus is easily proved for Cases 1, 2, and 4, such that

$$\frac{dE\Pi^{case3}}{dk} = \ln\frac{\bar{v}}{c} - 2q < 0\text{, and} \tag{A19}$$

$$\frac{dE\Pi^{case\,5}}{dk} = \ln\frac{\bar{v}}{c} - \frac{4-3\gamma}{(1-\gamma)\sqrt{4-2\gamma}}q\,. \tag{A20}$$

Because $$\frac{4-3\gamma}{(1-\gamma)\sqrt{4-2\gamma}} > \frac{4-4\gamma}{(1-\gamma)\sqrt{4-2\gamma}} = \frac{4}{\sqrt{4-2\gamma}} > 1\,,\ \frac{dE\Pi^{case\,5}}{dk} < 0. \tag{A21}$$

Fourth, the upper bound effect $(\bar{v})$ requires again that g denotes the positive coefficient of the $-kq$ term. In general, $\frac{dE\Pi}{d\bar{v}} = 2 - \frac{2c}{\bar{v}} + \frac{g(c + c\ln(\bar{v}/c) - \bar{v})}{\bar{v}q} = h_3(\bar{v}, c)$. This function is a decreasing function of c. The minimum is obtained when $c = \bar{v}$, and $h_3(\bar{v}, c) = 0$. Therefore, the upper bound parameter has a positive effect on expected profit.

Model	Optimal Ad Spending in the First Period A_1*	Optimal Ad Spending in the Second Period A_2*	Total Optimal Expected Profit $E\Pi^*$
Case 1. Myopic	kq	kq	$2\bar{v}-2c-2c\ln(\frac{\bar{v}}{c})-4kq$
Case 2. Monopoly, Exogenous Retention	$kq\sqrt{\frac{1}{1-\alpha}}$ $if\ \alpha\in[0,\frac{\sqrt{5}-1}{2})$ $kq\sqrt{1+\frac{1}{\alpha}}$ $if\ \alpha\in[\frac{\sqrt{5}-1}{2},1)$	$kq\left(1-\alpha\sqrt{\frac{1}{1-\alpha}}\right)$ $if\ \alpha\in[0,\frac{\sqrt{5}-1}{2})$ 0 $if\ \alpha\in[\frac{\sqrt{5}-1}{2},1)$	$2\bar{v}-2c-2c\ln\frac{\bar{v}}{c}-2kq(1+\sqrt{1-\alpha})$ $2\bar{v}-2c-2c\ln\frac{\bar{v}}{c}-2kq\left(\sqrt{1+\frac{1}{\alpha}}\right)$
Case 3: Monopoly, Endogenous Retention	kq	$k(q-\ln\frac{\bar{v}}{c})$	$2\bar{v}-2c-2c\ln\frac{\bar{v}}{c}+k\ln\frac{\bar{v}}{c}-4kq$
Case 4. Competition, Exogenous Retention	$kq\sqrt{\frac{(1-\gamma)(2-\gamma)}{2+\alpha(\gamma-2)-2\gamma}}$ $if\ \alpha\in[0,M)$ $\frac{kq}{2(1-\gamma)}\sqrt{\frac{2(1+\alpha)(2-\gamma)}{\alpha}}$	$kq\left(\frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)}-\alpha\sqrt{\frac{(1-\gamma)(2-\gamma)}{2+\alpha(\gamma-2)-2\gamma}}\right)$ $if\ \alpha\in[0,M)$ 0 otherwise	$2\bar{v}-2c-2c\ln\frac{\bar{v}}{c}-kq\left(\sqrt{\frac{2+\alpha(\gamma-2)-2\gamma}{(1-\gamma)^3(2-\gamma)}}+(1-\alpha)\sqrt{\frac{(1-\gamma)(2-\gamma)}{2+\alpha(\gamma-2)-2r}}+\frac{4-3\gamma}{(1-\gamma)\sqrt{4-2\gamma}}\right)$ $2\bar{v}-2c-2c\ln\frac{\bar{v}}{c}-\frac{kq(4-3\gamma)}{1-\gamma}\sqrt{\frac{1+\alpha}{4\alpha-2\alpha\gamma}}$
Case 5: Competition, Endogenous Retention	$\frac{kq\sqrt{2(2-\gamma)}}{2(1-\gamma)}$	$k(\frac{\sqrt{2(2-\gamma)}}{2(1-\gamma)}q-\ln\frac{\bar{v}}{c})$	$2\bar{v}-2c-2c\ln\frac{\bar{v}}{c}+k\ln\frac{\bar{v}}{c}-\frac{2kq(4-3\gamma)}{(1-\gamma)\sqrt{4-2\gamma}}$

Notes: $q\equiv\sqrt{\frac{2\bar{v}-2c-c\ln(\bar{v}/c)(2+\ln(\bar{v}/c))}{k}}$, and $M=\frac{\gamma-2+\sqrt{20+\gamma(-68+\gamma(97+16(-4+\gamma)\gamma))}}{4(1-\gamma)^3}$.

Table 1: Summary of Results