Adverse Selection and Non-Exclusive Contracts[*]

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Abstract

This paper studies the Rothschild and Stiglitz (1976) adverse selection environment, relaxing the assumption of exclusivity of insurance contracts. There are three types of agents that differ in their risk level, their riskiness is private information and known before any contract is signed. Agents can engage in multiple insurance contracts simultaneously, and the terms of these contracts are not observed by other firms. Insurance providers behave non-cooperatively and compete offering menus of insurance contracts from an unrestricted contract space. We derive conditions under which a separating equilibrium exists and fully characterize it. The unique equilibrium allocation consists of agents with a lower probability of accident purchasing no insurance and agents with higher accident probability buying the actuarially-fair level of insurance. The equilibrium allocation also constitutes a linear price schedule for insurance. To sustain the equilibrium allocation, firms must offer latent contracts. These contracts are necessary to prevent deviations by other firms; in particular they can prevent cream-skimming strategies. As in Rothschild and Stiglitz (1976), pooling equilibrium still fails to exists.

1 Introduction

In this paper we address the question of what type of insurance contracts emerge when insurance providers compete among themselves. We are interested in environments where the insured have private information on their risk probability and can sign without being observed multiple insurance contracts with different insurance providers.

Insurance contracts are written to offset the risk associated with a wide variety of events. Examples of different types of insurance contracts include insurance against person-related events (medical, life, annuities), property events (car, home), and financial events (credit default swaps).[1] These insurance contracts share two common properties: the realization of uncertainty can be verified, and subscribers might have additional private information about the *probabilities* that an event realizes. However, due to different regulatory oversight, a feature that varies greatly amongst them is the ability of the insurer to enter into additional contracts with other insurance providers. This possibility of non-exclusive insurance holding, while rare in property insurance, is a definite possibility – for example, in the case of credit default swaps.[2]

Motivated by the above observations we investigate the restrictions on the equilibrium insurance contracts that arise once we dispense with the exclusivity assumption. We consider a variation of the standard Rothschild and Stiglitz (1976) (RS henceforth) environment. Agents are subject to uncertainty regarding their endowment realization, the endowment can be either high or low. We consider three types of agents, each type features different probability of the high realization of the endowment. This probability is private information of the agent. Differently from RS we allow agents to engage in multiple insurance contracts simultaneously with multiple insurance providers. The key assumption is that the terms of these contracts are not observed by other insurance providers. Insurance providers behave non-cooperatively and compete offering menus of insurance contracts from an unrestricted contract space. We derive parameter restrictions under which a separating equilibrium exists and fully characterize it. The unique equilibrium allocation consists of agents with the lowest probability of receiving the high realization of endowment (the bad type) buying the actuarially-fair level of insurance. The other two types with medium and high probability of high endowment realization (the medium and good type) purchase no insurance.

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[1]For a review, refer to Duffie (1999). For recent empirical pricing evidence refer to Blanco, Brennan, and Marsh (2005).

[2]Until early 2009, credit default swaps were issued in private bilateral trades without any intermediation by any clearing house. On March 10, 2009 ICE Trust$^{\text{TM}}$ began operating as a central counter party clearing house for credit default swaps in North America.

Similarly to our paper, the equilibrium allocation in RS features full insurance at the actuarially fair price for the low type. Differently from our paper in RS (see also Wilson (1977)) the medium and high type receive a positive amount of insurance. Another key difference with respect to RS are the condition required for existence. In our environment these conditions are stronger, this is due to the nature of non exclusive competition that allows for additional types of deviation by entrants that are not present in RS. When an equilibrium exists we find that latent contracts must be offered by insurance providers. These are contract offered in equilibrium but not chosen by any type. These contracts are necessary to prevent cream-skimming deviations by entrants and also deviations by incumbents.[3] This highlights the dual role that non-exclusivity plays in our environment. First, by allowing agents to sign additional insurance contracts, it constitutes a constraint on what an insurance providers can offer hence limiting the availability of insurance and in certain cases leading to non existence of equilibrium. Second, non exclusivity enables insurance providers to sustain equilibrium contracts. Deviations from equilibrium can by prevented with the threat that any agent can combine latent contracts with the ones offered following a deviation. This behavior makes it impossible for an entrant to separate different agent types.

Related Literature

This paper is related to two large and growing literatures. The first one studies problems with adverse selection. It originates from the pioneering work of Akerlof (1970), Rothschild and Stiglitz (1976), Wilson (1977) and Miyazaki (1977).[4] From this literature we take our basic setup where agents seek insurance and are privately informed on their risk type. The second one is a more recent literature focusing on non exclusive contracting, see for example the work of Epstein and Peters (1999), Peters (2001), Martimort and Stole (2002) and references therein. In these papers a principal cannot prevent an agent to contract with other competing principals. In addition contracts are private information of the agent and the counter-party. From this literature, as in Biais, Martimort, and Rochet (2000), we adopt the approach to equilibrium characterization. We consider the case where each individual insurance provider offers a set of menus of contracts and delegates to the agent the choice of which insurance contract to pick.

The closest paper to hours is Attar, Mariotti, and Salanié (2014) independently developed during the same time as ours. The two papers share many similarities and some differences. There are three main differences in terms of modeling assumptions. First, their paper re-

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[3]For the use of latent contracts to prevent deviation by entrants also refer to Arnott and Stiglitz (1991) for the case of moral hazard and Attar, Mariotti, and Salanié (2011) for the case of adverse selection.

[4]For a review refer to Dionne, Doherty, and Fombaron (2001).

stricts the analysis to the two types case, while we consider agents with three privately observed types. Second, they consider the case without free entry while we consider the case with free entry. Finally, although their preference specification is more general nesting both a pure trade environment and an insurance environment, the restriction on insurance purchase is different in the two papers. In our paper any insurance purchase that does not lead to negative consumption is allowed. On the other hand Attar, Mariotti, and Salanié (2014) considers either the case either with arbitrary amount of insurance (without non-negativity constraint on consumption) or the case with only positive insurance (in the appendix). Both papers reach similar results: pooling equilibrium fails to exists, the agent with the highest risk reach full insurance, while for everybody else no insurance is provided. An equilibrium may fail to exist altogether. In the body of the paper we highlight additional similarities and differences.

This paper is also related to a series of papers that analyze the effect of non-exclusive contracting in the purchase of goods. One of the first papers to do so is Biais, Martimort, and Rochet (2000). The authors consider an environment where competing traders provide liquidity to a risk-averse agent who is privately informed on the value of an asset. As in this paper, the agent is not restricted in trading with only one trader. Moreover traders compete among each other using menus of possible trades. Differently from our paper, they consider an environment where goods are being traded rather than insurance and they consider the case where the privately observed type can take a continuum of values while we consider a finite (three) number of types. Ales and Maziero (2014) study a dynamic environment with private information (but unlike this paper, the realization of private information happens after agents sign the contract) where agents can engage in multiple non-exclusive contracts for both labor and credit relationships. The paper shows that a unique equilibrium always exists and that latent contracts are necessary. As in this paper, the equilibrium can be implemented using linear contracts for wages and bonds. In a recent paper, Attar, Mariotti, and Salanié (2011) extend the environment of Akerlof (1970) (with linear preferences and a capacity constraint for the informed agent) to include non-exclusive contracting. Differently than our paper they show that an equilibrium always exists. Similarly to our environment, when the equilibrium exists, it is unique in terms of allocations, it involves linear prices, and it is sustained by latent contracts. Arnott and Stiglitz (1991) and Bisin and Guaitoli (2004) study static moral hazard environments where agents trade in non exclusive relationships. In particular, the latter shows that latent contracts are necessary to sustain the equilibrium and lead to positive profit for the insurance providers. In our paper insurance provider generate zero profits. Finally Parlour and Rajan (2001) and Attar, Campioni, and Piaser

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(2006) study the effect of non exclusive relationships for the provision of credit either under limited commitment (the former) or under moral hazard (the latter).

In spirit, this paper is also related to an earlier literature focused on modifying the equilibrium concept first studied by Rothschild and Stiglitz (1976). In particular Wilson (1977) extends the equilibrium concept used in RS beyond static Nash equilibrium by allowing insurance providers to take into account how a change in their policy offers might affect the set of policies offered by other insurance providers. In our paper, latent contracts play a similar role to these non stationary expectations by enabling a reaction of insurance providers to deviations of other insurance providers. On a similar note are the papers that study inter-firm communication in insurance settings, such as Jaynes (1978) and Hellwig (1988).[5] The first considers a static adverse selection economy and allows firms the choice to disclose or not information on who accepts the insurance contract. It shows that some firms share information leading to a separating equilibrium (that always exists), while in the case where no information is shared, no equilibrium exists. Sharing information allows a firm to offer an insurance contract that is contingent on additional purchases of insurance an agent might accept. In our paper, firms gather information on insurance purchased by also offering latent contracts, which allows us, in contrast to Jaynes, to have an equilibrium even without any information being shared directly. Latent contracts have the same role as information sharing, since they enable firms to react to deviation of incumbent firms. Hellwig (1988) highlights that the ability to react is the key to equilibrium existence rather than inter-firm communication. In particular, inter-firm communication enables firms to react only if the equilibrium concept considered in Jaynes (1978) is implicitly assuming a non-stationary expectation similar to Wilson (1977). Along similar lines, Picard (2009) considers the case where the contracts offered by the insurance providers feature participating clauses so that the payout is conditional on the profits of the insurance provider. In this setup it is shown that an equilibrium always exists and coincides with the Miyazaki-Spence-Wilson allocation.

This paper is organized as follows: Section 2 describes the environment. Characterization and implementation are studied respectively in Sections 3 and 4. Section 5 concludes.

2 Environment

The economy is populated by a continuum of measure one of agents and infinite but countably many insurance providers. Following Rothschild and Stiglitz (1976) we assume free-entry in

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[5]For an extension to a moral hazard environment, also look at Hellwig (1983).

the insurance market. Agents are ex ante heterogeneous. We consider three types of agents. There is a fraction p_g of type g agents (the good type) a fraction p_b of type b (the bad type) and a remaining fraction $p_m = 1 - p_g - p_b$ of type m (the medium type).[6] We assume that $p_b > 0$, $p_m > 0$ and $p_g > 0$.[7] The economy lasts for 1 period. Agents' utility u is defined over consumption c. Assume $u : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable, increasing, and strictly concave function. At time 1, an agent of type $j = b, m, g$ receives an endowment ω_H with probability π_j and ω_L with probability $1 - \pi_j$. Let $\omega_H > \omega_L$ and denote $\omega = (\omega_L, \omega_H)$. The realization of the endowment is publicly observed. Assume that $\pi_g > \pi_m > \pi_b$, that these probabilities are private information of the agent and that these probabilities are known to the agent before signing any insurance contract. Define the market (average) probability of high realization as $\pi = p_g \pi_g + p_m \pi_m + p_b \pi_b$. The probability of high endowment realization averaged across any two types is given by $\pi_{i,j} = \frac{p_i \pi_i + p_j \pi_j}{p_i + p_j}$ with $i, j = b, m, g$. In much of the characterization with three types it is be important to distinguish if $\pi_m \geq \pi$ or if $\pi_m < \pi$. Respectively theses two cases represent the instance in which the medium type is less risky ($\pi_m \geq \pi$) or more risky ($\pi_m < \pi$) than the overall average market. In the former case it is easy to see that $\pi_{b,g} \leq \pi$ while in the latter $\pi_{b,g} > \pi$.

In this environment, each agent seeks to insure himself against the uncertain realization of the endowment. Insurance is provided by insurance providers (referred to as firms in the body of the paper). Denote by I the number firms active in equilibrium. Each firm $i \in \{1, \ldots, I\}$ offers insurance contracts to agents. A contract prescribes consumption transfers conditional on the realization of the endowment. The key feature of our environment is that agents can simultaneously sign contracts with more than one firm and that the terms of the contract between an agent and any firm are not observed by other firms. We restrict the analysis to bilateral mechanisms between a principal and an agent; as described in Martimort and Stole (2002) and Peters (2001) in a single agent environment and in Han (2006) in a multi-agent setting,[8] we can restrict the analysis to menu games. In a menu game, each firm i offers a menu: a set C^i in $\mathcal{P}(\mathbb{R}^2)$ (the power set of $\mathbb{R}^2$). We focus on pure-strategy equilibria.

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[6]The original insurance problem discussed in Rothschild and Stiglitz (1976) focuses on two types only. Refer to Wilson (1977) for the case with multiple types.

[7]The case with one of the $p_j = 0$ with $j = b, m, g$, reduces the environment to one with only two types of agents. This case was studied in a previous version of this paper. All of the results of this paper hold in that case also.

[8]Han (2006) extends the delegation principle to a multi-agent setting in which the contracts offered by principals to an agent can depend on the messages sent by this agent and not on the messages sent by other agents. The paper shows that a pure-strategy equilibrium of any bilateral mechanism can be implemented as a pure-strategy equilibrium of a menu game. Considering multilateral mechanisms that allow an agent's allocation to depend upon the reports of other agents might expand the set of equilibria allocations as shown in Yamashita (2010).

Elements of C^i are transfers pairs, (τ_L, τ_H), conditional on a realization respectively of ω_L or ω_H. To assure the agent's problem has a solution, we restrict the menus offered to be compact sets. We do not impose any additional restriction on the type of menus offered by firms besides requiring that each firm offers the null transfer $(0,0)$.[9] If agents of type j choose the transfer pair (τ_L, τ_H) in the menu, its impact on the profit of the firm is given by $-p_j(\pi_j\tau_H + (1 - \pi_j)\tau_L)$.

We focus on symmetric equilibria so that agents of the same type make the same choices.[10] Denote by $\tau^{j,i}$ the choice of agents of type j within C^i. Given the number of firms active in equilibrium, I, and the menus offered by these firms, $C = \{C^1, \ldots, C^I\}$, the problem for an agent of type $j = b, m, g$ is defined as:

$$\max_{\{(\tau_L^{j,i}, \tau_H^{j,i}) \in C^i\}_{i=1}^I} \left[\pi_j u\left(c_H^j\right) + (1 - \pi_j)u\left(c_L^j\right) \right] \tag{1}$$

$$\text{s.t.} \quad c_k^j = \omega_k + \sum_{i=1}^I \tau_k^{j,i} \quad \text{for } k = H, L,$$

$$c_k^j \geq 0, \quad \text{for } k = H, L.$$

Two observations are worth emphasizing. First the optimal choices of the agents are not necessarily unique. In equilibrium (defined below) firms take as given agents' optimal choice and multiple equilibria might arise given the multiplicity of agents' optimal choices. Second, best replies of the agents depend not only on the menu offered by firm i but on all the menus offered by other firms.

The objective of firms is to maximize profits by optimally choosing a menu. Each firm takes as given the menus offered by other firms, denoted by C^{-i}, and the optimal choices of the agents (their best reply) defined in (1) to solve the following:

$$\max_{C^i \in \mathcal{P}(\mathbb{R}^2)} - \sum_{j=b,m,g} p_j[\pi_j\tau_H^{j,i}(C^i, C^{-i}) + (1 - \pi_j)\tau_L^{j,i}(C^i, C^{-i})] \tag{2}$$

$$(0,0) \in C^i$$

$$\tau^{j,i}(C^i, C^{-i}) \text{ solves (1)}$$

The constraints on the firm's problem require that, for any menu a firm offers, the choice of the agents are according to their best reply. It also highlights the dependence on the menus

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[9]In particular we allow firms to offer negative insurance: offer menus containing contracts that imply a negative transfer upon the realization of the low state. See also discussion following Proposition 2.

[10]This assumption is made for convenience and plays no crucial role for our results.

offered by other firms by taking into account the fact that whenever a firm $-i$ changes the menu offered, it might imply that agents' choices in the menu offered by firm i also changes. We denote by $\Pi(C', C)$ the profit to a firm of offering menu C' when C is the set of menus offered by the other firms active in equilibrium.

We can now define our free-entry equilibrium in the menu game.

Definition 1 (**Equilibrium**). *A pure strategy symmetric equilibrium of the menu game is: the number of firms active in equilibrium, I, a collection of menus C^i for all $i \in \{1, \ldots, I\}$ and agents' optimal choices $\{(\tau_L^{j,i}, \tau_H^{j,i})\}_{i=1}^I$, for all $j = b, m, g$ such that:*

1. *For each $j = b, m, g$, $\{(\tau_L^{j,i}, \tau_H^{j,i})\}_{i=1}^I$ is a solution of the agent problem (1).*

2. *For each $i \in \{1, \ldots, I\}$, taking as given C^{-i} and agents' optimal choices (their best replies) $\{(\tau_L^{j,i}, \tau_H^{j,i})\}_{i=1}^I$ for each $j = b, m, g$, C^i solves (2).*

3. *There is no $C' \in \mathcal{P}(\mathbb{R}^2)$ such that $\Pi(C', C) > 0$.*

The definition of equilibrium is standard, agents maximize expected utility and in particular conditions *1.* and *2.* together imply that no contract offered in equilibrium makes negative profits. Given the free-entry nature of our environment there is always a firm not active in equilibrium; given this, condition *3.* implies that there is no contract left outside of the equilibrium that would otherwise earn positive profits.[11] Note that a menu offered in equilibrium might contain more alternatives than the number of types, given our focus on symmetric equilibria this implies that some alternatives are not be chosen in equilibrium. We denote a contract as *latent* if it is offered in equilibrium by some firm and is not chosen in equilibrium by any agent. As it will be shown in the following section these contract are necessary to ensure the existence of equilibrium.

For notational convenience we consider, in the body of the paper, utility and profits derived from consumption transfers rather than by menus. Let $c = (c_L, c_H)$ denote the consumption in the low and high state realization. For a given type $j = b, m, g$, denote by $U^j(c) = \pi_j u(c_H) + (1 - \pi_j) u(c_L)$ the expected utility from consumption c, where $c_k = \omega_k + \sum_{i=1}^I \tau_k^i$ for $k = L, H$. We denote by $U^j(\omega)$ the expected utility in autarky for agents of type j. Given $\tau = (\tau_L, \tau_H)$, let $\Pi^j(\tau) = -\pi_j \tau_H - (1 - \pi_j)(\tau_L)$. Conditional on agents of type $j \in \{b, m, g\}$ accepting contract τ, profits for a firm offering τ are equal to $p_j \Pi^j$. Similarly let $\Pi^{i,j}(\tau) = -\pi_{i,j} \tau_H - (1 - \pi_{i,j}) \tau_L$, if multiple types i and j accept contract τ profits will

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[11]In Attar, Mariotti, and Salanié (2014) instead the number of firms in the environment is fixed.

be equal to $(p_i + p_j)\Pi^{i,j}(\tau)$. If all three types accept contract τ then we denote profits by $\Pi(\tau) = -\pi\tau_H - (1-\pi)\tau_L$. It is convenient to define aggregate profits as the sum of all profits originating from the aggregate consumption allocation. Aggregate profits Π are defined as:

$$\Pi = \sum_{j=b,m,g} p_j \Pi^j (c^j - \omega). \tag{3}$$

3   Characterization of Equilibrium

A first straightforward result is that in any equilibrium profits for the firms must be non-negative: for all i, $\Pi(C^i, C^{-i}) \geq 0$, this is because a firm can always secure zero profit by only offering the null contract $(0,0)$. This implies that aggregate profits must be weakly positive: $\Pi \geq 0$. Similarly for all agents $j = b, m, g$ the consumption allocation c^j must be weakly preferable to the autarky one: $U^j(c^j) \geq U^j(\omega)$. The concavity assumption on preferences implies a positive demand for insurance. In particular since $\omega_H > \omega_L$ we have that:

$$\frac{1 - \pi_j}{\pi_j} \frac{u'(\omega_L)}{u'(\omega_H)} > \frac{1 - \pi_j}{\pi_j}, \qquad j = b, m, g.$$

The above relationship implies that each agent is willing to purchase small positive amounts of insurance at a price which generates positive profits for the insurance provider. On the other hand, the same relationship implies that each agent is willing to purchase small amounts of negative insurance at a price which generates negative profits for the insurance provider if only such agent purchases it.

   To characterize equilibrium, we consider two cases. The first case is a *pooling equilibrium*. In this case agents of all three types receive the same equilibrium allocation. In the next subsection we show that this type of equilibrium never exists. A second type of equilibrium, a *separating equilibrium*, occurs when at least two types receive a different consumption allocation. In subsection 3.2, we show that the unique equilibrium consumption allocation of the menu game is a separating equilibrium with the following characteristics: agents of type b (the bad type) receive full insurance against the realization of the endowment at their actuarially-fair price, while agents of types m and g, receive no insurance.

3.1   Pooling Equilibrium

We first determine necessary conditions that any pooling equilibrium must satisfy. In this subsection we let $c = (c_L, c_H)$ be the candidate polling equilibrium level of consumption.

Lemma 1. *For any pooling equilibrium allocation for consumption* $c = (c_L, c_H)$*, the following conditions must hold:*

$$c_L \geq c_H, \tag{4}$$

$$if \quad \pi_m \leq \pi, \quad \frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} \geq \frac{1 - \pi}{\pi}, \tag{5}$$

$$if \quad \pi_m > \pi, \quad \frac{1 - \pi_m}{\pi_m} \frac{u'(c_L)}{u'(c_H)} \geq \frac{1 - \pi}{\pi}. \tag{6}$$

Proof. In appendix A. □

Equation (4) implies that pooling equilibrium must be (weakly) in the overinsurance region. This condition is equivalent to the following relation:

$$\frac{1 - \pi_b}{\pi_b} \frac{u'(c_L)}{u'(c_H)} \leq \frac{1 - \pi_b}{\pi_b},$$

the above implies that the marginal rate of substitution between consumption in the two states for the b agent is less than or equal to the actuarially-fair price for the insurance if only b agents accept it. This relation provides an intuition for the necessary condition (4). If it were not to hold, c cannot be an equilibrium since a profitable entry opportunity is always available. A deviating firm can provide additional insurance for agents of type b charging a price slightly higher then the actuarially-fair one. Such deviation is always profitable for the firm and increases expected utility for the agents of type b. Similarly, the necessary conditions (5) and (6) require that the marginal rate of substitution between consumption in the two states for agents of type g and m respectively is greater than the price for insurance when all agents accept the contract. If not, entrant insurance providers can profitably provide additional insurance to agents of type g and m. A direct implication of Lemma 1 is that there is no pooling equilibrium, since there is no allocation that simultaneously satisfies the conditions described in equations (4), and either (5) or (6).

Proposition 1. *There is no pooling equilibrium.*

Proof. Suppose there exists a pooling equilibrium $c = (c_L, c_H)$. The equilibrium allocation must satisfy conditions (4) and either (5) or (6). In the first case with $\pi_m \leq \pi$, from (4) and (5) we have $\frac{1 - \pi_g}{\pi_g} \geq \frac{1 - \pi}{\pi}$. So that $\pi_g \leq \pi$, this is a contradiction since $\pi_b < \pi_m < \pi_g$ implies $\pi_g > \pi$. In the second case with $\pi_m > \pi$, from (4) and (6) we have that $\frac{1 - \pi_m}{\pi_m} \geq \frac{1 - \pi}{\pi}$. This implies $\pi_m \leq \pi$, a contradiction. □

The key intuition for the proof of non existence of a pooling equilibrium with non-exclusive contracts is that latent contracts cannot prevent entrants from providing additional positive (or negative) insurance. Under exclusive contracts this type deviation does not need to be considered. With exclusive contracts, following an entrant's deviation, agents must choose whether to stay with the incumbent or to accept the entrant's contract but cannot do both as is in the case with non-exclusive contracts. Nonetheless Rothschild and Stiglitz (1976) and Wilson (1977) show that there is no pooling equilibrium when contracts are exclusive by showing that there is always an alternative contract that can be offered by an entrant that is profitable and attracts only good types (a cream-skimming deviation). It is interesting to notice that with non-exclusive contracts some cream-skimming deviations might be prevented. We provide a graphical example for the two-types case in Figure 1. In this figure, the solid lines represent the indifference curves for agents of type b (the steeper curve) and g (the flatter curve) at the *best* pooling equilibrium.[12] Consider a contract $\hat{\tau}$ that allows agents to reach a consumption level in the shaded area starting from the endowment point. Contract $\hat{\tau}$ is preferred to the pooling equilibrium by agents of type g but not by agents of type b. In addition $\hat{\tau}$ it is profitable for a firm as long as only agents of type g accept it. If contracts are exclusive as in Rothschild and Stiglitz (1976), $\hat{\tau}$ constitutes a profitable cream-skimming deviation. This is not necessarily the case under non-exclusivity. Consider the following tentative equilibrium: the incumbents offer a pooling equilibrium and $\tau^L = L - \omega$ with point L as in Figure 1. Consider a candidate equilibrium where only τ^L and the pooling allocation is offered. In this scenario τ^L is latent, agents of type g and b accept the pooling equilibrium. This latent contract τ^L makes $\hat{\tau}$ unprofitable. This is because $\hat{\tau}$, when combined with τ^L, is now strictly preferred to the pooling equilibrium by agents of type b. Hence τ^L prevents an entrant from offering $\hat{\tau}$. However the pooling equilibrium in Figure 1 fails to be an equilibrium under non-exclusivity since agents of type b are instead attracted by deviations of entrants offering additional small positive amount of insurance. No latent contract is able to prevent such deviations.

3.2   Separating equilibrium

We now study separating equilibria. For each type $j = b, m, g$ denote the equilibrium consumption allocation by $c^j = (c_L^j, c_H^j)$. The separating equilibrium allocation is denoted by $c = \{c^b, c^m, c^g\}$. Since c is a separating equilibrium there must exist at least one j and j'

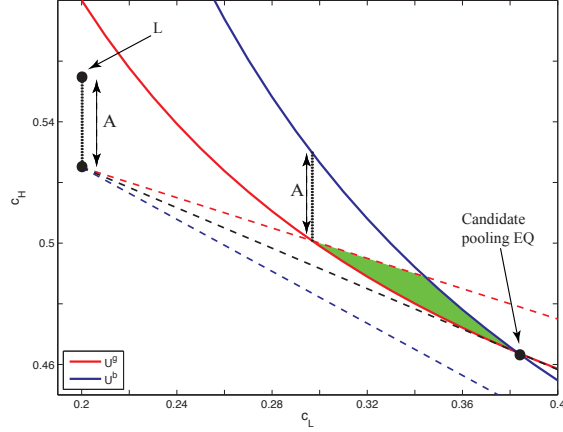[12]The pooling equilibrium that delivers the highest expected utility when agents are weighted equally.

Figure 1: Pooling Equilibrium and The Role of Latent Contracts.

such that $c^j \neq c^{j'}$. By revealed preferences, the following incentive constraints must hold:

$$U^j\left(c^j\right) \geq U^j(c^{j'}), \text{ for } j, j' = b, m, g. \tag{7}$$

Before fully characterizing the equilibrium allocations in Proposition 2, we provide some of the necessary conditions that any separating equilibrium must satisfy. Lemma 2 focuses on the magnitude of consumption for each type for each endowment realization.

Lemma 2. *Any separating equilibrium allocation $c = \{c^b, c^m, c^g\}$ must satisfy:*

$$c_H^g \geq c_L^g, \tag{8}$$

$$c_L^b \geq c_H^b, \tag{9}$$

$$c_H^g \geq c_H^m \geq c_H^b \quad and \quad c_L^g \leq c_L^m \leq c_L^b. \tag{10}$$

Proof. In appendix A. □

The above Lemma implies that the allocation for the g type must be in the underinsurance region (condition (8)), while the allocation of the b type must be in the overinsurance region (condition (9)). The intuition for this result is as follows: if the agents of type g are in the overinsurance region, an entrant can offer a small negative insurance contract at a price worse than the actuarially fair one. Agents of type g accept this contract and it is profitable even if any other type accepts it. The intuition for the case when b is underinsured is similar:

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offering additional insurance to type b is always profitable even if other types also accept it. In addition, condition (10) in Lemma 2 states that consumption under the realization of the low endowment must be weakly decreasing as agent type increases, while must be weakly increasing in type upon the realization of the high endowment shock. This monotonicity property is derived from the equilibrium conditions (7) and the concavity of the utility function.[13]

The previous Lemma shows some of the restrictions imposed by non exclusivity on the equilibrium allocation. To show these restrictions we repeatedly use the following argument: if any of the restriction does not hold, agents accept a deviation offered by an entrant together with their original choice of contracts with the incumbents. These deviations occur if they cannot be made unprofitable. In the next Lemma we continue with this logic, however we now consider the fact that an agent might switch his original choice of contracts with the incumbents upon a particular entry. This logic introduces pairwise restrictions on the equilibrium allocation for consumption. Lemma 3 provides some of these restrictions.[14] In appendix A we show some additional conditions that must hold in any separating equilibrium and are used in the complete characterization of the equilibrium allocation.

Lemma 3. *Any separating equilibrium allocation* $c = \{c^b, c^m, c^g\}$ *must satisfy:*

$$\forall\, j = g, m, \qquad \Pi^b(c^b - c^j) \leq 0; \tag{11}$$

$$\Pi^g(c^g - c^m) \leq 0; \tag{12}$$

$$\Pi^{b,m}(c^m - c^g) \leq 0. \tag{13}$$

Proof. In appendix A. $\qquad\qquad\square$

Condition (11) is determined by the behavior of agents of type b following a deviation of the entrant. This condition imposes that a contract allowing agents to reach consumption level c^b starting from either c^m and c^g must be unprofitable (weakly) if agents of type b choose it.[15] The intuition is clear in this case: if not, an entrant would provide such contract (with a small additional transfer in one of the two states) and agents of type b would accept the contract. Profits to the entrant are positive irrespective of which other type also accepts the contract. Condition (12) is the mirror image of (11), focusing on the behavior of agents of

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[13]Lemma 2 applies to any equilibrium not only the separating one. For the pooling equilibrium, conditions (8) and (9) imply that $c_L = c_H$ this immediately implies a violation of conditions (5) and (6) in Lemma 1 so that a pooling equilibrium does not exist.

[14]Refer also to Proposition 1 in Attar, Mariotti, and Salanié (2014) for similar characterizations.

[15]Note that given Lemma 2 such contract constitutes positive insurance.

type g. It states that a contract allowing an agent to go from c^m to c^g must be unprofitable if agents of type g accept it. To understand this condition we first recall that from Lemma 2 we have that $c_L^g \leq c_L^m$ so that the contract just considered is in fact a form of negative insurance (paying when the endowment is high). In this case agents of type g are the "worst" type for an entrant: generating the smallest profits. Given this, if condition (12) is violated we immediately have an opportunity for a deviation that is always profitable and that is accepted by agents of type g. Finally condition (13) looks at the behavior of agents of type m. As before, this condition considers the relationship between c^m and c^g. It considers a contract (offering positive insurance) allowing agents to reach consumption level c^m starting from c^g. In this case if (13) is violated an entrant attracts agents of type m offering a contract that allows agents to reach c^m from c^g (with a small additional transfer in one of the two states). This contract, since is accepted by agents of type m, remains profitable if agents of type b or g (or both) accept it.

Condition (11) immediately allows us to rule out the equilibrium allocation characterized in Rothschild and Stiglitz (1976) and Wilson (1977). When contracts are exclusive, under certain parameter restrictions, there exists a unique separating equilibrium (referred to as RSW from here onwards). The RSW separating equilibrium consumption allocation is (r^b, r^g) where $r^b = (\omega^b, \omega^b)$ and $r^g = (r_L^g, c_H^g)$ such that $U^b(r^b) = U^b(r^g)$ and $\Pi^g(r^g - \omega) = 0$. The RSW equilibrium allocation is displayed in Figure 2.

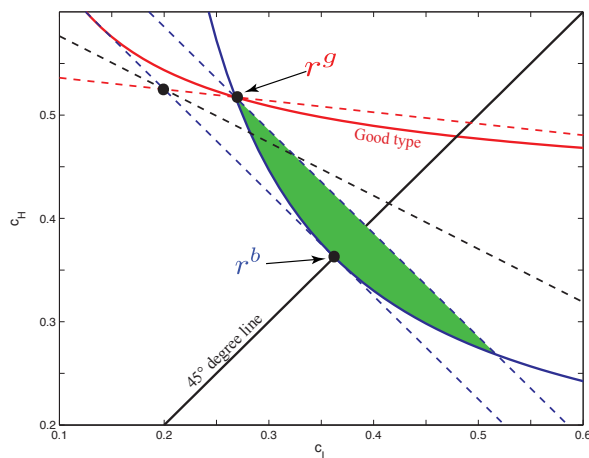

Figure 2: The Rothschild-Stiglitz-Wilson Equilibrium: r^g consumption for agents of type g, r^b consumption for agents of type b.

In the RSW equilibrium, given the consumption allocation of agents of type b, it is

clear that the positive insurance purchased by agents of type g implies that $\Pi^b(r^b - r^g) > 0$ contradicting (11). Indeed given the RSW equilibrium allocation with non exclusive contracts an entrant can offer additional insurance $\hat{\tau}$ for agents of type b at a price slightly worse than the actuarially fair price. As soon as $\hat{\tau}$ is offered, agents of type b will accept the additional insurance together with the allocation r^g. Given the price charged for insurance, this entry is always profitable. The shaded area in Figure 2 displays the set of consumption that can be achieved with an entrant offering $\hat{\tau}$.

We now complete the characterization of the equilibrium. Proposition 2 below shows that, if an equilibrium exists, there exist a unique equilibrium allocation for consumption. This allocation (which is our candidate equilibrium) is given by $c^b = (\omega^b, \omega^b)$ with $\omega^b = \pi_b\omega_H + (1 - \pi_b)\omega_L$ and $c^m = c^g = (\omega_L, \omega_H)$: as the RSW equilibrium allocation the candidate equilibrium provides full insurance at the actuarially-fair price to agents of type b; differently than the RSW equilibrium, the candidate equilibrium allocation provides no insurance for agents of type m and g.

Proposition 2. *Any equilibrium allocation of the menu game satisfies:* $c^b = (\omega_b, \omega_b)$, *where* $\omega_b = \pi_b\omega_H + (1 - \pi_b)\omega_L$; $c^m = c^g = (\omega_L, \omega_H)$.

Proof. In appendix A. □

Lemmas 2 and 3 impose strong restrictions on the equilibrium allocations (c^b, c^m, c^g). In particular with the additional results of Lemma 4 in the Appendix it follows that the equilibrium allocation must be of the type described in Jaynes (1978), Hellwig (1988) and Glosten (1994) so that $\Pi^{b,m}(c^m - c^g) = 0$ and $\Pi^b(c^b - c^m) = 0$.[16] These restrictions however, still do not rule out the fact that either c^m or c^g might generate positive profits when chosen respectively by agents of type m or g. For this to happen either agents of type m or g must receive a positive amount of insurance. The proof of Proposition 2 shows that indeed c^m and c^g cannot generate positive profits and that agents of type m and g purchase no insurance and remain in autarky. At the heart of the proof there is a familiar argument using cream-skimming strategies. The idea is that *if* agents of type g or m generate positive profits in the aggregate, then an entrant will try to attract them by replicating one of the profitable contracts offered by the incumbents to one of these two types. The proof is quite lengthy and the main difficulties are to consider all the possible deviations of the agents that might occur following a deviation of the entrant and second to show that in equilibrium there cannot be a contract being offered that can simultaneously prevent a cream-skimming

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[16]For the two type case Attar, Mariotti, and Salanié (2014) show that any separating equilibria must be of Jaynes-Hellwig-Glosten type.

strategy and be latent. As an example, consider the graphical argument used to discuss the pooling equilibrium: Figure 1. Suppose that the pooling equilibrium in the figure is now a pooling allocation between agent of type m and b only. As before contract τ^L allowing to reach consumption allocation L from the endowment point makes $\hat{\tau}$ (allowing to reach a consumption point in the shaded area) unprofitable. However it is easy to see that τ^L cannot be latent in equilibrium. An entrant can exploit τ^L offering additional positive insurance at a rate actuarially fair for agents of type b. This deviation is accepted by both agents of type m and b improving on their original allocation.

Note that the proof of Proposition 2 relies on the ability of insurance providers to provide negative insurance, specifically the arguments used on Lemma 3. This assumption has been shown to be non essential in the two type case considered in Attar, Mariotti, and Salanié (2014).[17]

As in Rothschild and Stiglitz (1976), a separating equilibrium may fail to exist. In our environment, we require the following necessary condition on primitives to guarantee the existence of an equilibrium.[18]

Assumption 1.

$$\frac{1-\pi_g}{\pi_g}\frac{u'(\omega_L)}{u'(\omega_H)} \leq \frac{1-\pi}{\pi}, \tag{14}$$

$$\frac{1-\pi_m}{\pi_m}\frac{u'(\omega_L)}{u'(\omega_H)} \leq \frac{1-\pi_{b,m}}{\pi_{b,m}}. \tag{15}$$

Condition (14) states that the indifference curve for g agents at the equilibrium allocation is flatter than the market zero profits line (actuarially- fair price of insurance if all agents buy it). It implies that the g agent only accept any additional insurance relative to the endowment (either positive or negative) at a price that is not profitable if all agents accept it also. Condition (15) states that the indifference curve for m agents at the equilibrium allocation is flatter than the zero profits line of insurance if only agents b and m accept it. It implies that agents m accept additional insurance only at price that is unprofitable if agents b and m accept it. As an implication of (14) and (15) we have that agents of type g and m prefer to remain in autarky rather than accept either the allocation of agents of type b. The above conditions are satisfied if, for example, π_g is large relative to π_m and π_b or if the spread

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[17]It is also worth emphasizing that Propositions 1 and 4 do not depend on the firms' ability to offer negative insurance.

[18]An interesting extension left for future research is the characterization of equilibrium using random menus. See Dasgupta and Maskin (1986a,b) for the study of existence of equilibrium in the case with exclusive contracts. And also Carmona and Fajardo (2009) and Monteiro and Page (2008) for the case with non-exclusive contracts.

between ω_L and ω_H is sufficiently small. Assumption 1 is necessary for an equilibrium to hold. As the following Proposition shows if any of the conditions where to be violated, then there is always a profitable deviation for an insurance provider selling additional insurance at either agents of type g and m at a price which is always profitable.

Proposition 3. *If Assumption 1 does not hold, there is no equilibrium.*

Proof. Suppose (14) in Assumption 1 is violated. Consider an entrant firm offering a menu containing $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small, and α satisfying $\frac{1-\pi_g}{\pi_g}\frac{u'(\omega_L)}{u'(\omega_H)} > \alpha > \frac{1-\pi}{\pi}$. The contract $\hat{\tau}$ is accepted by agents of type g since $U^g(\omega + \hat{\tau}) > U^g(\omega)$. This implies that $\pi_g[u(\omega_H - \alpha\varepsilon) - u(\omega_H)] + (1-\pi_g)[u(\omega_L+\varepsilon) - u(\omega_L)] > 0$. Since $\varepsilon > 0$ and $\pi_m < \pi_g$ it follows that $\pi_m[u(\omega_H - \alpha\varepsilon) - u(\omega_H)] + (1-\pi_m)[u(\omega_L+\varepsilon) - u(\omega_L)] > 0$, so that $U^m(\omega + \hat{\tau}) > U^m(\omega)$. Given this, minimum profits are achieved when also agents of type b accept $\hat{\tau}$. From the definition of α this deviation is always profitable. This implies that the consumption allocation for agents of type g and m is not $c^m = c^g = \omega$, which contradicts Proposition 2.

If (15) in Assumption 1 is violated, then there exists an $\alpha > 0$ so that $\frac{1-\pi_m}{\pi_m}\frac{u'(\omega_L)}{u'(\omega_H)} > \alpha > \frac{1-\pi_{b,m}}{\pi_{b,m}}$. Consider an entrant firm offering a menu $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small. $\hat{\tau}$ is accepted by agents of type m. Minimum profits are achieved when only agents of type b and m accept the contract and from the definition of α the entrant makes positive profits which is a contradiction. $\square$

The necessary conditions for existence of equilibrium in Assumption 1 are stronger than those found in Rothschild and Stiglitz (1976) and Wilson (1977). The previous proposition provides an intuition on why this is the case. Relative to the case with exclusive contracts, the non-exclusivity assumption introduces additional opportunities for profitable deviations. These deviations cannot be prevented and are severe enough that might induce profits for the incumbents to became strictly negative. The lack of existence result is also confirmed in Attar, Mariotti, and Salanié (2014) in an environment with two types and without free entry. Their environment, focusing on two types and without free entry, features the same necessary conditions as in Assumption 1. One fundamental issue that leads to non-existence result present in both our paper and RS is the lack of any capacity constraint. This issue has been raised by Inderst and Wambach (2001) where the standard RS environment is complemented with capacity constraints in the amount of insurance that an insurance can provide. In this case it is shown that an equilibrium exists.[19] In our environment (see also the discussion in Attar, Mariotti, and Salanié (2014) section 3.5) each insurance provider

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[19]Similarly, in Guerrieri, Shimer, and Wright (2010) the competitive search environment introduces a sort of capacity constraint. In this paper an equilibrium always exists.

can service the entire market. Hence, an entrant can exploit this by forcing an incumbent firm to provide insurance to a larger number of types than it originally planned for.[20]

4   Implementation of Equilibrium

We now show that if Assumption 1 holds an equilibrium exists. The following proposition shows, by construction, that the allocation (c^b, c^m, c^g) characterized in Proposition 2 can be sustained in equilibrium. Recall that in the proposed equilibrium $c^g = c^m = \omega$ and $c^b = \omega^b$ with $\omega^b = \pi_b \omega_H + (1 - \pi_b)\omega_L$.

Proposition 4. *Let* $\{\pi_g, \pi_m, \pi_b, \omega_H, \omega_L, u, p_g, p_m, p_b\}$ *satisfy Assumption 1, then there exists an equilibrium of the menu game.*

The complete proof of Proposition 4 is provided in Appendix B. In what follows we show the result in the simpler case with two types and when a deviating firm offers a contract that attracts only agents of type g.[21] Set $p_m = 0$. In this case Assumption 1 reduces to condition (14). Consider the following strategies by firms. Without loss of generality, let firms $i = 1, 2$ offer the menu $C^i = L_b$ where the set L_b is defined as follows:

$$L_b = \left\{ x_L \geq 0, x_H \leq 0 \;\middle|\; -\pi_b x_H - (1 - \pi_b)x_L = 0 \right\}.$$

The set L_b constitutes the set of positive insurance at the actuarially fair price for agents of type b. All remaining firms $i \neq 1, 2$ offer the menu: $C^i = (0, 0)$. Let $\tau_L^b = \pi_b(\omega_H - \omega_L)$, $\tau_H^b = (1 - \pi_b)(\omega_L - \omega_H)$, it is easy to show that under Assumption 1, there exist an equilibrium where agents of type b choose $\left(\tau_L^b/2, \tau_H^b/2\right)$ from both firms 1 and 2 and $(0, 0)$ from remaining firms; type g chooses $(0, 0)$ from all firms. In this equilibrium, all firms make zero profits. Suppose that firm i deviates and offers the contract $\tau^g = (\tau_L^g, \tau_H^g)$ targeting the good types only. Consider the case with $\tau_L^g < 0$: negative insurance is being offered to agents of type g. Since $U^g(\omega + \tau^g) > U^g(\omega)$, if $\tau_L^g < 0$, it follows that $\Pi^g(\tau^g) < 0$. This implies that the profits from the deviation are negative, hence we rule out the case with $\tau_L^g < 0$. Consider now the case with $\tau_L^g > 0$: positive insurance is being offered to agents of type g. Consider the following transfer:

$$\hat{\tau} = \left(\pi_b(\omega_H - \omega_L + \tau_H^g - \tau_L^g), -(1 - \pi_b)(\omega_H - \omega_L + \tau_H^g - \tau_L^g) \right).$$

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[20]In the pure trade environment with linear preferences non exclusivity of Attar, Mariotti, and Salanié (2011) an equilibrium always exists. In this case the capacity constraint is in the form of the limited supply of goods that the seller is endowed with.

[21]We thank an anonymous referee for providing insights in simplifying the proof in this case.

We have that $\hat{\tau} \in L_b$[22] and can be chosen from either firm $i = 1, 2$ (for this step it is crucial to have at least two firms offering L_b in equilibrium, so that following a deviation of any firm i, L_b is still available to agents of type b). When combined with τ^g, contract $\hat{\tau}$ provides full insurance for agents of type b: consumption in both states is given by $\omega_b + \pi_b \tau_H^g + (1 - \pi_b)\tau_L^g$. Since $\tau_L^g > 0$, from Assumption 1 we have that $\Pi(\tau^g) < 0$; this also implies that $\Pi^b(\tau^g) < 0$. Hence $\omega_b + \pi_b \tau_H^g + (1 - \pi_b)\tau_L^g > \omega_b$: agents of type b can reach higher level of consumption by choosing τ^g and $\hat{\tau}$ than their original consumption allocation. If τ^g is offered it is chosen by both agents of type g and b delivering negative profits to the entrant. This argument is displayed in Figure 3. The general proof for the case with $p_m \neq 0$ and when a firm offers
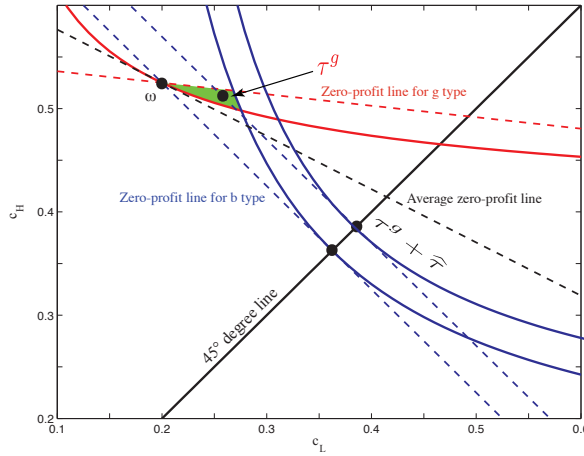

Figure 3: Sketch of proof of Proposition 4 for $p_m = 0$.

contracts to all the three types is in Appendix B.[23]

From the proof of existence of equilibrium a key fact emerges. As in a Bertrand-like environment where firm compete with exclusive contracts, at least two firms must be active in equilibrium and offer contracts different than the null one. The reason is twofold. First, as in the case with exclusive contracts, the fact that neither of the two firm is necessary to reach consumption level c^b implies that neither firm can deviate offering a contract to agents of type b generating higher profits. Second, and differently than the case with exclusive contracts, since neither firm is necessary to reach c^b it implies that neither incumbents nor

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[22]To show that $\hat{\tau}$ is in L_b we must have that $\omega_H + \tau_H^g - \omega_L - \tau_L^g \geq 0$. From the definition of τ^g we have: $U^g(\omega + \tau^g) \geq U^g(\omega^b)$ and $U^b(\omega^b) \geq U^b(\omega + \tau^g)$. Since $U^g(\omega^b) = u(\omega^b)$ we have that $(\pi_g - \pi_b)[u(\omega_H + \tau_H^g) - u(\omega_L + \tau_L^g)] \geq 0$, hence $\omega_H + \tau_H^g - \omega_L - \tau_L^g \geq 0$.

[23]Refer to Attar, Mariotti, and Salanié (2014) for a more general treatment of the two type case.

entrant can deviate and offer additional insurance to agents of type m or g.

To expand on this last point consider any implementation of the equilibrium allocation in wich $\omega^b = \sum_{i=1}^{I^b} \tau^{b,i} + \omega$ and for any $i' \in I^b$ there exists a set of incumbent I' so that $i' \notin I'$ and $\omega^b = \sum_{i=1}^{I'} \tau^{b,i} + \omega$. Suppose that either i' or an entrant deviates and offers additional insurance to agents of type m given by $\tau^m = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small and where $\alpha > 0$ satisfies:

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(\omega_L)}{u'(\omega_H)} > \alpha > \max \left\{ \frac{1 - \pi_m}{\pi_m}, \frac{1 - \pi_g}{\pi_g} \frac{u'(\omega_L)}{u'(\omega_H)} \right\}.$$

Note that such $\alpha > 0$ always exists since $\omega_L < \omega_H$ and $\pi_m < \pi_g$. It follows that $U^m(\omega + \tau^m) > U^m(\omega)$. Given the definition of α, this deviation is not accepted by agents of type g. Assumption 1 implies that $\frac{1 - \pi_{b,m}}{\pi_{b,m}} > \alpha$, hence the deviation is not profitable if agents of type b accept it together with type m agents. Consider the following deviation of agents of type b accept τ^m together with trades leading to c^b (which is always available even following a deviation of an incumbent firm). We have that from Assumption 1 $U^b(c^b + \tau^m) > U^b(c^b)$ so that agents of type b accept the deviation making it unprofitable.

5   Conclusion

In this paper we characterize the equilibrium of a standard adverse selection economy in which agents can sign simultaneous insurance contracts with more than one firm. We consider an environment with free entry in the insurance market and with three types of agents: a good a medium and a bad type. Worse types represent a higher probability of receiving the low endowment. Agents are privately informed on their own types prior to signing any insurance contract. In this environment we show that there is no pooling equilibrium and that under certain parameter restrictions there is a unique equilibrium consumption allocation. When those parameter restrictions are violated an equilibrium fails to exists. In the unique equilibrium, the bad type receives full insurance at his actuarially-fair price. The good and medium type receive no insurance. Overall in this environment, when an equilibrium exists, the amount of insurance provided in equilibrium is reduced when compared with the environment in which agents sign exclusive contracts as in Rothschild and Stiglitz (1976). An important message of this paper is that non-exclusivity of contracts imposes strong restrictions on the insurance contracts that are offered, reducing drastically the provision of insurance. The non-exclusivity friction discussed in this paper can then be viewed as a positive institutional foundation for the strong regulations observed in data against the multiplicity of insurance contracts observed in several insurance markets, such as property

and health insurance.

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Appendix

A   Proofs of Section 3

Proof of Lemma 1

Proof. Suppose a pooling equilibrium c is such that equation (4) does not hold. In this case we have that $c_H > c_L$. This implies there exists an $\alpha > 0$ so that:

$$\frac{1 - \pi_b}{\pi_b} \frac{u'(c_L)}{u'(c_H)} > \alpha > \frac{1 - \pi_b}{\pi_b}. \tag{16}$$

Consider a firm not originally active in equilibrium, an entrant, deviating and offering a menu comprised of the null contract $(0,0)$ and $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ for some small $\varepsilon > 0$ with α defined above. The contract $\hat{\tau}$ is chosen by agents of type b together with the original pooling equilibrium. To see this: $U^b(c + \hat{\tau}) = \pi_b u(c_H - \alpha\varepsilon) + (1 - \pi_b) u(c_L + \varepsilon)$, expanding for small values of ε we have: $U^b(c + \hat{\tau}) = U^b(c) + \varepsilon\left[-\pi_b u'(c_H)\alpha + (1 - \pi_b) u'(c_L)\right] + \mathcal{O}(\varepsilon^2)$, from the first inequality in (16), ε can be chosen small enough so that $U^b(c + \hat{\tau}) > U^b(c)$. Let $\overline{\Pi}$ be the profit of the entrant. Since $\varepsilon > 0$, $\hat{\tau}$ constitutes positive insurance hence minimum profits for the entrant occur when only agents of type b accept $\hat{\tau}$. So that $\overline{\Pi} \geq \Pi^b(\hat{\tau}) = \pi_b\alpha\varepsilon - (1 - \pi_b)\varepsilon > 0$. Where the strict inequality follows from the second inequality of (16). Since a profitable deviation exists we reach a contradiction with c being a pooling equilibrium.

Let $\pi_m \leq \pi$. Suppose a pooling equilibrium c exist where (5) does not hold. This implies there exists and $\alpha > 0$ so that

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} < \alpha < \frac{1 - \pi}{\pi}. \tag{17}$$

As in the previous case, consider an entrant offering $\hat{\tau} = c - \omega + (-\varepsilon, \alpha\varepsilon)$ for some small $\varepsilon > 0$ and α defined above. In this case, $\hat{\tau}$ is accepted by agents of type g. To see this we have $U^g(\omega + \hat{\tau}) = \pi_g u(c_H + \alpha\varepsilon) + (1 - \pi_g) u(c_L - \varepsilon)$ expanding for small values of ε:

$$U^g(\omega + \hat{\tau}) = U^g(c) + \varepsilon\pi_g u'(c_H)\left[\alpha - \frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)}\right] + \mathcal{O}(\varepsilon^2) > U^g(c),$$

where the strict inequality follows from the first inequality in (17). Let $\overline{\Pi}$ be the profits of the entrant. Let π_x the probability of receiving a high realization of the endowment given the types of agents that accept the entrant's menu. By definition $\overline{\Pi}$ can be rewritten as:

$$\begin{aligned}
\overline{\Pi} &= \pi_x(\omega_H - c_H - \alpha\varepsilon) + (1 - \pi_x)(\omega_L - c_L + \varepsilon) \\
&= \pi_x(\omega_H - c_H) + (1 - \pi_x)(\omega_L - c_L) + \varepsilon(1 - \pi_x - \pi_x\alpha).
\end{aligned}$$

Since agents of type g accept $\hat{\tau}$, π_x is equal to one of the following $\{\pi_g, \pi, \pi_{b,g}, \pi_{m,g}\}$. From equation (4) it follows that $(\omega_H - c_H) > (\omega_L - c_L)$, this implies that for small enough ε, profits are increasing in π_x. From our assumption of $\pi_m \leq \pi$ we have that $\pi \leq \pi_{b,g}$. Minimum profits are achieved when $\pi_x = \pi$: all agents accept the entrant contract. This implies: $\overline{\Pi} \geq \Pi(c - \omega) + \varepsilon(1 - \pi - \pi\alpha) > \Pi(c - \omega) \geq 0$, where the second inequality is given by the second inequality in equation (17) and the third inequality from the condition on aggregate equilibrium profits $\Pi(c - \omega)$ being non negative. Since a profitable deviation exists we reach a contradiction with c being a pooling equilibrium.

Let $\pi_m > \pi$. Suppose (6) does not hold. This implies there exists an $\alpha > 0$ so that:

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(c_L)}{u'(c_H)} < \alpha < \frac{1 - \pi}{\pi}, \tag{18}$$

similarly to the previous case, consider an entrant offering $\hat{\tau} = c - \omega + (-\varepsilon, \alpha\varepsilon)$ with $\varepsilon > 0$ and small. Given (18) and the fact that $\frac{1 - \pi_g}{\pi_g} < \frac{1 - \pi_m}{\pi_m}$ there exist an α satisfying equation (17). Proceeding as in the previous case we can show that the entrant contract is accepted by both agents of type m and g. In this case minimum profits for the entrant are achieved when all agents accept $\hat{\tau}$. Hence, as in the previous case, the entrant always makes a strictly positive profit. Since a profitable deviation exists we reach a contradiction with c being a pooling equilibrium. $\qquad\square$

Proof of Lemma 2

Proof. Suppose condition (8) does not hold, so that $c_H^g < c_L^g$ (the consumption of type g is in the overinsurance region). If so, there exists an α so that:

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L^g)}{u'(c_H^g)} < \alpha < \frac{1 - \pi_g}{\pi_g}. \tag{19}$$

Consider an entrant offering $\hat{\tau} = (-\varepsilon, \alpha\varepsilon)$. This menu constitutes a form of negative insurance. For small enough ε, we have that $U^g(c^g + \hat{\tau}) > U^g(c^g)$, so that agents of type g accept the entrant's contract. Minimum profits from $\hat{\tau}$ occur when only agents of type g accept it. Profits from the deviation $\overline{\Pi}$, are such that $\overline{\Pi} \geq -\pi_g\alpha\varepsilon + (1 - \pi_g)\varepsilon > 0$, where the strict inequality follows from the second inequality in (19). We thus reach a contradiction having found a profitable deviation. The proof of condition (9) follows the same steps of the proof of (4) in Lemma 1.

We next prove the condition in equation (10). We focus on the relation between quantities for the agents of type g and m. The proof for the relation between quantities for agents of type m and b is analogous. By contradiction suppose that $c_H^g < c_H^m$, from (7) it must also be the case that $c_L^m < c_L^g$. In this case we have that:

$$u(c_H^m) - u(c_L^m) > u(c_H^g) - u(c_L^g). \tag{20}$$

From (7) we also have that $U^m(c^m) \geq U^m(c^g)$ and $U^g(c^g) \geq U^g(c^m)$, summing these two

inequalities we get $(\pi_m - \pi_g) [u(c_H^m) - u(c_L^m) - (u(c_H^g) - u(c_L^g))] \geq 0$. Substituting (20) we get $\pi_m \geq \pi_g$ a contradiction since by assumption $\pi_g > \pi_m$. $\qquad\square$

The following lemmas determine restrictions on the equilibrium allocation derived from profitable deviations of entrants. If any of the restrictions were not to hold, an entrant finds it profitable to offer either additional or substitute contracts. Additional contracts offer additional amounts of positive or negative insurance. These contracts are always chosen in combination with contracts already offered in equilibrium. Substitute contracts, as the name suggests, are accepted instead of the equilibrium allocation. We begin with Lemma 3. Conditions (3.B), (3.C) and (3.D) were discussed in the body of the paper. We also show condition (3.A) which states that the profits associated with the contract of the bad type cannot be positive. If is violated, an entrant can provide a substitute contract to agents of type b that provides additional insurance for type b and is always profitable.

Lemma 3. *Any separating equilibrium allocation* $c = \{c^b, c^m, c^g\}$ *must satisfy:*

$$\Pi^b(c^b - \omega) \leq 0; \tag{3.A}$$

$$\forall\, j = g, m, \qquad \Pi^b(c^b - c^j) \leq 0; \tag{3.B}$$

$$\Pi^g(c^g - c^m) \leq 0; \tag{3.C}$$

$$\Pi^{b,m}(c^m - c^g) \leq 0. \tag{3.D}$$

Proof.

Proof of (3.A)
Suppose (3.A) does not hold, we then have $\Pi^b(c^b - \omega) > 0$. An entrant can offer the following contract $\hat{\tau} = c^b - \omega + (\varepsilon, 0)$. With $\varepsilon > 0$ and small. We have that $U^b(\omega + \hat{\tau}) > U^b(c^b)$: agents of type b prefer $\hat{\tau}$ to their original equilibrium allocation. In addition, since c^b is in the overinsurance region from Lemma 2, profits for the entrant $(\overline{\Pi})$ are such that $\overline{\Pi} \geq \Pi^b(c^b - \omega) + \Pi^b((\varepsilon, 0)) > 0$ when ε is sufficiently small. We thus reach a contradiction having found a deviation that is always profitable.

Proof of (3.B)
Suppose equation (3.B) does not hold. We then have $\pi_b(c_H^j - c_H^b) + (1 - \pi_b)(c_L^j - c_L^b) > 0$. Consider an entrant deviating and offering $\hat{\tau} = c^b - c^j + (0, \varepsilon)$ with $\varepsilon > 0$ and small. $\hat{\tau}$ allows an agent to reach consumption level c^b starting from c^j. In this case we have $U^b(c^j + \hat{\tau}) = \pi_b u(c_H^b + \varepsilon) + (1 - \pi_b) u(c_L^b) > U^b(c^b)$ so that agents of type b pick $\hat{\tau}$ together with the allocation originally chosen by agents of type j. As in the proof of Lemma 1, let π_x be the probability of receiving a high realization given the types of agents that accept the entrant's menu. Profits for the entrant are given by $\overline{\Pi} = \pi_x(c_H^j - c_H^b - \varepsilon) + (1 - \pi_x)(c_L^j - c_L^b)$. For small enough ε profits are increasing in π_x. This follows from (10) and the fact that from the contradicting assumption we cannot simultaneously have $c_H^j = c_H^b$ and $c_L^j = c_L^b$; hence minimum profits are achieved when only agents of type b accept $\hat{\tau}$: $\overline{\Pi} \geq \pi_b(c_H^j - c_H^b - \varepsilon) + (1 - \pi_b)(c_L^j - c_L^b) > 0$. Where the last inequality follows from the contradicting assumption and ε sufficiently small.

We thus reach a contradiction having found a deviation that is always profitable.

Proof of (3.C)

Suppose by contradiction (3.C) does not hold: $\pi_g(c_H^m - c_H^g) + (1 - \pi_g)(c_L^m - c_L^g) > 0$. This implies that we cannot simultaneously have $c_L^m = c_L^g$ and $c_H^m = c_H^g$. In this case, consider the following contract offered by an entrant $\hat\tau = c^g - c^m + (0, \varepsilon)$ with $\varepsilon > 0$ positive and small. Since $U^g(c^m + \hat\tau) = \pi_g u(c_H^g + \varepsilon) + (1 - \pi_g) u(c_L^g) > U^g(c^g)$, agents of type g accept the entrant's contract. Profits for the entrant are given by: $\overline{\Pi} = \pi_x(c_H^m - c_H^g - \varepsilon) + (1 - \pi_x)(c_L^m - c_L^g)$, with π_x as above. From (10) we have that $c_H^g \geq c_H^m$ and $c_L^g \leq c_L^m$ together with the contradicting assumption, it implies that profit are decreasing in π_x, minimum profits are achieved when only the g type accepts $\hat\tau$, so that $\overline{\Pi} \geq \pi_g(c_H^m - c_H^g - \varepsilon) + (1 - \pi_g)(c_L^m - c_L^g) > 0$. Where the last inequality follows from the contradicting assumption and a sufficiently small ε. We reach a contradiction having found a profitable entry.

Proof of (3.D)

Suppose that by contradiction (3.D) is violated, then: $\Pi^{b,m}(c^m - c^g) > 0$. Consider an entrant deviating and offering the contract $\hat\tau = c^m - c^g + (0, \varepsilon)$ with $\varepsilon > 0$ and small. This contract is accepted by m types together with c^g. From the contradicting assumption we have that $\Pi^{b,m}(c^m - c^g) > 0$ so that we cannot simultaneously have $c_L^m = c_L^g$ and $c_H^m = c_H^g$, then from (10) and for ε sufficiently small it follows that profits are positive for any additional type that also accepts the contract. We then reach a contradiction having found a deviation that is always profitable. $\square$

We now move to Lemma 4. Condition (4.A) determines that agents of type m must be purchasing either no insurance or positive insurance, else aggregate profits (defined in (3)) must be negative. The second condition (4.B) shows that the profits associated with the allocation of either types m or g must be weakly positive if all agents accept it. This result is a direct implication of conditions (3.B) and (3.D). We also show in (4.C) that aggregate profits must be equal zero, if not, additional insurance can be offered to types m and g. The last conditions, (4.D) and (4.E), strengthen the results showed in Lemma 3.

Lemma 4. *The equilibrium allocation satisfies:*

$$c_L^m \geq \omega_L; \qquad c_L^g \geq \omega_L. \tag{4.A}$$

$$\forall\, j = g, m, \qquad \Pi(c^j - \omega) \geq 0. \tag{4.B}$$

$$\Pi = \sum_{j=b,m,g} p_j \Pi^j (c^j - \omega) = 0. \tag{4.C}$$

$$\Pi^b(c^b - c^m) = 0. \tag{4.D}$$

$$\Pi^{b,m}(c^m - c^g) = 0. \tag{4.E}$$

Proof.

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Proof of (4.A)

We first show that $c_L^m \geq \omega_L$. Suppose not. From Lemma 2 it follows that $c_L^g < \omega_L$. Since $U^j(c^j) \geq U^j(\omega)$ for $j = m, g$, it follows that $\Pi^j(c^j - \omega) < 0$ for $j = m, g$. In addition, since $\Pi^b(c^b - \omega) \leq 0$ from (3.A), aggregate profits $\Pi = \sum_{j=b,m,g} p_j \Pi^j(c^j - \omega) < 0$. Having reached a contradiction, we conclude $c_L^m \geq \omega_L$.

We next show that $c_L^g \geq \omega_L$. Suppose not. Write aggregate profits as $\Pi = p_g \Pi^g(c^g - \omega) + (p_b + p_m)\Pi^{b,m}(c^m - \omega) + p_b \Pi^b(c^b - c^m)$. Since $c_L^g < \omega_L$ and $U^g(c^g) \geq U^g(\omega)$, we have that $\Pi^g(c^g - \omega) < 0$. Also $\Pi^b(c^b - c^m) \leq 0$ from (3.B) and $\Pi \geq 0$ this implies that $\Pi^{b,m}(c^m - \omega) > 0$. From the previous step, it follows $c_L^m > \omega_L$. Consider an entrant offering $\tau^m = c^m - \omega + (\varepsilon, 0)$ with $\varepsilon > 0$ and small. The entry is accepted by agents of type m and remains profitable for any types that accepts it. We thus reach a contradiction.

Proof of (4.B)

To show (4.B) holds for $j = g$, rewrite aggregate profits defined in (3) as:

$$\Pi = p_b \Pi^b(c^b - c^m) + (p_b + p_m)\Pi^{b,m}(c^m - c^g) + \Pi(c^g - \omega). \tag{21}$$

The above can be interpreted as the profits originating from all agents choosing c^g; agents of type b and m choosing the transfer $c^m - c^g$ and finally agents of type b choosing the transfer $c^b - c^m$. Using conditions (3.B) and (3.D) in the above together with $\Pi \geq 0$ implies that $\Pi(c^g - \omega) \geq 0$. To show (4.B) holds for $j = m$, rewrite aggregate profit as $\Pi = p_b \Pi^b(c^b - c^m) + \Pi(c^m - \omega) + p_g \Pi^g(c^g - c^m)$. Using conditions (3.B) and (3.C) in the previous equation together with $\Pi \geq 0$ implies that $\Pi(c^m - \omega) \geq 0$.

Proof of (4.C)

Suppose (4.C) does not hold: $\Pi > 0$. Consider an entrant offering a menu $\hat{\tau} = \{\tau^g, \tau^m\}$, where $\tau^g = c^g - \omega + (\varepsilon, 0)$ and $\tau^m = c^m - \omega + (\varepsilon, 0)$, where $\varepsilon > 0$ and small. Suppose that agents of type g accept τ^g. From (3.D) and (4.A) it follows that minimum profits are achieved when agents of type b and m accept τ^m. In this case the profits of the entrant $(\overline{\Pi})$ satisfy:

$$\overline{\Pi} = \Pi(c^m - \omega) + p_g \Pi^g(c^m - c^g) + \Pi((\varepsilon, 0)) \geq \Pi + \Pi((\varepsilon, 0)) > 0.$$

Where the weak inequality follows from rewriting profits as $\Pi = \Pi(c^m - \omega) + p_g \Pi^g(c^m - c^g) + p_b \Pi^b(c^b - c^m)$ and $\Pi^b(c^b - c^m) \leq 0$ from (3.B) for $j = m$. The last (strict) inequality follows from the contradicting assumption $\Pi > 0$ together with ε chosen small enough.

Suppose now that agents of type g prefer τ^m to τ^g. In this case, the entrant offers $\hat{\tau} = \tau^m$. Agents of type g and m accept τ^m. We have that $\Pi = \Pi(c^m - \omega) + p_b \Pi^b(c^b - c^m) + p_g \Pi^g(c^g - c^m)$. Since $\Pi > 0$ from the contradicting assumption and $\Pi^b(c^b - c^m) \leq 0$ from (3.B), using (3.C) it follows that $\Pi(c^m - \omega) > 0$. Since from (4.A) $c_L^m \geq \omega_L$, profits from the entry satisfy $\overline{\Pi} \geq \Pi(c^m - \omega) + \Pi((\varepsilon, 0)) > 0$: the deviation makes positive profits for small enough ε. Having found a profitable entry we reach a contradiction so that (4.C) holds.

Proof of (4.D)

If (4.D) does not hold, from (3.B) for $j = m$, it follows that $\Pi^b(c^b - c^m) < 0$. Consider an entrant offering a menu $\hat{\tau} = \{\tau^g, \tau^m\}$, where $\tau^g = c^g - \omega + (\varepsilon, 0)$ and $\tau^m = c^m - \omega + (\varepsilon, 0)$, where $\varepsilon > 0$ and small. We have that $U^j(\tau^j + \omega) > U^j(c^j)$ for $j = m, g$. Suppose, as a first case, that agents of type g prefer τ^g over τ^m. From (3.D) it follows that minimum profits are achieved when agents of type b and m accept τ^m. In this case (if agents of type b do not accept any contract from the entrant the proof follows similarly) total profits for the entrant ($\overline{\Pi}$) satisfy the following: $\overline{\Pi} \geq \Pi - p_b \Pi^b(c^b - c^m) - \Pi((\varepsilon, 0)) > \Pi \geq 0$, where the strict inequality follows from $\Pi^b(c^b - c^m) < 0$ and ε sufficiently small. Hence the entry makes strictly positive profits.

Suppose now agents of type g prefer τ^m to τ^g. Consider an entrant offering $\hat{\tau} = \tau^m$. Suppose first that $c_L^m > \omega_L$. Since agents of type m accept τ^m, minimum profits are achieved when agents of type b also accept τ^m. Profits for the entrant satisfy: $\overline{\Pi} \geq \Pi(\tau^m) > 0$, where the second inequality follows from (4.B) together with $\Pi \geq 0$ and the assumption that $\Pi^b(c^b - c^m) < 0$. Suppose now that $c_L^m = \omega_L$. From (4.A) this implies $c_L^g = \omega_L$ and $c_H^m = c_H^g = \omega_H$. We can write aggregate profits for the incumbent as $\Pi = p_b \Pi^b(c^b - \omega)$. From the contradicting assumption follows that $\Pi < 0$, reaching a contradiction. Having found a profitable entry for every case, we reach a contradiction so that (4.D) holds.

Proof of (4.E)

If (4.E) does not hold, from (3.D) it must be the case that $\Pi^{b,m}(c^m - c^g) < 0$. We have that $\Pi = \Pi(c^g - \omega) + (p_b + p_m)\Pi^{b,m}(c^m - c^g) + p_b \Pi^b(c^b - c^m) \geq 0$. From (4.C), (4.D) and the contradicting assumption it follows that $\Pi(c^g - \omega) > 0$.

Suppose first $\pi_m \leq \pi$. Consider an entrant offering $\tau^g = c^g - \omega + (\varepsilon, 0)$. The entrant always make positive profits since under the assumption of $\pi_m \leq \pi$, $\Pi^{b,g}(\tau^g) \geq \Pi(\tau^g) > 0$. In addition $\min\{\Pi^g(\tau^g), \Pi^{m,g}(\tau^g)\} > \Pi(\tau^g) > 0$, so the deviation is always profitable. Having found a profitable entry we reach a contradiction.

Suppose now $\pi_m > \pi$. If $c_L^m = c_L^g$ the result is immediate. Suppose this is not the case, from Lemma 2, it follows that $c_L^m > c_L^g$. We consider two cases $c_L^m > c_H^m$ and $c_L^m \leq c_H^m$. Suppose first $c_L^m > c_H^m$. Define the following transfers: $\tau^m = c^m - \omega + (-\varepsilon_m, \alpha\varepsilon_m)$ with $\varepsilon_m > 0$ and α such that $\frac{1-\pi_m}{\pi_m}\frac{u'(c_L^m)}{u'(c_H^m)} < \alpha < \frac{1-\pi_m}{\pi_m}$; $\tau^g = c^g - \omega + (\varepsilon_g, 0)$ with $\varepsilon_g > 0$. Given these transfers, $U^m(\tau^m + \omega) > U^m(c^m)$ and $U^g(\tau^g + \omega) > U^g(c^g)$. Given that $\pi_m > \pi$, from (4.B) it follows that $\Pi^m(c^m - \omega) > 0$. We reach a contradiction (by choosing ε_m and ε_g sufficiently small) unless agents of type g prefer τ^g to τ^m and b, m prefer τ^m to τ^g. In this case suppose an entrant offers $\{\tau^m, \tau^g\}$. Profits from the deviation can be written as: $\overline{\Pi} = \Pi + p_g \Pi^g((\varepsilon_g, 0)) + (p_b + p_m)\Pi^{b,m}((-\varepsilon_m, \alpha\varepsilon_m)) > 0$. Where the last inequality follows from (4.C), the definition of α and by choosing ε_g small enough.

Consider now the case with $c_L^m \leq c_H^m$. This implies $c_L^g < c_H^g$ hence agents of type g are in the region of under-insurance. Consider the following contracts $\tau^m = c^m - \omega + (\varepsilon_m, 0)$ and $\tau^g = c^g - \omega + (\varepsilon_g, -\alpha_g \varepsilon_g)$, with $\varepsilon_g > 0$ and α_g such that $\frac{1-\pi_g}{\pi_g}\frac{u'(c_L^g)}{u'(c_H^g)} > \alpha_g > \frac{1-\pi_g}{\pi_g}$. If agents of type m accept a deviation comprised by τ^g we reach a contradiction. Suppose this is not

the case and consider a deviation comprised by $\{\tau^m, \tau^g\}$.

Suppose agents of type g accept τ^g. Suppose first that agents of type b accept τ^m. In this case profits for the entrant are given by: $\overline{\Pi} = \Pi + p_g \Pi^g((\varepsilon_g, -\alpha_g \varepsilon_g)) + (p_b + p_m)\Pi^{b,m}((-\varepsilon_m, 0)) > 0$. Where the last inequality follows from (4.C), the definition of α_g and by choosing ε_m small enough. Suppose now b and g choose τ_g and m chooses τ_m. Profits are $\overline{\Pi} = (p_b + p_g)\Pi^{b,g}(\tau^g) + p_m \Pi^m(\tau^m)$. Adding and subtracting $p_b \Pi^b(c^m - c^g)$ we get:

$$\overline{\Pi} = p_g \Pi^g(c^g - \omega) + (p_b + p_m)\Pi^{b,m}(c^m - \omega) - p_b \Pi^b(c^m - c^g) + \mathcal{O}(\varepsilon),$$

which (since from (4.D) $\Pi^b(c^b - c^m) = 0$) is equal to: $\overline{\Pi} = \Pi - p_b \Pi^b(c^m - c^g) + \mathcal{O}(\varepsilon) > 0$, where the inequality follows from the fact that $0 > \Pi^{b,m}(c^m - c^g) > \Pi^b(c^m - c^g)$ and choosing ε sufficiently small.

Finally suppose now agents of type g accept τ^m (in this case only τ^m is offered). The deviation is still profitable unless agent of type b accept τ^m and $\Pi(c^m - \omega) = 0$. In this case however from (4.C) and (4.D) it follows that $\Pi^g(c^g - c^m) = 0$. In this case an entrant offering $\tau^g = c^g - c^m + (\varepsilon_g, -\alpha_g \varepsilon_g)$ with ε_g and α_g defined above will always make a profit, from the definition of α_g and the fact that $c_L^g < c_L^m$.

$\square$

The previous Lemma implies the following:

Corollary 1. *In equilibrium: (i) $\Pi(c^g - \omega) = 0$. (ii) If $c_L^m > c_L^g$ then $\Pi(c^m - \omega) > 0$.*

Proof. To show *(i)* write Aggregate profits as $\Pi = \Pi(c^g - \omega) + (p_b + p_m)\Pi^{b,m}(c^m - c^g) + p_b \Pi^b(c^b - c^m)$. From (4.C), (4.D) and (4.E) it follows that $\Pi(c^g - \omega) = 0$. Condition *(ii)* follows immediately from *(i)* and (4.E). $\square$

A.1 Proof of Proposition 2

Before proving Proposition 2 we prove a supporting lemma. Proposition 2 argues that the unique equilibrium allocation features agents of type g and m pooling at the endowment point $c^g = c^m = \omega$ and agents of type b reaching full insurance at the actuarially fair price for their own type. The proof is by contradiction. For this purpose Lemma 5 characterizes an implication that originates if the equilibrium differs from the proposed one and additional properties that must hold in equilibrium: condition (5.A) considers the case when agent of type g and m are not pooling. Also (5.B) shows that agents of type b receive more insurance than agents of type m, while (5.C) shows that agents of type must receive full-insurance.

Lemma 5.

(A) *If in equilibrium $c_L^m > c_L^g$, then:*

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(c_L^m)}{u'(c_H^m)} = \frac{1 - \pi_{b,m}}{\pi_{b,m}}. \tag{5.A}$$

(B) *In any equilibrium:*

$$c_L^m < c_L^b. \tag{5.B}$$

(C) *In any equilibrium:*

$$c_L^b = c_H^b. \tag{5.C}$$

Proof.

Proof of (5.A)

Suppose condition (5.A) is violated. As a first case suppose there exists an $\alpha > 0$ so that: $\frac{1-\pi_m}{\pi_m} \frac{u'(c_L^m)}{u'(c_H^m)} > \alpha > \frac{1-\pi_{b,m}}{\pi_{b,m}}$. Given this α, consider an entrant offering $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small. The contract $\hat{\tau}$ is accepted by agents of type m. Minimum profits are realized when only agents of type b and m accept the contract. In this case, from the definition of α, the entrant makes positive profits which is a contradiction. Suppose now that there exist an $\alpha > 0$ such that:

$$\frac{1-\pi_m}{\pi_m} \frac{u'(c_L^m)}{u'(c_H^m)} < \alpha < \frac{1-\pi_{b,m}}{\pi_{b,m}}. \tag{22}$$

From (4.E) we have that $\Pi^{b,m}(c^m - c^g) = 0$. Suppose an entrant offers $\tau^m = c^m - c^g + (-\varepsilon, \alpha\varepsilon)$ with $\varepsilon > 0$ and small; agents of type m strictly prefer $c^g + \tau^m$ to c^m. Since $c_L^m > c_L^g$ for small enough ε we have that τ^m constitutes positive insurance. Minimum profits are realized when also agents of type b accept τ^m. In this case we have: $\overline{\Pi} = (p_b + p_m)\Pi^{b,m}(c^m - c^g) + (p_b + p_m)\Pi^{b,m}((-\varepsilon, \alpha\varepsilon)) > 0$, where the strict inequality follows from $\Pi^{b,m}(c^m - c^g) = 0$ and the definition of α.

Proof of (5.B)

Suppose not, then from (10) in Lemma 2, $c_L^m = c_L^b$. Given the choices of the agents, this implies $c_H^m = c_H^b$. From (10) in Lemma 2 and Proposition 1 it follows that $c_L^m > c_L^g$. Using (5.A) we conclude that $c_L^m < c_H^m$ since $\pi_{b,m} < \pi_m$. This implies $c_L^b < c_H^b$ contradicting (9) in Lemma 2.

Proof of (5.C)

Suppose not, then from (9) in Lemma 2, $c_L^b > c_H^b$. So that there exists an α such that $\frac{1-\pi_b}{\pi_b} \frac{u'(c_L^b)}{u'(c_H^b)} < \alpha < \frac{1-\pi_b}{\pi_b}$. Consider an entrant offering the contract $\hat{\tau} = c^b - c^m + (-\varepsilon, \alpha\varepsilon)$. Since $c_L^b > c_L^m$ from (5.B), then $\hat{\tau}$ constitutes positive insurance. Minimum profits are given when only agents of type b accept it and are given by $\overline{\Pi} = \Pi^b(c^b - c^m) + \Pi^b(-\varepsilon, \alpha\varepsilon) > 0$, where the inequality follows from (4.D) and the definition of α.

$\square$

We now go to the proof of Proposition 2.

Proof. The proof is divided in two steps. In Step 1 we show that $c^g = \omega$, in Step 2 we show that $c^m = \omega$. The result then follows from (4.C) and (5.C).

Step 1 Let $c^g = \sum_{i \in I^g} \tau^{g,i} + \omega$ with $I^g \subseteq I$. Let $I^+ \subseteq I^g$ be the set of i for which $\tau_L^{g,i} > 0$, similarly let $I^- \subseteq$ be the set of i for which $\tau_L^{g,i} < 0$. We first show that if $I^+ \neq \emptyset$ then there exists an $i \in I^+$ such that $\Pi(\tau^{g,i}) \geq 0$. Suppose not, so that for all $i \in I^+$ we have that $\Pi(\tau^{g,i}) < 0$. Since from from Corollary 1, $\Pi(c^g - \omega) = 0$. It follows that $I^- \neq \emptyset$ and $\tau^- = \sum_{i \in I^-} \tau^{g,i}$ is such that $\Pi(\tau^-) > 0$ and $\tau_L^- < 0$.

Consider an entrant offering the following two contracts: $\hat{\tau}^m = \tau^- + (\varepsilon, 0) + c^m - c^g$ and $\hat{\tau}^b = \tau^- + (\varepsilon, 0) + c^b - c^g$ with $\varepsilon > 0$ and small. For $j = m, g$, we have that $\hat{\tau}^j$ plus additional transfers from all incumbents in I^+ is preferred to c^j by agents of type j. If types b and m respectively get $\hat{\tau}^b$ and $\hat{\tau}^m$, profits from the deviation are:

$$\overline{\Pi} \geq \Pi(\tau^-) + (p_b + p_m)\Pi^{b,m}(c^m - c^g) + p_b\Pi^b(c^b - c^m) + \Pi^{b,m}((\varepsilon, 0)) > 0.$$

where the inequalities follows from $\Pi^{b,m}(\tau^-) \geq \Pi(\tau^-) > 0$, from (4.D), (4.E) and choosing ε sufficiently small. Profits are also positive if agents choose different $\hat{\tau}^j$.

We next show that $I^+ = \emptyset$. Suppose not, from the previous step let i' be the incumbent offering $\tau^{g,i'}$ such that $\tau_L^{g,i'} > 0$ and $\Pi(\tau^{g,i'}) \geq 0$. From (5.B), (5.C) and (10) in Lemma 2, we have that agents of type g are strictly in the underinsurance region: $c_H^g > c_L^g$. Suppose an entrant offers $\hat{\tau}^g = \tau^{g,i'} + (\varepsilon_g, -\alpha\varepsilon_g)$ with $\varepsilon_g > 0$ and small and α satisfies $\frac{1-\pi_g}{\pi_g}\frac{u'(c_L^g)}{u'(c_H^g)} > \alpha > \frac{1-\pi_g}{\pi_g}$. Agents of type g accept this deviation together with contracts $I^g \setminus i'$ available with the incumbents. If the deviation is not accepted by agents of type b, it is profitable thus reaching a contradiction. Suppose then agents of type b accept $\hat{\tau}^g$. There must exist τ' so that $U^b(\hat{\tau}^g + \tau' + \omega) \geq U^b(c^b)$. This implies that $U^b(\tau^{g,i'} + \tau' + \omega) \geq U^b(c^b)$.

Suppose first $U^b(\tau^{g,i'} + \tau' + \omega) > U^b(c^b)$. Let $\tau' = \sum_{j \in I'} \tau'^j$, with $I' \subseteq I$. Incumbent i' must be necessary to reach τ'. That is, $i' \in I'$ and does not exist another $I'' \subseteq I \setminus i'$ so that $\tau' = \sum_{j \in I''} \tau^j$, for some τ^j offered by firm j. If not, then agents of type b, by trading with i' and firms in I'', are able to reach $\tau^{g,i'} + \tau'$, which is strictly preferred to c^b, reaching a contradiction. Since i' is necessary to reach τ', i' can deviate and instead of $\tau^{g,i'}$ offer $\hat{\tau}^g$. Given the condition on α, it is accepted by type g but not by type b and generates positive profits for firm i'. This is a contradiction, since aggregate profits are zero and since no firm makes strictly negative profit, it follows that every active firm in equilibrium makes zero profits.

Suppose that $U^b(\tau^{g,i'} + \tau' + \omega) = U^b(c^b)$. Suppose an entrant deviates and offers $\hat{\tau} = \{\hat{\tau}^b, \hat{\tau}^g\}$ with $\hat{\tau}^b = c^b - c^m + (\varepsilon_b, 0)$ and $\hat{\tau}^g$ as defined before.[24] If agents of type g accept $\hat{\tau}^g$ and agents of type b accept $\hat{\tau}^b$, minimum profits are achieved when agents of type m also accept $\hat{\tau}^g$.[25] In this case profits for the entrant are given by: $(p_m + p_g)\Pi^{m,g}(\hat{\tau}^g) + p_b\Pi^b(c^b - c^m) + p_b\Pi^b((\varepsilon_b, 0)) > 0$. Where the strict inequality follows from the fact that $\Pi^{m,g}(\hat{\tau}^g) > \Pi(\hat{\tau}^g) > 0$ and $\varepsilon_g, \varepsilon_b$ small enough. We next show that is possible to choose ε_b so that agents of type b

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[24]The two type deviation adopted is similar in spirit to Attar, Mariotti, and Salanié (2014). The key difference here is that the response of each type needs to be considered in a case by case basis. Refer also to Attar, Mariotti, and Salanié (2014) Section 5.3.

[25]If agents of type g accept $\hat{\tau}^b$ profits are positive from (4.D) and (5.B).

prefer $\hat{\tau}^b$ to $\hat{\tau}^g$. We have that $U^b(\hat{\tau}^g + \omega + \tau') - U^b(c^b) = \varepsilon_g\{-\pi_b\alpha u'(\tau_H^{g,i'} + \omega_H + \tau_H') + (1 - \pi_b)u'(\tau_L^{g,i'} + \omega_L + \tau_L')\} + \mathcal{O}(\varepsilon_g^2)$. Also $U^b(\hat{\tau}^b + c^m) - U^b(c^b) = \varepsilon_m\{(1 - \pi_b)u'(c_L^b)\} + \mathcal{O}(\varepsilon_m^2)$. So $U^b(\hat{\tau}^b + c^m) > U^b(\hat{\tau}^g + \omega + \tau')$ if:

$$\varepsilon_b > \varepsilon_g \max\left\{\frac{-\pi_b\alpha u'(\tau_H^{g,i'} + \omega_H + \tau_H') + (1 - \pi_b)u'(\tau_L^{g,i'} + \omega_L + \tau_L')}{(1 - \pi_b)u'(c_L^b)}, 1\right\}.$$

Note that ε_b is of the same order of ε_g. Having found a profitable deviation we reach a contradiction.

This implies that $I^+ = \emptyset$. From (4.A) we must have that $I^- = \emptyset$. This implies that $c^g = \omega$. In addition, for all $i \in I^g$, $\tau^{g,i} = (0, 0)$.

Step 2 We now show $c^m = \omega$. Suppose not, so that $c_L^m > \omega_L$. From the previous step and (4.E) it follows $\Pi^{b,m}(c^m - \omega) = 0$ so that $\Pi^m(c^m - \omega) > 0$. Let $c^m = \sum_{i \in I^m} \tau^{m,i} + \omega$ Since $\Pi^m(c^m - \omega) > 0$, it must exist at least one $i' \in I^m$ for which $\Pi^m(\tau^{m,i'}) > 0$. Suppose first that $\tau_L^{m,i'} < 0$. Since firm i' makes zero profits and since from the previous step $\tau^{g,i'} = (0, 0)$, it follows that $\Pi^b(\tau^{b,i'}) < 0$. Consider the deviation of i' in which it withdraws $\tau^{b,i'}$. In this case either agents of type b accept $\tau^{m,i'}$ or (without loss of generality) accept $\tau^{g,i'}$. Since $\Pi^{b,m}(\tau^{m,i'}) > 0$, under both cases the profits for firm i' are increased leading to a contradiction.

Suppose now that $\tau_L^{m,i'} > 0$. Suppose an entrant offers $\hat{\tau}^m = \tau^{m,i'} + (\varepsilon_m, -\alpha\varepsilon_m)$ where α satisfies $\frac{1-\pi_m}{\pi_m}\frac{u'(c_L^m)}{u'(c_H^m)} > \alpha > \frac{1-\pi_m}{\pi_m}$. Since from (5.B), (5.C) and (10) in Lemma 2, agents of type m are underinsured, for small enough ε_m, $U^m(\hat{\tau}^m + \sum_{i \in I^m\setminus i'} \tau^{m,i} + \omega) > U^m(c^m)$. The deviation is profitable unless agents of type b also accept it (recall that $\tau_L^{m,i'} > 0$ so that $\Pi^{m,g}(\tau^{m,i'}) > \Pi^m(\tau^{m,i'}) > 0$). Hence, there must exist additional contracts available with the incumbents summing to a net trade τ', so that $U^b(\hat{\tau}^m + \omega + \tau') \geq U^b(c^b)$. Since $\hat{\tau}^m$ is arbitrarily close to $\tau^{m,i'}$, it must be the case that $U^b(\tau^{m,i'} + \omega + \tau') \geq U^b(c^b)$. Consider first the case in which $U^b(\tau^{m,i'} + \omega + \tau') > U^b(c^b)$. As in step 1, incumbent i' must be necessary to reach τ'. If not then agents of type b trading with i' and firms in I'' are able to reach $\tau^{m,i'} + \tau'$, which is strictly preferred to c^b, reaching a contradiction. Since i' is necessary to reach τ', i' can deviate and instead of $\tau^{m,i'}$ offer $\hat{\tau}^m$. Given the condition on α, it is accepted by type m. Since profits of i' are zero and since $\Pi^{m,g}(\hat{\tau}^{m,i'}) > 0$, the deviation is profitable.

We now consider the case in which $U^b(\tau^{m,i'} + \omega + \tau') = U^b(c^b)$. An entrant can deviate offering $\hat{\tau} = \{\hat{\tau}^b, \hat{\tau}^m\}$, where $\hat{\tau}^m$ is defined above and $\hat{\tau}^b = c^b - c^m + (\varepsilon_b, 0)$ with $\varepsilon_b > 0$ and small. Since $U^b(\hat{\tau}^b + c^m) > U^b(c^b)$, agents of type b choose either $\hat{\tau}^m$ or $\hat{\tau}^b$ upon entry. If only agents of type b choose $\hat{\tau}^b$ and only agents of type m choose $\hat{\tau}^m$, profits for the entrant are $\overline{\Pi} = p_m\Pi^m(\hat{\tau}^m) + p_b\Pi^b(c^b - c^m) + p_b\Pi^b((\varepsilon_b, 0)) > 0$ where the strict inequality follows from the assumption $\Pi^m(\hat{\tau}^m) > 0$; $\Pi^b(c^b - c^m) = 0$ from (4.D) and choosing ε_b sufficiently small. We now show that is possible to choose ε_b and ε_m so that $U^b(\hat{\tau}^b + c^m) > U^b(\hat{\tau}^m + \omega + \tau')$. We have that $U^b(\hat{\tau}^m + \omega + \tau') - U^b(c^b) = \varepsilon_m\{-\pi_b\alpha u'(\tau_H^{m,i'} + \omega_H + \tau_H') + (1 - \pi_b)u'(\tau_L^{m,i'} + \omega_L + \tau_L')\} + \mathcal{O}(\varepsilon_m^2)$.

Also $U^b(\hat{\tau}^b + c^m) - U^b(c^b) = (1 - \pi_b)\varepsilon_b u'(c_L^b) + \mathcal{O}(\varepsilon_b^2)$. Let:

$$\varepsilon_b > \varepsilon_m \max \left\{ \frac{-\pi_b \alpha u'(\tau_H^{m,i'} + \omega_H + \tau_H') + (1 - \pi_b)u'(\tau_L^{m,i'} + \omega_L + \tau_L')}{(1 - \pi_b)u'(c_L^b)}, 1 \right\}.$$

Note that ε_b is of the order of ε_m. In addition the max in the definition of ε_b is required since the term $U^b(\hat{\tau}^m + \omega + \tau') - U^b(c^b)$ can be negative (for example if $\alpha = \frac{1 - \pi_b}{\pi_b}$ and $\hat{\tau}^m + \tau'$ is in the underinsurance region). For a given ε_m, the above choice of ε_b ensures that agents of type b accept $\hat{\tau}^b$. When agents of type g and/or m accept $\hat{\tau}^b$ the entry makes strictly positive profits. This is because from (5.B) $c_L^b > c_L^m$. Finally, if agents of type g accept $\hat{\tau}^m$ profits are increased since $\tau_L^{m,i'} > 0$. Hence no τ' exists so that $U^b(c^m + \tau') = U^b(c^b)$.

Given the above results, the only possible equilibrium allocation is one in which $c^g = c^m = \omega$. Then from (4.C) and (5.C), the allocation for type b agents is $c^b = \omega^b$, concluding the proof. $\qquad\square$

B   Proof of Proposition 4

Proof.
The proof is by construction. We first describe strategies adopted by firms, and choices of agents; then we show that no incumbent or entrant wishes to deviate from the proposed equilibrium. Consider the following strategies for the firms. Let firms $i = 1, 2$ offer the menu $C^i = L_b$ where the set L_b is defined as follows:

$$L_b = \left\{ x_L \geq 0, x_H \leq 0 \ \middle| \ -\pi_b x_H - (1 - \pi_b)x_L = 0 \right\}.$$

L_b is the set of all positive insurance contracts priced at the actuarially fair price for agents of type b. All remaining firms $i \neq 1, 2$ offer the menu: $C^i = (0, 0)$. Let $\tau_L^b = \pi_b(\omega_H - \omega_L)$, $\tau_H^b = (1 - \pi_b)(\omega_L - \omega_H)$, it is easy to show that under Assumption 1, the agents strategies are (without loss of generality): agents of type b choose $(\tau_L^b/2, \tau_H^b/2)$ from both firms 1 and 2 and $(0, 0)$ from remaining firms; types m and g choose $(0, 0)$ from all firms. In this equilibrium, all firms make zero profits, and agents of type b, m and g receive allocations c^b, c^m and c^g respectively.

Suppose that firm i deviates and offers a menu containing $\{\tau^b, \tau^m, \tau^g\}$, without loss of generality suppose that agents of type b, m and g pick contracts denoted respectively by τ^b, τ^m and τ^g (we don't exclude the case that any two might be equal or that any might equal $(0, 0)$). For firm i, profits are given by $\overline{\Pi} = p_b \Pi^b(\tau^b) + p_m \Pi^m(\tau^m) + p_g \Pi^g(\tau^g)$. We can rewrite profits as:

$$\overline{\Pi} = \Pi(\tau^g) + (p_m + p_b)\Pi^{b,m}(\tau^m - \tau^g) + p_b \Pi^b(\tau^b - \tau^m).$$

Our goal is to show that $\overline{\Pi} < 0$, we do this by showing that all the above terms are either weakly or strictly negative.

We first show that $\Pi^b(\tau^b - \tau^m) \leq 0$, suppose not. If $\tau_L^m = \tau_L^b$ the result follows immediately

since it must also be the case that $\tau_H^m = \tau_L^b$. Suppose first that $\tau_L^m < \tau_L^b$.[26] Denote by $\lambda^i(\tau) \in L_b$ the additional insurance that agent of type i accepts within L_b having accepted transfer τ from the entrant. Consider the following choice from agents of type b: accept τ^m from firm i and pick an additional transfer $\hat{\tau} + \lambda^b(\tau^b) \in L_b$ where $\hat{\tau} = [(\tau_L^b - \tau_L^m), -(1 - \pi_b)/\pi_b(\tau_L^b - \tau_L^m)]$. Consumption in the low state is $\hat{c}_L = \omega_L + \tau_L^b + \lambda_L^b(\tau^b)$: this strategy provides the same amount of consumption in the low state as the original choice of agents of type b accepting τ^b. Consumption in the high state is $\hat{c}_H = \omega_H + \frac{1-\pi_b}{\pi_b}(\tau_L^m - \tau_L^b) + \tau_H^m + \lambda_H^b(\tau^b)$. Since by the contradicting assumption $\Pi^b(\tau^b - \tau^m) > 0$ we have that $\frac{1-\pi_b}{\pi_b}(\tau_L^m - \tau_L^b) > \tau_H^b - \tau_H^m$ so that $\hat{c}_H > \omega_H + \tau_H^b + \lambda_H^b(\tau^b)$: this strategy provides strictly higher consumption in the high state. Hence we reach a contradiction with τ^b being preferred to τ^m by agents of type b. Suppose now that $\tau_L^m > \tau_L^b$, in this case the proof proceeds as before but now agents of type m accept τ^b and choose additional transfers in L_b. Having reached a contradiction it follows that $\Pi^b(\tau^b - \tau^m) \leq 0$.

We next show that $\Pi(\tau^g) < 0$. We first show that it must be the case that $\Pi^g(\tau^g) \geq 0$. Suppose not so that $\Pi^g(\tau^g) < 0$, since $\Pi^b(\tau^b) < 0$ it follows that $\Pi^m(\tau^m) > 0$ so $\tau_L^m > 0$. We have that $\overline{\Pi} < (p_b + p_m)\Pi^{b,m}(\tau^m) + p_b\Pi^b(\tau^b - \tau^m)$. From the previous step: $\Pi^b(\tau^b - \tau^m) \leq 0$, and also $\Pi^{b,m}(\tau^m) < 0$. To see this, from (15) in Assumption 1 and $\tau_L^m > 0$ it follows that $\Pi^{b,m}(\tau^m + \lambda^m(\tau^m)) < 0$. Since $\lambda^m(\tau^m) \in L_b$ and $\Pi^{b,m}(\lambda^m(\tau^m)) \geq 0$ which implies $\Pi^{b,m}(\tau^m) < 0$. This implies $\overline{\Pi} < 0$, reaching a contradiction hence it must be the case $\Pi^g(\tau^g) \geq 0$. Since $\Pi^g(\tau^g) \geq 0$ and $U^g(\omega + \tau^g + \lambda^g(\tau^g)) \geq U^g(\omega)$, then $\tau_L^g > 0$ and $\tau_H^g < 0$. Since $\tau_L^g > 0$ from (14) in Assumption 1 it thus follows that $\Pi(\tau^g + \lambda^g(\tau^g)) < 0$, because $\lambda^g(\tau^g) \in L_b$ it implies $\Pi(\lambda^g(\tau^g)) \geq 0$ hence $\Pi(\tau^g) < 0$.

Finally we show that $\Pi^{b,m}(\tau^m - \tau^g) < 0$. Given (15) in Assumption 1 and $\tau_L^g > 0$ we have:
$$\frac{1 - \pi_m}{\pi_m}\frac{u'(\omega_L + \tau_L^g)}{u'(\omega_H + \tau_H^g)} \leq \frac{1 - \pi_{b,m}}{\pi_{b,m}}. \tag{23}$$

This implies that $\lambda^m(\tau^g) = (0,0)$. This is because the slope of the indifference curve of agents of type m at $\omega + \tau^g$ is flatter than the slope of the zero profit line for contracts chosen by both b and m. Given the choices of agents of type m it must be the case that:

$$U^m(\omega + \tau^m + \lambda^m(\tau^m)) \geq U^m(\omega + \tau^g).$$

The above, together with equation (23), then implies that $\Pi^{b,m}(\tau^m + \lambda^m(\tau^m) - \tau^g) < 0$. Since $\lambda^m(\tau^m) \in L_b$, it follows that $\Pi^{b,m}(\lambda^m(\tau^m)) \geq 0$ so $\Pi^{b,m}(\tau^m - \tau^g) < 0$. Since $\Pi^{b,m}(\tau^m - \tau^g) < 0$, combining with the previous results that $\Pi(\tau^g) < 0$ and $\Pi^b(\tau^b - \tau^m) \leq 0$ it implies that profits of the deviation are strictly negative: $\overline{\Pi} < 0$ reaching a contradiction. Since there is no profitable deviation we conclude that (c^b, c^m, c^g) constitutes an equilibrium allocation.

$\square$

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[26]The relative ordering of τ_L^m and τ_L^b cannot be established as we have done in Lemma 2 since now final consumption for agents of type b and m also depends on the additional transfers available in L_b.