

# TARGETED VS. COLLECTIVE POSTING IN SOCIAL PLATFORMS\*

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## Abstract

We introduce a simple two-period game of endogenous network formation and private information sharing (i.e., posting) for reasoning about the optimal design of social platforms like Facebook, Google+, or Twitter. We distinguish between unilateral or bilateral connections, and between targeted or collective postings. Agents value being connected to other agents, and they value making and receiving posts. We study how the design of the social platform and the utility specifications affect welfare. Surprisingly, we find that in general, targeted posting is not necessarily better than collective posting. However, if all agents are “friends”, in equilibrium, targeted posting is always better than collective posting, while the comparison between unilateral and bilateral platforms remains ambiguous.

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# 1. Introduction

Online (social) platforms are used by roughly one in four people worldwide and the number of users is estimated to reach 2.55 billions by the end of 2017 (eMarketer, 2013). Excluding dating websites, there are more than 200 notable platforms that are currently active. Among these platforms, some have been more successful than others in attracting a very large number of users.<sup>1</sup> In particular, with more than 500 million registered users each, Facebook, Google+ and Twitter are among the largest platforms.<sup>2</sup> However, even when restricting attention to only these three platforms, little is know about how various platform design options affect their success.

Facebook, Google+, and Twitter essentially evolved over time, often following a “trial and error” approach, and their design was shaped by substantive lessons from both failures and successes. In some cases, less prominent options that one platform made available to its users have been adopted by another platform, further developed, and then given prominent status. For example, Facebook had an option called “friends lists”, which allowed a user to group a subset of his friends into a list and then share information only with the people in this list. However, this option was hidden deep in the user interface and Facebook did not take any steps to promote it or to encourage its usage. Indeed, before 2010, only 5% of Facebook’s users have used “friends lists” (Eldon, 2010). In contrast, on it’s launch in early 2011, Google+ showcased as one of its main features an option that allowed users to group their contacts into meaningful groups called “circles”. Furthermore, it encouraged its users to share information selectively using these circles. Nowadays, although the friends list and the circles have roughly the same functionality, the emphasis placed on these options by each network remains very different. This is particularly interesting since these options are key in determining the information shared and received by users, and thus significantly influence the platform’s owner revenue from content specific advertising. Note that these revenues are impressive by any metric.<sup>3</sup>

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<sup>1</sup>For example, platforms that are estimated to have in excess of 100 million registered users include specialized platforms such as Twitter (microblogging), LinkedIn (professional), Snapchat (impermanent photos) or general platforms such as Facebook, Google+, Bebo, Habbo, Netlog, Orkut, Qzone, Renren or Tagged.

<sup>2</sup>Another measure is the number of monthly active users (MAUs). According to the latest official quarterly reports, Facebook and Twitter had an average of 1.5 billion and 320 million MAUs, respectively (Facebook Inc., 2015; Twitter Inc., 2015). Google+ reportedly attracted by mid 2013 around 359 million MAUs (eMarketer, 2013).

<sup>3</sup>Facebook and Twitter obtained in the first 9 months of 2015 a revenue of \$12 and \$1.5 billions, respectively (Facebook Inc., 2015; Twitter Inc., 2015). Companies in the United States alone spent an estimated \$6.1 billion in 2013 on ads in social media (BIA/Kelsey, 2014).

We propose a new framework for reasoning about the optimal design of some of the options that social platforms like Facebook, Google+, or Twitter, make available to their users. In particular, we are interested in the design of the options that specify how agents may form connections and post information. Our model consists of a simple two-period game.

In the first period, we consider a process of endogenous network formation in which agents choose with what other agents to connect with. Given the type of connections that agents are allowed to form, we distinguish between unilateral and bilateral networks. In a unilateral network an agent can establish a connection with another agent without the agreement of the later. In this case, the former agent “follows” the later. In contrast, in a bilateral network, each connection requires the consent of both agents involved in the link. In practice, the typical connection in Twitter is unilateral, whereas in Facebook and Google+ it is bilateral.

In the second period, after the connections are formed, each agent observes some (private) information and has the option of sharing it with his connections by making a post. An agent’s information is interpreted as anything that may constitute the source of a typical post on a social platform: an event in the agent’s life, an opinion or photo of his, etc.<sup>4</sup> Given the constraints upon posting, we distinguish between targeted and collective posting. Targeted posting allows an agent to perfectly discriminate with whom to share his information among his connections. In contrast, collective posting requires an agent to share his information with all his connections.

Agents have quasilinear utilities from being connected to other agents and for making and receiving posts. In general (i.e., without any restrictions on the utilities of the agents), an agent who makes a post might value it differently than the one who receives it. Furthermore, the value of a post made or received need not be positive. For instance, an employee may have a negative value for sharing a private photo with his boss, or some posts that an agent may view in his newsfeed might be undesired or irrelevant, essentially spamming him. However, for so called social platforms for “friends”, we restrict the utilities of the agents as we require a post to be valued similarly by the agent who’s making it and any agent who is receiving it. The platform owner’s revenue is derived from advertisement, which is proportional with the

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<sup>4</sup>We require that posts can be made or received only among directly connected agents. This assumption rules out certain reshares of posts. For instance, if Al is connected to Bob, and Bob to Carl, but Al and Carl have no direct connection, a situation where Al’s post is reshared by Bob and Carl receives it too is ruled out. However, in practice, this situation can only appear if Al’s post setting is “public” and can therefore be viewed by anyone. In contrast, if Al uses the default settings for his post, these settings implicitly impose some restrictions, and even if Bob reshares Al’s post, this post cannot be received by any new agents who were not among Al’s connections to begin with. In particular, Carl does not receive it.

postings made and received in the platform.

We use subgame perfect Nash equilibrium as a solution concept for our two-period game. For all possible platform designs, we derive agents' equilibrium strategies and we show that for our two-period game, an equilibrium is always guaranteed to exist (Theorem 1). We then turn to studying how the platform design and the utility specifications affect the set of connections, the welfare of the agents, and the revenue of the platform owner in equilibrium. In general, we find that the comparison between targeted and collective posting is ambiguous (Example 1). To see this intuitively, imagine an analogy between targeted posting and first-degree price discrimination and collective posting and the law of one price; that the flexibility offered under targeted posting is welfare enhancing is not surprising. However, in sharp contrast with the economic intuition that flexibility is always welfare enhancing, our example also shows that for the most general utility specification, for either unilateral or bilateral networks, targeted is not necessarily better than collective posting for either the agents or the platform owner. Intuitively, it could be that under targeted posting an agent  $i$  shares his information with agent  $j$  but excludes another agent  $k$  who values a lot receiving the post, while under collective posting both agents  $j$  and  $k$  receive  $i$ 's post. If agent  $i$ 's value for posting is smaller than  $j$ 's value for receiving it, than collective posting is superior to targeted posting. Thus, our example shows that in general there is no unambiguous recommendation for what design options an owner seeking to maximize his utility should choose. We then restrict attention to platforms for "friends", in which we require that the post made by an agent and received by another is valued similarly by both agents. For social platforms for friends, we prove that targeted posting is always weakly better than collective posting for both the agents and the platform owner (Theorems 2 and 3). We also show that in platforms with either targeted or collective information sharing, when comparing platforms with unilateral versus bilateral connections, the welfare effects are ambiguous (Theorem 4).

## 2. Related Literature

In our modelling, information can be thought of as a club good (Buchanan, 1965), i.e., a good that is excludable and non-rivalrous, and our two-period game is in line with the game theoretical literature on multi-stage games with incomplete information (Fudenberg and Tirole, 1991).

The first stage of our game connects with the literature on strategic network formation (Bala and Goyal, 2000; Galeotti, Goyal, and Kamphorst, 2006; Galeotti et al., 2010; Jackson and Wolinsky, 1996). Jackson and Zenou (2013) provide an excellent overview of this literature. Closest to our work, Bala and Goyal (2000) propose a non-cooperative model of network formation and they characterize the architectures of the networks that arise in strong Nash equilibria. In contrast to their main results, we compare the set of connections, the utilities of the agents, and the revenue of the network, across the equilibria in different types of social platforms. We also assume that agents receive benefits not only from incoming connections, i.e., through receiving information, but also from outgoing connections, i.e., through sharing information. These type of benefits do not appear neither in Bala and Goyal (2000) nor in any other known to us paper.

The second stage of our game is constructed around the idea of studying information sharing, an idea that is inspired, although not directly relatable to, the studies on selective information disclosure (Austen-Smith, 1994; Okuno-Fujiwara, Postlewaite, and Suzumura, 1990; Ostrovsky and Schwarz, 2010; Pagano and Jappelli, 1993; Rao and Segal, 2010), and some of the intuition in our example goes back to the literature on discrimination in contracting with externalities (Segal, 2003; Segal and Whinston, 2003).

Finally, we note that social platforms such as Facebook recently started to generate a lot of interest in the economic literature (Batzilis et al., 2014; Chen et al., 2011; Hartline, Mirrokni, and Sundararajan, 2008; Kleinberg and Ligett, 2013; Tarbush and Teytelboym, 2012, forthcoming). However, with the exception of Mueller-Frank and Pai (2014) who compare classical display advertising to advertising in social platforms from a theoretical point of view, most of this literature is centred around empirical investigations. Closest to our work, Kleinberg and Ligett (2013) analyze information sharing and privacy in social platforms. They study the behavior of rational agents in a network where an information shared by an agent may spillover from the agent who is the intended recipient to other agents, with potentially unpleasant consequences for the agent who initially shared the information. In their model, the utility of an agent takes one of two fixed values to reflect his relation with other agents, “friends” or “enemies”, with whom he is in the same component. Thus, theirs is a simultaneous game concerned with the formation of Nash stable components. In contrast, in our setting agents are simultaneously concerned with the information shared and received. Our model is a multi-stage game with observed actions and incomplete information in which we study not just what connections are formed among agents but also what information gets shared. The network

structure does not influence our results.

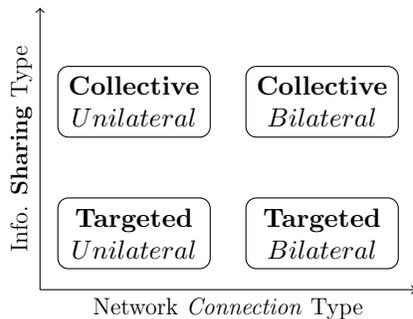
### 3. Model and Definitions

Conceptually, we consider a two–period game played by a finite set of agents. In each period, the agents choose their actions simultaneously. The actions chosen in the first period are observed by all agents and then form the history of the game, constraining the actions available in the second period. In the second period, before choosing his actions, each agent observes a private information. Payoffs are given by a function that assigns a utility for each terminal history and private information combination.

Concretely, there is a (platform) *owner*, denoted by 0, and a finite set of *agents*  $\mathcal{I} = (1, \dots, I)$  who are using this platform. Let  $2^{\mathcal{I}}$  denote the power set of  $\mathcal{I}$ . The owner is not a player but he chooses the “rules” of the platform, which specify in the first period of the game how agents may form connections, and then in the second period how they are allowed to share their private information via these connections.

*Period 1 (network formation)*: Each agent  $i \in \mathcal{I}$  chooses any subset  $Ch_i \in 2^{\mathcal{I} \setminus \{i\}}$  of other agents that he wishes to connect with. We denote the choices of all agents except  $i$  by  $Ch_{-i}$ . The choices of all agents are simultaneous and denoted by  $Ch \equiv \{Ch_i\}_{i \in \mathcal{I}}$ . We consider two possible rules, or platform designs. In platforms with *unilateral connections*, for a connection  $ij$  to be formed, it is enough for an agent  $i$  to choose to follow another agent  $j$ . In contrast, in platforms with *bilateral connections*, for a connection  $ij$  to be formed, it is not enough for  $i$  to choose  $j$ ; in addition, it also has to be the case that  $j$  chooses  $i$ . That is, we require agents  $i$  and  $j$  to choose each other. The choices of all agents are revealed at the end of the period and determine a finite *network*  $N$ . We write  $ij \in N$  for a connection  $ij$  in network  $N$ . Let  $N_{i \rightarrow} \equiv \{j : ij \in N \text{ and } j \in Ch_i\}$  denote the set of agents that agent  $i$  follows at  $N$ . Symmetrically, let  $N_{i \leftarrow} \equiv \{j : ij \in N \text{ and } i \in Ch_j\}$  denote the set of agents who follow agent  $i$  at network  $N$ . Observe that with unilateral connections, in general,  $N_{i \rightarrow} \neq N_{i \leftarrow}$ . Meanwhile, with bilateral connections, we always have  $N_{i \rightarrow} = N_{i \leftarrow} = N_i$ . The set of all possible networks is  $\tilde{N}$ .

*Period 2 (information sharing)*: Each agent  $i \in \mathcal{I}$  observes some (*private*) *information*  $x^i$ , with  $x^i$  independently distributed according to some function  $h^i$  with support  $X^i$ . We interpret



**Figure 1.** Platform types.

the private information as anything that may constitute the source of a *typical* Facebook or Google+ post. For example,  $x^i$  could be an event that happens in agent  $i$ 's life, an opinion of his, or a photo from his collection. We do not restrict  $X^i$  in any way. Each agent knows  $x^i$  but not  $x^{-i}$ . Let  $\mathbf{x} \in X$  denote the profile of information, where  $\mathbf{x} \equiv \{x^i\}_{i \in \mathcal{I}}$  and  $X \equiv \prod_{i=1}^I X^i$ . Each agent  $i$  may share his information with his followers  $N_{i\leftarrow}$  and may receive information from the agents that he is following  $N_{i\rightarrow}$ . We define a binary action of agent  $i$  regarding sharing his information  $x^i$  with agent  $j$  via connection  $ij$  at network  $N$  as  $\mathbf{1}_{ij}(x^i, N) : X^i \times \tilde{N} \rightarrow \{0, 1\}$ , where  $\mathbf{1}_{ij}(x^i, N) = 1$  if  $j \in N_{i\leftarrow}$  and  $i$  shares his information; and  $\mathbf{1}_{ij}(x^i, N) = 0$  otherwise. We consider two possible rules, or platform designs. In networks with *targeted posting*, an agent can select individually with whom to share his information among his connections; formally, there are no restrictions on actions  $\mathbf{1}_{ij}$ . In contrast, in networks with *collective posting*, each agent can share his information only with all his connections at once; formally,  $\mathbf{1}_{ij} = \mathbf{1}_{ij'}$  for all  $j, j' \in N_{i\leftarrow}$ . We denote agent  $i$ 's actions at each possible history and at any information profile by  $\mathbf{1}_i \equiv \{\mathbf{1}_{ij}(\cdot, \cdot)\}_{j \in \mathcal{I} \setminus \{i\}}$ , and the actions of all agents are  $\mathbf{1} \equiv \{\mathbf{1}_i\}_{i \in \mathcal{I}}$ .

In our model, the network formation precedes the information sharing because we consider that connection decisions are more long term than posting decisions. Thus, the long term decisions are taken as given when the short term decisions are made. We summarize the possible types of platforms in Figure 1. A *pure strategy* of agent  $i$  is specified by a pair of actions  $(Ch_i, \mathbf{1}_i)$ .

### 3.1. Preferences

Each agent  $i$  has a constant value  $c^{ij} \in \mathbb{R}$  for being connected to another agent  $j$ . We interpret  $c^{ij}$  as the fixed cost or benefit derived by  $i$  for being seen as being associated with or following

$j$ . For each agent  $i$ , his value for sharing his information  $x^i$  with agent  $j$  is  $s^{ij}(x^i) \in \mathbb{R}$ , and his value for receiving information  $x^j$  from  $j$  is  $r^{ij}(x^j) \in \mathbb{R}$ . We allow for  $s^{ij}(x^i) \leq 0$  to capture situations in which sharing  $x^i$  with  $j$  may harm  $i$ . For instance, an employee  $i$  may have a negative value for sharing a private photo  $x^i$  with his boss  $j$ . Analogously, we allow for  $r^{ij}(x^j) \leq 0$  to capture situations in which  $x^j$  is irrelevant for, or otherwise unwanted by, agent  $i$ . For instance, some posts made by  $j$  may appear in  $i$ 's newsfeed and be viewed by  $i$ , although  $i$  is not interested into them (i.e., spam posts); other posts may be denigratory or unpleasant, etc. Next, we define the utilities of the agents *after* and *before* observing their private information.

For each agent  $i \in \mathcal{I}$  who observes  $x^i$  at network  $N$ , given that in Period 2 he follows action  $\mathbf{1}_i$  while the other agents follow actions  $\mathbf{1}_{-i}$ , his *ex-post utility* is  $U_i(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N)$ , and we define  $U_i(\cdot)$  taking into account the various platform rules possible.

In platforms with unilateral connections, each agent  $i$  may post to his followers  $j \in N_{i\leftarrow}$ , has a constant value  $c_{ij}$  for following agent  $j$ , and may receive information from the agents  $j \in N_{i\rightarrow}$  that he is following. Thus, denoting by  $E$  the mathematical expectation operator, each agent  $i$ 's utility is

$$U_i^u(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N) \equiv \sum_{j \in N_{i\leftarrow}} s^{ij}(x^i) \mathbf{1}_{ij}(x^i, N) + \sum_{j \in N_{i\rightarrow}} (c^{ij} + E(r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N))).$$

In platforms with bilateral connections, each agent has a constant value for being connected with another agent, and any two connected agents can freely share and receive information between them. Each agent  $i$ 's utility is

$$U_i^b(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N) \equiv \sum_{j \in N_i} (c^{ij} + s^{ij}(x^i) \mathbf{1}_{ij}(x^i, N) + E(r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N))).$$

For each agent  $i \in \mathcal{I}$ , before he observes his private information, but after the network  $N$  is formed, depending if the platform allows for unilateral or bilateral connections, his *ex-ante utility* at  $N$  is  $U_i^u(N, \mathbf{1}) \equiv E U_i^u(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N)$  or  $U_i^b(N, \mathbf{1}) \equiv E U_i^b(\mathbf{1}_i, \mathbf{1}_{-i}|x^i, N)$ , respectively.

The utility of the network owner is derived from content specific advertising, which is proportional to the amount of information exchanged among agents.<sup>5</sup> Thus, the utility of the

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<sup>5</sup>Content specific advertising is a type of advertising whereby the information that users share and receive

network owner is the sum of agents' ex-ante benefits corresponding to information exchange:

$$U_0^u(N, \mathbf{1}) \equiv E \sum_{i \in \mathcal{I}} \left( \sum_{j \in N_{i \leftarrow}} s^{ij}(x^i) \mathbf{1}_{ij}(x^i, N) + \sum_{j \in N_{i \rightarrow}} r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N) \right).$$

$$U_0^b(N, \mathbf{1}) \equiv E \sum_{i \in \mathcal{I}} \sum_{j \in N_i} (s^{ij}(x^i) \mathbf{1}_{ij}(x^i, N) + r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N)).$$

We assume that the network owner and the agents are all utility maximizers.

### 3.2. Subgame Perfect Nash Equilibrium

We use subgame perfect Nash equilibrium as a solution concept. Formally, a profile of strategies  $(Ch, \mathbf{1})$  is a *subgame perfect Nash equilibrium* if

1.  $U_i(Ch_i, Ch_{-i}, \mathbf{1}) \geq U_i(Ch'_i, Ch_{-i}, \mathbf{1})$  for each  $i \in \mathcal{I}$ , each  $Ch'_i \subseteq \mathcal{I}$ , and
2.  $\mathbf{1}_i(\mathbf{x}^i, N) \in \operatorname{argmax}_{S_i \in \Omega_i(N_{i \leftarrow})} U_i(S_i, \mathbf{1}_{-i} | x^i, N)$  for each  $x^i \in X^i$ , each  $N \in \tilde{N}$ , where for targeted posting  $\Omega_i(N_{i \leftarrow}) = \{0, 1\}^{N_{i \leftarrow}}$  and for collective posting  $\Omega_i(N_{i \leftarrow}) = \{0, 1\}$ .

For situations which result in identical payoffs, we assume that ties are broken consistently by employing the following *tie-breaking* rule. If an agent is indifferent between: (1) forming or not a connection with another agent, then he forms the connection, (2) sharing or not his information with the other agent(s), then he shares his information. For short, we refer to a subgame perfect equilibrium that satisfies this tie-breaking rule simply as an *equilibrium*.

## 4. Equilibrium Strategies

We start by analyzing the strategic behavior of the agents in Period 2, after all connections have been formed and the private information has been observed. The fixed costs or benefits are determined and agents cannot influence receiving information. However, agents may choose whether to share their private information so as to maximize their expected ex-ante utility.

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influences which adverts are shown and to whom. Common examples include “native advertising” such as Facebook Sponsored Stories or Twitter Promoted Tweets. Native advertising accounts for 39% of the total US firms expenditure on social media advertising (see BIA/Kelsey, 2013).

Under targeted posting, each agent  $i$  shares his information  $x^i$  with any follower  $j \in N_{i\leftarrow}$  if and only if  $s^{ij}(x^i) \geq 0$ . Thus, agent  $i$ 's expected ex-ante utility from posting is

$$E\left(\sum_{j \in N_{i\leftarrow}} \max(s^{ij}(x^i), 0)\right). \quad (1)$$

Under collective posting, each agent  $i$  shares his information  $x^i$  with all his followers  $N_{i\leftarrow}$  if and only if  $\sum_{j \in N_{i\leftarrow}} s^{ij}(x^i) \geq 0$ . Thus, agent  $i$ 's expected ex-ante utility from posting is

$$E\left(\max\left(\sum_{j \in N_{i\leftarrow}} s^{ij}(x^i), 0\right)\right). \quad (2)$$

Given that the maximum function is a convex function, we make the following remark.

**Remark 1.** *Given the same set of followers, each agent's benefit from sharing information in platforms with targeted posting is weakly higher than in platforms with collective posting.*

Recall that our tie-breaking rule requires that if an agent is indifferent between sharing or not his information with his follower(s), then he shares it. Thus, a Nash equilibrium for the Period 2 subgame is guaranteed to exist. For platforms with targeted posting, the Nash equilibrium is unique. For platforms with collective posting, we may have a multiplicity of optimal sets, but the agents' utility in equilibrium is unique. We fix agents' continuation payoffs to be equal with their Nash equilibrium payoffs. We now analyze the strategic behavior in Period 1.

In platforms with unilateral connections and targeted posting, agent  $i$  cannot control who follows him. However,  $i$  can choose whom to follow. In order to follow another agent  $j$ , agent  $i$ 's value  $r^{ij}(x^j)$  for receiving  $j$ 's information  $x^j$  has to exceed the fixed cost  $c^{ij}$ . Furthermore, agent  $i$  has to expect  $j$  to actually share with him his information  $x^j$ . That is, agent  $i$  follows any agent  $j$  if and only if

$$c^{ij} + E(\max(r^{ij}(x^j)\mathbf{1}_{j_i}(x^j, i), 0)) \geq 0. \quad (3)$$

In platforms with unilateral connections and collective posting, agent  $i$  chooses to follow another agent  $j$  if  $i$ 's value  $r^{ij}(x^j)$  for receiving  $j$ 's information  $x^j$  exceeds the fixed cost  $c^{ij}$ . Furthermore, agent  $i$  has to expect  $j$  to actually share his information with all his followers  $N_{j\leftarrow}$ . That is,

agent  $i$  follows the set of agents that is obtained by maximizing the following expression:

$$\max_{Ch_i \subseteq \mathcal{I}} \left( \sum_{j \in Ch_i} c^{ij} + E(\max(\sum_{j \in Ch_i} r^{ij}(x^j) \mathbf{1}_{j_i}(x^j, N_{j \leftarrow}), 0)) \right). \quad (4)$$

The optimization problems above are discrete maximization problems. Hence, they are well defined. This establishes the existence of a subgame perfect Nash equilibria for platforms with unilateral connections.

In platforms with bilateral connections, the Nash equilibrium is a very weak equilibrium concept. To see that it is always guaranteed to exist for both targeted and collective posting, consider the following example: In Period 1, no connections are formed, while in Period 2, agents choose the actions which induce Nash equilibrium payoffs. The above analysis proves the following theorem.

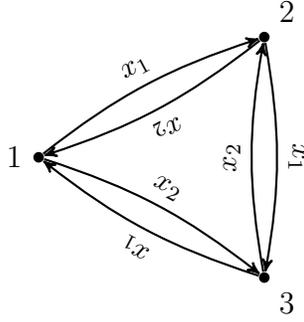
**Theorem 1.** *For all possible platform rules, the two-period game of network formation and information sharing has an equilibrium in pure strategies.*

## 5. Welfare Analysis

In this section we analyze the welfare implications of the various platform rules. We show that in general, comparing the agents' utility in equilibrium under targeted posting to their utility in equilibrium under collective posting does not yield clear results. Moreover, a similar statement applies for the platform owner.

### Example 1. Targeted versus collective posting.

Let 0 denote the platform owner and let  $\mathcal{I} = \{1, 2, 3\}$ . We assume that agents form the complete network with either unilateral or bilateral connections. Let  $c^{ij} = 0$  for all  $i \neq j$  where  $i, j \in \mathcal{I}$ . For convenience, for each agent  $i \in \mathcal{I}$ , we require  $X^i$  be a subset of  $\mathbb{R}^2$  and we represent  $i$ 's private information by a two-dimensional vector  $\mathbf{x}^i = (x_1^i, x_2^i)$  that is uniformly distributed within half of the unit circle:  $\{(x_1^i, x_2^i) : (x_1^i)^2 + (x_2^i)^2 \leq 1, x_1 + x_2 \geq 0\}$ . Agents' benefits from information sharing and receiving are linear:  $s^{ij}(\mathbf{x}) = \mathbf{s}^{ij} \mathbf{x}$  and  $r^{ij}(\mathbf{x}) = \mathbf{r}^{ij} \mathbf{x}$  with  $\mathbf{s}^{ij}, \mathbf{r}^{ij} \in \mathbb{R}^2$  for all  $i \neq j$ . The information sharing weights, which can be thought of as reflecting the agents' values for sharing individual dimensions of their multidimensional private information, are:



**Figure 2.** Nodes represent agents. An arc reflects that the “tail” agent wants to share the part of his private information that is specified on the label of the arc with the “head” agent.

$\mathbf{s}^{12} = \mathbf{s}^{23} = \mathbf{s}^{31} = (1, 0)$  and  $\mathbf{s}^{13} = \mathbf{s}^{32} = \mathbf{s}^{21} = (0, 1)$ . We specify the information receiving weights  $\mathbf{r}^{ij}$  later. Since agents are symmetric, we consider only agent 1. For convenience, we omit the specific index of the agent. Figure 2 gives a visual representation of our example so far. Next, we analyze each agent’s incentives in Period 2.

In platforms with targeted posting, since  $s^{12} = (1, 0)$  and  $s^{13} = (0, 1)$ , agent 1 prefers to share his information  $\mathbf{x} = (x_1, x_2)$  with agent 2 if  $x_1 \geq 0$ , and with agent 3 if  $x_2 \geq 0$  (see the left panel on Figure 3). Thus, the ex-ante utility for posting for agent 1 is  $E(\max(x_1, 0)) + E(\max(x_2, 0)) = \frac{2(2+\sqrt{2})}{3\pi}$ .

In platforms with collective posting, since  $s^{12} = (1, 0)$  and  $s^{13} = (0, 1)$ , agent 1 prefers to share his information  $\mathbf{x} = (x_1, x_2)$  with agents 2 and 3 if  $x_1 + x_2 \geq 0$  (see the right panel on Figure 3). Thus, the ex-ante utility for posting for agent 1 is  $E(\max(x_1 + x_2, 0)) = \frac{4\sqrt{2}}{3\pi}$ .

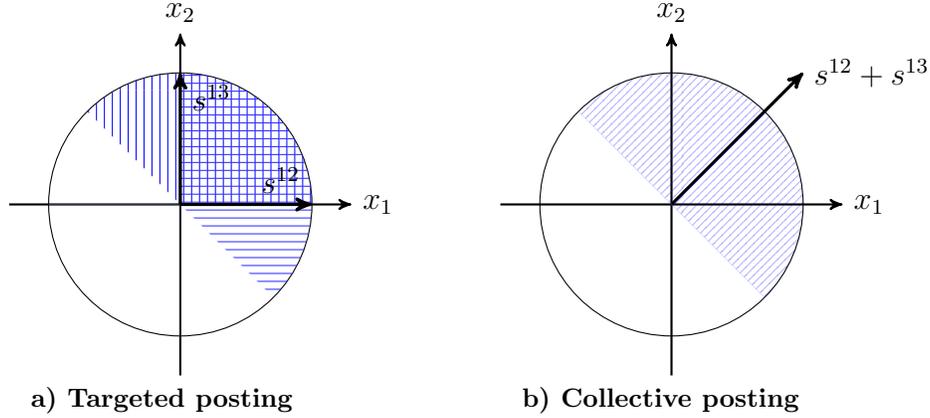
Since  $\frac{2(2+\sqrt{2})}{3\pi} > \frac{4\sqrt{2}}{3\pi}$ , for agent 1, his ex-ante utility for sharing his information is higher under targeted than under collective posting. Next, we consider his ex-ante utility from receiving information.

Let the information receiving weights be  $r^{ij} = (1, 1)$  for all  $i \neq j$ . Since agents 2 and 3 use the same information sharing strategy as agent 1, agent 1’s expected utility from receiving information in platforms with targeted posting is

$$Pr(x_1 \geq 0)E(x_1 + x_2 | x_1 \geq 0) + Pr(x_2 \geq 0)E(x_1 + x_2 | x_2 \geq 0) = \frac{4(1+\sqrt{2})}{3\pi},$$

while his expected utility from receiving information in platforms with collective posting is

$$2Pr(x_1 + x_2 \geq 0)E(x_1 + x_2 | x_1 + x_2 \geq 0) = \frac{8\sqrt{2}}{3\pi}.$$



**Figure 3.** Targeted versus collective posting. *Agent 1's information is uniformly distributed within half of the unit circle. Left panel: agent 1 prefers to share with 2 the information shaded with horizontal lines, and with 3 the information shaded with vertical lines. Right panel: agent 1 prefers to share with both 1 and 2 the information shaded with diagonal lines.*

Our calculations are summarized in Table 1.

	<i>Targeted</i>	<i>Collective</i>
Sharing benefits:	$\frac{2(2+\sqrt{2})}{3\pi}$	$\frac{4\sqrt{2}}{3\pi}$
Receiving benefits:	$\frac{4(1+\sqrt{2})}{3\pi}$	$\frac{8\sqrt{2}}{3\pi}$
Agent's/Owner's utility:	$\frac{8+6\sqrt{2}}{\pi}$	$\frac{12\sqrt{2}}{\pi}$

**Table 1.** The utility of the agents and of the owner for targeted and for collective posting.

Table 1 shows that given some network (irrespective of whether connections are unilateral or bilateral), the utilities of the agents and that of the owner may be smaller in platforms with targeted posting than in platforms with collective posting. If we maintain the assumption that for each agent the value of being connected to another agent is null and agents 2 and 3 have no information to share, then the same example also applies to endogenously formed networks. Hence, in general, there is no clear relation between the utility of the agents in platforms with targeted posting and their utility in platforms with collective posting. A similar statement applies for the owner. ■

Example 1 shows that in general, if a platform owner who seeks to maximize his own utility has to choose between allowing either targeted or collective posting, his choice is unclear.

Furthermore, the unclarity persists even if the owner is an altruistic social planner who wishes to maximize the welfare of the agents.

## 6. Social Platforms for Friends

In this section, we introduce a natural restrictions on the preferences of the agents. Consider an environment where the values for sharing and receiving an information coincide, i.e., for each  $i, j \in \mathcal{I}$  with  $i \neq j$  and any  $x^i \in X^i$ , we require  $s^{ij}(x^i) = r^{ji}(x^i)$ . We call such agents “*friends*”, and a platform in which all agents are friends is a *social platform for friends*. In the reminder of this section, we restrict our attention to social platforms for friends and we analyze the welfare implications of the various platform rules.

### 6.1. Targeted vs. Collective Posting

Since the value of an agent for sharing his private information is identical with the value that each agent connected to him has for receiving it, the interests of any pair of connected agents in Period 2 are perfectly aligned: the individual optimum and the joint optimum coincide. Building on this insight, we analyse also the network formation in Period 1, and we obtain the following result.

**Theorem 2.** *For unilateral connections, in any equilibrium of the game with targeted posting, {the set of connections, each agent’s utility, and the utility of the owner} are weakly higher than in any equilibrium of the game with collective posting.*

Consider a platform for friends with unilateral connections. Consider some equilibrium of the game with collective posting and let  $N^*$  denote the set of connections at this equilibrium. Then, the set of agents followed by agent  $i$  is  $N_{i \rightarrow}^*$  and solves the maximization problem in (4). Observe that if  $|N_{i \rightarrow}^*| < 2$ , there is no distinction between collective and targeted posting. Assume  $|N_{i \rightarrow}^*| \geq 2$  and let  $k \in N_{i \rightarrow}^*$ . Then, using relation (4), we have the following set of

inequalities:

$$\begin{aligned}
\sum_{j \in N_{i \rightarrow}^* \setminus \{k\}} c^{ij} + E(\max(\sum_{j \in N_{i \rightarrow}^* \setminus \{k\}} r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N_{j \leftarrow}^*), 0)) + c^{ik} + E(\max(r^{ik}(x^k) \mathbf{1}_{ki}(x^k, N_{k \leftarrow}^*), 0)) &\geq \\
\sum_{j \in N_{i \rightarrow}^*} c^{ij} + E(\max(\sum_{j \in N_{i \rightarrow}^*} r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N_{j \leftarrow}^*), 0)) &\geq \\
\sum_{j \in N_{i \rightarrow}^* \setminus \{k\}} c^{ij} + E(\max(\sum_{j \in N_{i \rightarrow}^* \setminus \{k\}} r^{ij}(x^j) \mathbf{1}_{ji}(x^j, N_{j \leftarrow}^*), 0)), &
\end{aligned}$$

where the first inequality follows from the convexity of the maximum function, and the second from the optimality of the set  $N_{i \rightarrow}^*$ . Contrasting the first and the third expression, we obtain that

$$c^{ik} + E(\max(r^{ik}(x^k) \mathbf{1}_{ki}(x^k, N_{k \leftarrow}^*), 0)) \geq 0,$$

which since we have a platform for friends, we can rewrite as

$$c^{ik} + E(\max(\sum_{j \in N_{k \leftarrow}^*} s^{kj}(x^k), 0)) \geq 0.$$

Furthermore, we also have that

$$\begin{aligned}
c^{ik} + E(\max(\sum_{j \in N_{k \leftarrow}^* \setminus \{i\}} s^{kj}(x^k), 0)) + E(\max(s^{ki}(x^k), 0)) &\geq \\
c^{ik} + E(\max(\sum_{j \in N_{k \leftarrow}^*} s^{kj}(x^k), 0)) &\geq \\
c^{ik} + E(\max(\sum_{j \in N_{k \leftarrow}^* \setminus \{i\}} s^{kj}(x^k), 0)), &
\end{aligned}$$

where the first inequality follows from the convexity of the maximum function, and the second from the optimality of the set  $N_{k \leftarrow}^*$ . Contrasting the first and the third expression, we obtain that

$$c^{ik} + E(\max(s^{ki}(x^k), 0)) \geq 0,$$

which since agents  $i$  and  $k$  are friends implies that

$$c^{ik} + E(\max(r^{ik}(x^k)\mathbf{1}_{ki}(x^k, i), 0)) \geq 0,$$

which by inequality (3) implies that agent  $i$  chooses to follow  $k$  in any equilibrium of the two-period game with targeted posting. Hence, when the platform rules only allow for unilateral connections and all agents are friends, the set of connections in any equilibrium of the two-period game with collective posting is a subset of the set of connections in any equilibrium of the two-period game with targeted posting.

Let us now consider the utility of agent  $i$ . By Remark 1, given the same set of followers  $N_{i\leftarrow}^*$ , agent  $i$ 's benefit from sharing information in platforms with targeted posting is weakly higher than in platforms with collective posting. Furthermore, when posting is targeted, in equilibrium, agent  $i$  may be followed by some additional agents  $k \in \mathcal{I} \setminus N_{i\leftarrow}^*$ , which can only weakly increase his utility for posting. Now consider agent  $i$ 's benefit from receiving information. For any  $k \in N_{i\rightarrow}^*$ , as we established above, in any equilibrium of the two-period game with collective posting, agent  $k$  derives smaller benefits from sharing his information  $x^k$  with  $i$  than in any equilibrium of the game with targeted posting:

$$E(s^{ki}(x^k)\mathbf{1}_{ki}^*(x^k, N^*)) \leq E(\max(s^{ki}(x^k), 0)),$$

where  $\mathbf{1}_{ki}^*$  is an optimal posting strategy, i.e.,  $\mathbf{1}_{ki}^*(x^k, N^*) = 1$  if  $\sum_{l \in N_k^*} s^{kl}(x^k) \geq 0$  and 0 otherwise. Since agents  $i$  and  $k$  are friends, this implies that  $i$  also enjoys smaller benefits from receiving information when information is shared collectively. Furthermore, when posting is targeted, in equilibrium, agent  $i$  might follow by some additional agents  $j \in \mathcal{I} \setminus N_{i\rightarrow}^*$ , with each additional connection satisfying inequality (3). Overall, in any equilibrium of the two-stage game with collective information sharing, agent  $i$  obtains a smaller utility than in any equilibrium of the two-stage game with targeted information sharing. Summing across all agents, the same comparison for the utility of the platform owner follows straightforwardly. ■

**Theorem 3.** *For bilateral connections, in any equilibrium of the game with collective posting, {the set of connections, each agent's utility, and the utility of the owner} are weakly smaller than in any equilibrium of the game with targeted posting.*

Consider a friends network with bilateral connections. Consider some equilibrium of the game with collective posting and let  $N^*$  denote the set of connections at this equilibrium. Since

the network is bilateral, agent  $i$  needs the “approval” of other agents to form connections. Since all agents are friends, agent  $i$ ’s value for receiving information from  $j$  coincides with  $j$ ’s for posting it. Thus, the optimal set of  $i$ ’s connections  $N_i^*$  solves

$$\max_{N_i \subseteq N_{-i}^*} \left( \sum_{j \in N_i} c^{ij} + E(\max(\sum_{j \in N_i} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i} s^{ji}(x^j) \mathbf{1}_{ji}^*(x^j, N^*)) \right), \quad (5)$$

where  $\mathbf{1}_{ji}^*(x^j, N^*) = 1$  if  $\sum_{l \in N_j^*} s^{jl}(x^j) \geq 0$ , and 0 otherwise. Consider some  $k \in N_i^*$ . Then,

$$\begin{aligned} & \sum_{j \in N_i^* \setminus \{k\}} c^{ij} + E(\max(\sum_{j \in N_i^* \setminus \{k\}} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i^* \setminus \{k\}} s^{ji}(x^j) \mathbf{1}_{ji}^*(x^j, N^*)) + \\ & c^{ik} + E(\max(s^{ik}(x^i), 0)) + E(\max(s^{ki}(x^k), 0)) \geq \\ & \sum_{j \in N_i^*} c^{ij} + E(\max(\sum_{j \in N_i^*} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i^*} s^{ji}(x^j) \mathbf{1}_{ji}^*(x^j, N^*)) \geq \\ & \sum_{j \in N_i^* \setminus \{k\}} c^{ij} + E(\max(\sum_{j \in N_i^* \setminus \{k\}} s^{ij}(x^i), 0)) + E(\sum_{j \in N_i^* \setminus \{k\}} s^{ji}(x^j) \mathbf{1}_{ji}^*(x^j, N^*)), \end{aligned} \quad (6)$$

where  $\mathbf{1}_{ji}^*(x^j) = 1$  if  $\sum_{l \in N_j^*} s^{jl}(x^j) \geq 0$ , and 0 otherwise. The first inequality follows from convexity of the maximum function. The second inequality follows from the optimality of set  $N_i^*$ : agent  $i$  does not want to drop any of his connections. Comparing the first inequality with the third, we obtain

$$c^{ik} + E(\max(s^{ik}(x^i), 0)) + E(\max(s^{ki}(x^k), 0)) \geq 0. \quad (7)$$

Hence, each agent  $i$  proposing the set of connections  $N_i^*$  and following his optimal strategies under collective posting is also in the equilibrium of the game with targeted posting. Furthermore, in this equilibrium, the agent’s benefit from receiving and sharing information is at least as large as in the equilibrium under collective posting. ■

## 6.2. Unilateral vs. Bilateral Connections

**Theorem 4.** *For both targeted and collective posting, the comparison in terms of {the set of connections, each agent’s utility, and the utility of the network owner} between an equilibrium of*

*the game with unilateral connections and some equilibrium of the game with bilateral connections is ambiguous.*

Consider a friends network with either targeted or collective posting. It is straightforward to see that the equilibrium of a game with unilateral connections can be weakly better (i.e., yield more connections, a higher agent’s utility, and more utility for the owner) than an equilibrium of a game with bilateral connections. To see that there may exist an equilibrium of a game with bilateral connections that is weakly better than the unique equilibrium of the game with unilateral connections, consider the following example.

There is a platform owner and let  $\mathcal{I} = \{i, j\}$ . Let  $c^{ij} = c^{ji} = -3$ ,  $x^i \in X^i$ ,  $x^j \in X^j$ , and  $s^{ij}(x^i) = r^{ji}(x^i) = s^{ji}(x^j) = r^{ij}(x^j) = 2$ . With bilateral connections, there is only one equilibrium: agents  $i$  and  $j$  follow each other, each obtains a utility of 1, and the owner’s utility is 2. In contrast, with unilateral connections, the equilibrium above unravels: since  $c^{ij} + r^{ji}(x^j) \leq 0$  and  $c^{ji} + r^{ij}(x^i) \leq 0$ , each agent finds it profitable to stop following the other one. Thus, with unilateral connections, there is only one equilibrium: no connections are formed, and everyone’s utility is zero. ■

## 7. Conclusion

We introduced a *new* model for reasoning about the design of some of the most important options that social platforms make available to their users. Our model is deliberately parsimonious. We use the minimal formalism needed for a first-order approximation of the most important options related to network formation and information sharing that are made available to their users by platforms such as Facebook, Google+, or Twitter. Apart from the design options, our model is also the first to allow users’ preferences to account for not just the information shared, but also for the one received. Surprisingly, we find that *in general*, enhancing and promoting the tools for targeted information sharing is *not* necessarily beneficial for the users or for the platform owner. Figuring out what combination of design options is best suited for the owner of a social platform depends on the preferences of the typical user of his platform. While pinning down the preferences of the typical user is an empirical question which we cannot answer with certainty in absence of direct access to data, we conjecture that our social platform for friends with bilateral connections is well suited to proxy the preferences of a typical Facebook or Google+ user. For this environment, we found that in equilibrium

targeted is at least as good as collective posting, which may explain why Google+ chose to refine and emphasize its “circles” option and why Facebook is taking steps towards making similar options more accessible. When comparing social platforms for friends with *unilateral* and *bilateral* connections, we find that the comparison with respect to the set of connections, each agent’s utility, and network revenue, is ambiguous, which may explain why platforms that are essentially built on bilateral connections such as Facebook and Google+ can coexist with Twitter, which is essentially built around unilateral connections.

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