Optimal Income Taxation with Endogenous Prices∗

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Abstract

We study an optimal income taxation problem in a Mirrleesian setting with endogenous product prices and positive firm profits. In the presence of foreign ownership or a progressive distribution of profit shares, we show that the public authority favors smaller equilibrium prices in competitive markets leading to a more progressive taxation in the optimum. Using a calibrated model of the U.S. housing market, we quantify the price effect and show that it adds about 6 percentage points to income tax rates. With oligopolistic markets, market power creates an additional non-competitive effect that puts a downward pressure on optimal income tax rates. Using the U.S. housing model, we find that the price effect dominates the non-competitive effect leading to higher optimal marginal income taxes in less competitive markets. We also show that the price and non-competitive effects persist in the presence of commodity taxation.

Keywords: Optimal income taxation, endogenous prices, pecuniary externality, competitive markets, oligopolistic markets, housing market.

JEL Classification: H21, H23, D43.

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1 Introduction

The design of income taxes is one of the most important economic problems. At the same time, the income taxation literature commonly assumes that product markets are perfectly competitive. While this assumption is convenient for analysis it is far from an accurate description of most markets. According to recent studies (Azar et al., 2017; Grullon et al., 2017), one third of U.S. industries are highly concentrated and over 75% of U.S. industries are now operating in markets that are more concentrated than 20 years ago.\(^1\) Similar market structures are observed in Europe, where a typical market share of the top five food retailers ranges from 43 to 69% (European Commission, 2015).

What are implications for optimal income taxation if markets are not perfectly competitive? What is the size of the optimal income tax change due to the presence of endogenous prices? The goal of this paper is to answer these questions and, thus, to bridge a gap in public economics literature by incorporating endogenous prices into the analysis of income taxation.\(^2\)

We consider the standard Mirrlees (1971) framework with a continuum of agents who differ in their productivity types. Agents earn labor income and care about the consumption of two goods: a numeraire good and a “main” good. The numeraire good is produced with a constant returns to scale technology and has a perfectly elastic supply function, the main good is produced with a decreasing returns to scale technology and has a strictly increasing supply function. As a consequence, the price of the numeraire good is constant and the price of the main good is endogenously determined by the market equilibrium condition. Also, firms producing the numeraire good earn zero profits and firms producing the main good earn positive profits. We assume that firm profits are progressively distributed among agents inside the economy with more productive agents receiving larger profit shares, but we also allow for some firm profits to belong to foreigners or capitalists who have no labor income.\(^3\)

The objective of the public authority is to design the optimal income tax schedule that maximizes the sum of total agent utilities subject to three constraints: the resource constraint, which demands that the public authority raises a fixed level of public funds; the incentive compatibility constraint, which requires agents to reveal their productivity types; and the market equilibrium condition, which determines the price level.\(^4\)


\(^2\)Atkinson (2012, p. 775) offers an extensive discussion of how the existing taxation literature mostly fails to take into account the underlying market structure and endogenous prices.

\(^3\)See Wolff (2017) for evidence on the progressive distribution of wealth and financial resources in U.S. The foreign ownership assumption is motivated by empirical evidence that a substantial share of equities is held by foreigners in many countries. The share of foreign equity holdings amounts to 13.6% in U.S. (US Treasury, 2017) and, on average, 38% in European countries (Davydoff et al., 2013). While important, the assumption of foreign ownership is not the sole driver of our results as we show in Sections 3 and 4. The assumption is also convenient to illustrate the effect of endogenous prices in complete information settings.

\(^4\)We also consider a variant of the model where the part of firm profits that belongs to foreigners or capitalists
When agent productivity is perfectly observable, the public authority can impose a lump-sum tax tailored for each individual. If all firm profits are distributed among agents inside the economy, lump-sum taxation can achieve the first-best level of social welfare since any Pareto-efficient allocation can be supported with competitive prices after some income redistribution (the second welfare theorem). However, if not all firm profits are distributed among agents inside the economy, then a Pareto-efficient allocation can no longer be supported with competitive prices. The competitive equilibrium is associated with over-production (when the main good is normal). Motivated by efficiency concerns, the public authority then imposes a positive marginal income tax to correct for the over-production.

When agent productivity is private information, the distribution of firm profits becomes an additional important factor for optimal income tax policy. In this case, not any Pareto efficient allocation can be supported with competitive prices. To illustrate, let us consider an extreme example with all agents having the same productivity type but different firm profit shares. If agents have linear utility in income, the difference in profit shares does not create any difference in the agents’ optimal labor supply. Then, all agents have the same labor income in equilibrium and are subject to the same income tax and, hence, their income cannot be freely redistributed. Overall, a tax policy based on labor income does not allow for full discretion in redistributing total agent income, which leads to the failure of the second welfare theorem.\textsuperscript{5}

The failure of the second welfare theorem leads to a binding market equilibrium condition in the optimum. The public authority can use the price level as an additional redistributive tool: a decrease in price level benefits low productivity agents as they can afford to consume more products and hurts high productivity agents as their utility is mainly influenced by the decrease in firm profits.\textsuperscript{6} Motivated by equity concerns, the public authority then favors a lower price level and a more progressive income taxation in the optimum.

To estimate the price effect on optimal income tax rates, we consider the U.S. housing market. This market is particularly suitable to illustrate our theoretical results because housing costs comprise the largest share of overall household expenditures. In particular, housing costs account for one fourth of average household expenditures in the U.S.\textsuperscript{7} Using a calibrated model receives a separate welfare weight in the social objective function (see Appendix A.3).\textsuperscript{5}

\textsuperscript{5}This result holds also if a tax policy is based on total agent income including profit shares (see Appendix A.4). The second welfare theorem can be restored only if the public authority can expropriate all firm profits.\textsuperscript{6}

\textsuperscript{6}This argument is in line with recent empirical results connecting individual earnings growth and real stock returns. Guvenen et al. (2017) show that individual earnings growth is more sensitive to a change in real stock returns for high and low income levels. This relationship for high income individuals supports our reasoning that a decrease in firm dividends hurts more wealthy individuals. For low income individuals, the sensitivity of earnings growth to real stock returns can be explained by a high correlation of real stock returns and GDP growth, i.e. labor earnings are higher in booms and lower in recession.

\textsuperscript{7}For U.S., Consumer Expenditure Survey, 2017, Table 1203. Incomes before taxes: Annual expenditure means, shares, standard errors, and coefficients of variation and Bureau of Economic Analysis, 2016, Table 23.5U. Personal Consumption Expenditures by Major Type of Product and by Major Function report 25% for the share of housing costs (including utilities) in household expenditures. Housing costs account for a similar average share of household expenditures in Europe (Eurostat, 2016).
of housing market (based on Miles and Sefton (2018) and Saiz (2010)) we find that the price effect increases optimal marginal income taxes by approximately 6 percentage points at most income levels with foreign ownership and progressive profit distribution each responsible for 3 percentage points.

We also study markets with various forms of oligopolistic competition. In oligopolistic markets, the presence of market power leads to under-production in equilibrium. As a countermeasure, the public authority wants to stimulate labor income and, thus, aggregate demand by decreasing marginal income taxes. This non-competitive effect works in the direction opposite to that of the price effect of profit distribution. At the same time, the non-competitive effect is similar to a corrective subsidy in the case of commodity taxation (see Auerbach and Hines, 2001). Using the US housing model, we show that the price effect can dominate the non-competitive effect which leads to more progressive optimal income taxes in less competitive markets.

We also investigate whether commodity and profit taxation can influence the effect of endogenous prices on income taxation. For both competitive and oligopolistic markets, we show that if firm profits are unequally distributed the price effect and commodity taxation coexist in the optimum. This is in contrast to Atkinson and Stiglitz (1976) who show that there is no need for commodity taxation in the presence of optimal income taxation. Their result, however, holds only when firm profits are taxed at 100%, which differs from our main assumption. If we allow for full profit extraction, the price effect vanishes in competitive markets. In this case, the competitive market equilibrium is constrained Pareto-efficient, which is also in line with the production efficiency theorem of Diamond and Mirrlees (1971). In oligopolistic markets, the price and anti-competitive effects vanish only if 100% profit taxation is coupled with the optimal commodity taxation (see also Myles, 1996).

Next, we relate our findings to the three tests on policy relevance proposed by Diamond and Saez (2011). First, our results are based on an economic mechanism that the second welfare theorem does not hold in incomplete-information markets when only a limited set of tax instruments is available. Second, we show that the price effect is empirically relevant and of the first order by estimating it equal to 6 percentage points using U.S. housing market (assuming that only a quarter of the economy has endogenous prices). Our results are also robust to various market structures, model specifications, and simulation assumptions. Third, our tax policy recommendation favors more progressive income tax rates, which is socially acceptable on equity grounds. Lastly, we also note that the price and non-competitive effects are of relevance for any policy with implications for income redistribution such as the design of minimum wage, basic income, welfare benefits, pensions, etc.

We postpone a detailed literature review until Section 6 and, here, briefly outline our main contributions in relation to previous papers. Compared to the growing literature on optimal
income taxation with endogenous wages in labor markets (e.g., Rothschild and Scheuer, 2013; Sachs, Tsyvinsky, and Werquin, 2016), we consider endogenous prices in product markets. The price effect is driven by the distribution of firm profits among agents inside and outside the economy rather than general equilibrium effects analyzed in this literature.

In relation to optimal income taxation literature with externalities (e.g., Rothschild and Scheuer, 2016; Lockwood et al., 2017; Rothschild and Scheuer, 2014) agents in our paper do not impose any direct externality. In contrast, agents in our model impose only pecuniary externalities. The welfare theorems imply that pecuniary externalities should not be corrected in complete information markets. If agent productivity is not perfectly observable and taxes are based on labor income, we show that a constrained Pareto-efficient allocation cannot be generally supported by a competitive equilibrium. This leads to a binding market equilibrium constraint and the price effect on optimal income taxation.

Finally, we are not aware of any paper analyzing optimal income taxation in the presence of imperfect competition. The previous literature assumes agents are price-takers. This assumption is a reasonable approximation for labor markets, but not for many product markets that are often oligopolistic. Our analysis is, however, parallel to a few important papers studying commodity taxation in oligopolistic markets (Auerbach and Hines, 2001; Myles 1987; Reinhorn, 2005). In contrast to these papers, we consider an incomplete information setting and highlight the influence of profit distribution among agents on the optimal tax schedule.

The remainder of the paper is organized as follows. Section 2 introduces the model. In Section 3, we consider competitive markets with complete and incomplete information, analyze properties of the optimal marginal income tax schedule, and provide simulation results estimating the size of the price effect. Section 4 presents our analysis for oligopolistic markets. We investigate the robustness of our results to the presence of commodity and profit taxation in Section 5. Section 6 provides a detailed literature review and Section 7 concludes. The omitted proofs are postponed to Appendix A.1. Appendices A.2-A.6 contain the extensions of our main model and some additional simulation results.

2 Model

There is a continuum of agents indexed by their productivity type \( n \) that is distributed according to probability density function \( f(n) > 0 \) with support \([n, \bar{n}]\). Agent \( n \)'s labor income is given by \( z = n\ell \), where \( \ell \) is the number of hours worked. The labor cost is represented by an increasing and convex function \( c(\ell) \). The labor income is taxed according to schedule \( T(z) \). After income tax, the agent’s disposable income is equal to \( y = z - T(z) \).

In the economy, there are two goods: a numeraire good and good X (referred to as the main good in the introduction). The numeraire good is produced with a constant return to
scale technology that results in a fixed price normalized to 1 and zero firm profits. Good X is produced with a decreasing returns to scale technology yielding positive profits. We denote $p$ and $\Pi(p)$ as the price and the profit of firms producing good X.\(^8\)

We assume that firm profits are distributed to agents in the form of dividends with agent $n$ receiving share $\xi(n) \geq 0$. Since agents with higher labor income typically possess a larger share of firm profits, we consider the case when $\xi'(n) \geq 0$ (see Wolff, 2017). Motivated by empirical evidence, we also assume that a part of firm profits can be owned by foreigners or capitalists who have no labor income, i.e., $\int \xi(n)f(n)dn = \Xi \leq 1$ (US Treasury, 2017; Davydoff et al., 2013). We also do not consider profit taxation and assume it equal to zero.\(^9\)

Overall, an agent’s income comprises of labor income and dividends (profit shares) $\tilde{y}(n) = y(n) + \xi(n)\Pi(p)$. Agents preferences for consumption are represented by an indirect utility function $v(p, \tilde{y})$, which is concave and increasing in $\tilde{y}$. Agent’s net utility is defined by

$$U(p, \tilde{y}, \ell) = v(p, \tilde{y}) - c(\ell).\quad (1)$$

The social welfare function is given by\(^10\)

$$W = \int H(U(p, \tilde{y}(n), \ell(n)))f(n)dn,\quad (2)$$

where $H$ is the social value of utility. Assuming that the public authority might have equity concerns, we let $H$ be a differentiable, increasing, and weakly concave function.

The public authority wants to maximize the social welfare function $W$ subject to three constraints. The first one is resource constraint

$$\int T(z(n))f(n)dn = \int (n\ell(n) - \tilde{y}(n) + \xi(n)\Pi(p)) f(n)dn \geq R\quad (3)$$

that ensures that the public authority covers its own expenses $R \geq 0$ that are spent solely on

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\(^8\)We do not model explicitly why firms that produce the numeraire good do not switch to a more profitable production of good X. One could think about firms having a lack of technology or patents or facing other barriers to entry. The high degree of persistent performance differences even among similar firms is a well-documented phenomenon (see, for example, Syverson (2011) for a recent survey).

\(^9\)The tax rate on firm profit in the form of domestic (qualified) dividends in U.S. is rather flat with a rate equal to 15% for most income levels. Within our framework, the optimal profit tax would be 100% leading to the production efficiency result of Diamond and Mirrlees (1971). However, only a few countries have imposed 100% profit tax and only during extreme historical periods, such as war times (see p. 392 in Atkinson and Stiglitz, 2015). One reason that we do not normally observe 100% profit tax is that it is hard to distinguish pure firm profits from return to capital or the return to entrepreneurship. One could also imagine problems for firms locating in countries that impose 100% profit taxation. For other arguments why we do not observe 100% profit tax see Ales and Sleet (2016) and Scheuer and Werning (2016).

\(^10\)In Appendix A.3, we consider a variant of the model where foreigners or capitalists who possess the remaining share of firm profits $(1 - \Xi)\Pi(p)$ receive a separate social weight $\omega$. For clarity of the exposition, we assume $\omega = 0$ in the main text.
the numeraire good. The second one is incentive compatibility constraint

\[ U(p, z(n) - T(z(n)) + \xi(n)\Pi(p), \ell(n)) \geq U(p, z(m) - T(z(m)) + \xi(n)\Pi(p), z(m)/n), \tag{4} \]

for all \( n, m \in [n, \bar{n}] \), which ensures that an agent with productivity \( n \) does not want to seek the labor income of an agent with productivity \( m \).

The third constraint is a market equilibrium condition that determines price \( p \). This condition varies across market structures. In competitive markets (Section 3), where we assume that firms are price-takers, the market equilibrium condition requires the market supply equal to the market demand for good X. In oligopolistic markets (Section 4), where each firm takes into account its influence on the level of product price, the market equilibrium condition is determined by the firm profit maximization condition.

Overall, the main difference between our framework and Mirrlees (1971) is that prices are endogenously determined in the economy. In addition, firms obtain positive profits that are progressively distributed among agents inside and outside the economy. The effect of endogenous prices and profit distribution on optimal income taxation is the main subject of our subsequent analysis.

## 3 Competitive Market

In this section, we analyze the problem of optimal income taxation in competitive markets, in which the price of good X is determined by the market equilibrium condition

\[ S(p) = \int x(p, \tilde{y}(n))f(n)dn. \tag{5} \]

On the left-hand side we have market supply \( S(p) \) and on the right-hand side the market demand for good X, where \( x(p, \tilde{y}) \) is Walrasian demand of agent with disposable income \( \tilde{y} \). It can be determined using Roy’s identity as \( x(p, \tilde{y}) = -v_p(p, \tilde{y})/v_y(p, \tilde{y}) \). We consider an increasing supply function \( S'(p) > 0 \) and zero fixed costs so that total firm profits coincide with producer surplus \( \Pi(p) = \int_0^p S(\tilde{p})d\tilde{p} \). We also assume that the demand for good X satisfies the law of demand \( x_p < 0 \) and it is either convex or concave in income. In Appendix A.2, we show how our model can be supported with a labor market. We also explain why the constant return to scale technology for the numeraire good ensures that condition (5) clears all the product and labor markets in the economy.

To highlight main ideas, we start with the case of complete information when agents’ productivity is observable. We then proceed to the case of incomplete information.
3.1 Complete Information

Assume that productivity types and profit shares are observable and that the public authority can design type-specific taxes, i.e., $T(z, n)$. The public authority’s problem is then to find the price $p$, income schedule $\tilde{y}(n)$, and individual labor supply schedule $\ell(n)$ that maximize

$$\max_{p, \tilde{y}(n), \ell(n)} \int H(v(p, \tilde{y}(n)) - c(\ell(n)))f(n)dn \quad \text{subject to (3) and (5)}.$$

The Lagrangian of the public authority’s problem is given by

$$\int \{H(v(p, \tilde{y}(n)) - c(\ell(n))) + \lambda(n\ell(n) - \tilde{y}(n) + \xi(n)\Pi(p) - R) + \gamma(S(p) - x(p, \tilde{y}(n)))\}f(n)dn,$$

where $\lambda$ and $\gamma$ are multipliers corresponding to constraints (3) and (5), respectively. The first-order conditions are given by

$$\tilde{y}(n) : H'v_y - \lambda - \gamma x_y = 0,$$

$$\ell(n) : -H'c_\ell + \lambda n = 0,$$

$$p : \int (H'v_p + \lambda \xi(n)\Pi'(p) + \gamma(S'(p) - x_p))f(n)dn = 0.$$

The optimal marginal income tax $t(z, n) = T_z(z, n)$ is determined by the individual utility maximization problem $\max_z U(p, z - T_z(z, n) + \xi(n)\Pi(p), z/n)$ that implies $t = 1 - c_\ell/(nv_y)$.

Taking into account that $\int \xi(n)f(n)dn = \Xi$ and $v_p = -xv_y$ (Roy’s identity), we obtain from first-order conditions (6)–(8) the following result.

**Theorem 1.** In competitive markets with complete information, the optimal marginal income tax is determined by

$$\frac{t}{1 - t} = \frac{\gamma}{\lambda} x_y,$$

where

$$\gamma = \frac{\lambda S(p)(1 - \Xi)}{S'(p) - \int (x_p + x_y x)fdn}.$$

The theorem implies that if all profits are distributed among agents in the economy ($\Xi = 1$) the optimal marginal income tax has to be zero. In other words, the maximum value of social value function is achieved using a lump-sum taxation targeted to each agent. This is a well-known result in the public economics literature (see Atkinson and Stiglitz, 2015) that also follows from the second welfare theorem. More specifically, let us consider a Pareto efficient allocation that maximizes objective (2) subject to budget constraint (3) for a given level of firm profits. Then, by the second welfare theorem it is always possible to find a redistribution function $\Pi(p)$ such that

$$\Pi(p) = \frac{1}{ \int (x_p + x_y x)fdn}.$$

We maximize over $p$ because the price is implicitly determined by equation (5). If we had an explicit price function, we could use it into indirect utility and maximize only over $(\tilde{y}(n), \ell(n))$. 

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of agents’ income and price $p$ that support the aforementioned allocation in a competitive equilibrium. Hence, the market equilibrium condition (5) is not binding that results in $\gamma = 0$.

When $\Xi < 1$, the second welfare theorems does not hold any longer, i.e., not any Pareto efficient allocation can be supported with a competitive equilibrium. Hence, the market equilibrium condition (5) is binding. Intuitively, labor supply contributes to firm profits that partly belong to foreigners. Hence, a competitive equilibrium results in an inefficient production level. To correct for this inefficiency, the public authority imposes an additional income tax that aims to correct labor supply bringing the competitive equilibrium outcome closer to Pareto efficiency.

We also note that the market equilibrium multiplier has to be positive $\gamma > 0$ in the presence of foreign ownership $\Xi < 1$. This follows from the slope of market supply being strictly positive $S'(p) > 0$ and the law of compensated demand $h_p = x_p + x_y x \leq 0$, where $h$ is the Hicksian (compensated) demand function. Finally, since the multiplier of the resource constraint is positive $\lambda > 0$, the sign of the corrective tax term is determined by $x_y$. Therefore, the optimal marginal income tax (9) is positive for normal goods ($x_y > 0$) and negative for inferior goods ($x_y < 0$). Furthermore, our assumption that the demand is either convex or concave in income ($x_{yy} \geq 0$ or $x_{yy} \leq 0$ at all income levels) also implies that $x_y$ is increasing for luxury goods and decreasing for necessity goods (see the proof of Corollary 1).

**Corollary 1.** In competitive markets with complete information and foreign ownership, the optimal marginal income tax is positive for normal goods and negative for inferior goods, increasing for luxury goods and decreasing for necessity goods.

### 3.2 Incomplete Information

Now we consider the case when agent productivity is private information. The public authority cannot then condition tax payments on agent types and, therefore, has to take into account the incentive compatibility constraints (4). When agent preferences are non-homothetic and firm profits unequally distributed, agent indirect utility (1) can violate the single-crossing property (see, e.g., Mirrlees (1976)). For better tractability, we consider only the case of homothetic preferences in the main text. This case also corresponds to our numerical simulations in Section 3.3, where we consider constant elasticity of substitution (CES) preferences. Homothetic preferences imply linear indirect utility that we express as

$$v(p, y) = a(p)y,$$

12 More precisely $\lambda \geq 0$, but $\lambda > 0$ when the resource constraint is active (which we assume).

13 The proof also contains an example that our assumption that the demand is either convex or concave with respect to income is generally indispensable.

14 Recent empirical studies find small income effects on individual labour supply (see a discussion in Sorensen, 2009), which also justifies the use of homothetic preferences in our analysis.
where \(a(p)\) is some decreasing function. For the linear case, we also assume that welfare function \(H\) is *strictly concave*. The analysis of the non-homothetic case is relegated to Appendix A.3.

Let us denote an agent’s utility from revealing his productivity type truthfully as

\[
u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n).
\]

If the truthful revelation is optimal then

\[
u(n) = \max_m (v(p, y(m) + \xi(n)\Pi(p)) - c(z(m)/n)). \tag{10}
\]

Using the envelope theorem, we obtain the following first-order condition

\[
u'(n) = c\ell z(n)/n^2 + \xi'(n)a(p)\Pi(p). \tag{11}
\]

Note that the presence of profits does not change incentive compatibility condition in the linear case, because the term with profit shares can be taken out of the maximization problem (10). Hence, the second-order condition that ensures truthtelling to be optimal is income schedule \(y(n)\) being non-decreasing as in the standard case (see Mirrlees, 1976). At the same time, the profit distribution \(\xi(n)\) does influence the level of agent utility \(u(n)\). This will be a quite important factor since the public authority has equity concerns (\(H\) is strictly concave).

The public authority’s problem is now to maximize the social welfare function subject to the resource, market equilibrium, and incentive compatibility constraints:

\[
\max_{p,\tilde{y}(n),\ell(n)} \int H(v(p, \tilde{y}(n))) - c(\ell(n))f(n)dn \quad \text{subject to (3), (5), and (11)}.
\]

It is convenient to change optimization variables \(\{p, \tilde{y}(n), \ell(n)\}\) to \(\{p, u(n), \ell(n)\}\), where the levels of utility are defined by \(u(n) = a(p)\tilde{y}(n) - c(\ell(n))\). From the latter expression we can invert disposable income \(\tilde{y}(n)\) and express it as \(\tilde{y} = r(p, u, \ell) \equiv \frac{u(n) + c(\ell)}{a(p)}\). The maximization problem can then be written as

\[
\max_{p, u(n), \ell(n)} \int H(u(n))f(n)dn \\
\text{s.t.} \\
\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)]f(n)dn \geq R \quad \tag{12} \\
\int [S(p) - x(p, r(p, u(n), \ell(n)))]f(n)dn = 0 \quad \tag{13} \\
u'(n) - \xi'(n)a(p)\Pi(p) - c\ell(n)/n = 0 \quad \tag{14}
\]

Let \(\lambda, \gamma,\) and \(\mu(n)\) be multipliers corresponding to constraints (12), (13), and (14), respectively.
After the integration by parts and taking into account the transversality condition \( \mu(n) = \mu(\overline{n}) = 0 \), we express the Lagrangian of the maximization problem as

\[
\int \left\{ [H(u(n)) + \lambda n \ell(n) - r(p, u(n), \ell(n))] + \xi(n)\Pi(p) - R + \gamma(S(p) - x(p, r(p, u(n), \ell(n))))\right\} f(n) - \\
\mu'(n)u(n) - \mu(n)(\xi'(n)a(p)\Pi(p) + c_\ell(\ell(n)/n))\right\} dn.
\]

Given \( r_u = 1/v_y, r_\ell = c_\ell/v_y, \) and \( r_p = -v_p/v_y = x \), the first-order conditions are

\[
u(n) : \left[H_u - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0 \tag{15}
\]

\[
\ell(n) : \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y}\right] f(n) - \mu(n)(c_\ell + c_\ell\ell(n))/n = 0 \tag{16}
\]

\[
p : \int \left\{ [\lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_yx)] f(n) - \\
\mu(n)\xi'(n)(a_p\Pi(p) + a\Pi'(p))\right\} dn = 0. \tag{17}
\]

From the individual maximization problem we have that the optimal marginal income tax satisfies \( t = 1 - c_\ell/(nv_y) \) (see p. 8). Taking into account that \( \Pi'(p) = S(p) = \int x(p, \tilde{y}) f(n) dn, \int \xi(n) f(n) dn = \Xi \), and that \( 1 + \ell c_\ell/c_\ell = (1 + E^u)/E^c \), where \( E^c \) is the elasticity of compensated labor supply, and \( E^u \) is the elasticity of uncompensated labor supply (see the proof of Theorem 2), we obtain the following result.

**Theorem 2.** In competitive markets, the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = \frac{v_y\mu + E^u}{\lambda nf} + \frac{\gamma x_y}{\lambda}, \tag{18}
\]

where

\[
\gamma = \frac{\lambda S(p)(1 - \Xi) + (a_p\Pi(p) + a\Pi'(p))\int \mu(n)\xi'(n) dn}{S'(p) - \int (x_p + x_yx)f(n) dn}. \tag{19}
\]

The optimal income tax formula has two terms in the case of incomplete information. The first one is the standard Mirrleesian term that balances work incentives with the public authority’s redistributive and budgetary objectives (see Mirrlees 1971, 1976). The second term captures the price effect that also appears in Theorem 1 for the complete information case.

The first part of the price effect \( \lambda S(p)(1 - \Xi)/(S' - \int (x_y + x_yx)f dn) \) corrects inefficiency associated with the presence of foreign ownership. In contrast to the complete information case, however, the price effect does not vanish in the absence of foreign ownership, because the public authority has also equity concerns (the second part). To see this, note \( \int \mu(n)\xi'(n) dn = \).
\( \int H_u(1 - \xi(n)) f(n) dn \). The latter expression is positive when the public authority has equity concerns \( H_u > 0 \) and profits are unequally distributed among agents \( \xi'(n) > 0 \). In addition, agent utility equals \( a(p)(y + \xi(n)\Pi(p)) \). Hence, \( a_p\Pi + a\Pi_p \) corresponds to a change in utility associated with the presence of dividends, which is also positive since

\[
\Pi'(p) = S(p) = \int x(p, \hat{y}) f(n) dn = -\frac{a_p(p)}{a(p)} \int \hat{y}(n) f(n) dn \geq -\frac{a_p(p)}{a(p)} pS(p) \geq -\frac{a_p(p)}{a(p)} \Pi(p),
\]

where the first equation follows from the definition of firm profits \( \Pi(p) = \int_{0}^{p} S(\hat{p}) d\hat{p} \), the second from market equilibrium condition, and the third from Roy’s identity. The first inequality follows from an agent’s individual budget constraint, and the second one from \( \int_{0}^{p} S(\hat{p}) d\hat{p} \leq pS(p) \). Intuitively, a decrease in equilibrium prices benefits low productivity agents because they can afford to consume more products. At the same time, a decrease in equilibrium price hurts high productivity agents, whose utility is mostly influenced by a level of firm profit shares. Overall, the public authority uses the equilibrium price level on par with income taxation to achieve its equity objectives in the optimum.

**Corollary 2.** In competitive markets with incomplete information, the price effect arises because of both efficiency and equity concerns. The price effect term does not vanish in the absence of foreign ownership.

One could also explain the price effect in the absence of foreign ownership by the failure of the second welfare theorem in the presence of incomplete information. Note that agents’ income consists of two parts: labor income and profit shares. However, income redistribution can only be based on the first part – labor income. Because of a limited set of tax instruments, the second welfare theorem does not hold. To illustrate, consider a simple example when all agents have the same level of productivity but different profit shares. Linear utility implies that they all choose the same level of labor and earn the same level of labor income. Then, irrespective of their total income all agents pay the same amount of tax which restricts the set of distributional objectives that the government can achieve.

Overall, Theorem 2 suggests that pecuniary externalities need to be corrected in the markets with incomplete information, unequal distribution of firm profits, and the presence of equity concerns. In Appendix A.4 we also establish that this insight is robust if we assume that income tax is based on total agent income (labor income plus dividends) instead of labor income alone.  

The failure of the second welfare theorem leads to the competitive equilibrium condition

\[15\text{Since agent indirect utility is linear, terms } x_y \text{ and } v_y \text{ are constants. Hence, (15) implies that } \int H_u f(n) dn = \lambda + \gamma x_y / v_y \text{ and } \int \mu(n) \xi(n) dn = - \int \mu' \xi(n) dn = \int H_u (1 - \xi(n)) f(n) dn.\]

\[16\text{However, this result relies on the assumption that profit shares cannot be taxed away. If firm profits can be taxed, it is optimal to apply 100% profit tax restoring the constrained-Pareto efficiency of competitive equilibrium.}\]
being binding in the optimum. We now establish that $\gamma > 0$. As discussed above, when $\xi'(n) > 0$ and $\Xi < 1$ we have $\lambda S'(p)(1-\Xi) > 0$ and $\int \mu(n)\xi'(n)dn = \int H_u(1-\xi(n))f(n)dn > 0$. As in the case of complete information, we also have $S'(p) > 0$ and $h_p = x_p + x_y^p \leq 0$. Hence, the properties of the price term are determined by $x_y$ and we obtain the following result.\footnote{A similar analysis also applies to the environment when agents have non-labor endowments $e(n)$ instead of profit shares $\xi(n)\Pi(p)$. In the absence of foreign ownership $\Xi = 1$, we then have $\gamma < 0$ because $(a(p)e(n))_p < 0$. Hence, the price effect on optimal income tax is negative. Intuitively, an additional endowment increases the aggregate demand. Hence, it raises the equilibrium price and decreases willingness to work for agents. In order to induce the same amount of labor the public authority has to reduce marginal income taxes.}

**Corollary 3.** In competitive markets with incomplete information, the price effect term is positive for normal goods and negative for inferior goods.

We do not have an analog of the second part of Corollary 1 because term $x_y$ is constant in income when agent indirect utility is linear. We finally note that the profit redistribution term in (19) disappears only in the absence of foreign ownership $\Xi = 1$ and when firm profits are equally distributed $\xi'(n) = 0$. In this case the income tax instrument can correct for the firm profit distribution by imposing an equal lump-sum tax on all agents.

Another corollary of Theorem 2 is that the seminal end-point results of Sadka (1976) and Seade (1977) no longer hold with price endogeneity. When prices are fixed, the optimal tax formula has only the incentive term and the transversality condition implies that the optimal marginal tax for the most productive agents is zero. Intuitively, if the marginal tax were positive, the public authority could instead impose zero tax on any income in excess of the income of the most productive agents. Then, these agents would respond by exerting an additional effort to achieve the previously not feasible level of utility. Since the amount of tax collected would not change while the most productive agents would obtain higher utility, the overall welfare would increase. With the binding market equilibrium constraint, the aforementioned tax relief would have implications for product prices. Therefore, the public authority cannot increase the utility of the most productive agents without influencing the other agents.

### 3.3 Numerical Simulations

In this subsection, we provide numerical simulations to estimate the size of the price effect on optimal income taxes. To accomplish this goal we consider the U.S. housing market that suits particularly well to quantify our results because housing expenditure (including utilities) is the largest consumption item and accounts for about 25% of total household expenditures.\footnote{See Bureau of Economic Analysis, 2017, Table 2.3.5U. *Personal Consumption Expenditures by Major Type of Product and by Major Function* and Consumer Expenditure Survey, 2017, Table 1203. *Income before taxes: Annual expenditure means, shares, standard errors, and coefficients of variation.* Similar numbers are also observed in the European Union, where Eurostat (2016) reports 24.4% for an average housing expenditure share.}

To model housing demand, we consider the setting of Miles and Sefton (2018) that has
constant elasticity of substitution (CES) utility function

\[ u(x, g) = (ag^{1-1/\rho} + (1 - a)x^{1-1/\rho})^{\frac{\rho}{\rho-1}}, \]  

(20)

where \( x \) denotes the consumption of housing, \( g \) denotes the consumption of the other goods, \( a \) is the weight on the other goods (relative to housing consumption) in utility, and \( \rho \) is the elasticity of substitution between housing and the other goods. Following Miles and Sefton (2018) we take \( a = 0.85 \) and \( \rho = 0.6. \)\(^{19}\) This utility specification results in the absolute value of the price elasticity of demand of 0.7 and the unit income elasticity of demand in our simulations, which are consistent with empirically estimated elasticities in the literature.\(^{20}\)

We model the supply of housing using the standard constant price elasticity function \( S(p) = sp^\varepsilon \), where \( s \) is a scale parameter and \( \varepsilon \) is the price elasticity of supply.\(^{21}\) We calibrate scale parameter \( s \) to match the average share of housing expenditure of 25%, which renders \( s = 0.5 \). The estimates of the price elasticity of supply \( \varepsilon \) vary significantly across countries and even across cities. In particular, Saiz (2010) show that \( \varepsilon \) highly depends on geographical and regulatory constraints within U.S. metropolitan areas. Drawing on his estimates for the average U.S. metropolitan area, we take \( \varepsilon = 1.75. \)\(^{22}\) In Appendix A.6, we also present the results for inelastic supply \( \varepsilon = 0 \) that better describes housing supply in large U.S. coastal cities (e.g., Boston, San-Francisco) and in countries with a rigid housing planning system (e.g., United Kingdom).\(^{23}\) We also present results for the price elasticity of supply \( \varepsilon = 3 \) that is closer to the estimates obtained in Epple and Romer (1991) and Green et al. (2005).

We use lognormal function \( \ln(m, \sigma) \) for the distribution of agent productivities \( F(n) \). In particular, following Kanbur and Tuomala (2013), we consider the parameter values of mean \( m = e^{-1} \) and standard deviation \( \sigma = 0.7 \) that are found to offer a good match with the empirical distribution of personal income in the U.S. In Appendix A.6, we also show that our estimation results remain unchanged if we consider a composite lognormal-Pareto distribution that fits better the upper tail of income distribution (Diamond and Saez, 2011; Sachs et al., 2016). Following Saez et al. (2012) and Kanbur and Tuomala (2013), we assume the labor supply elasticity of 1/3, cost function \( c(\ell) = \ell^4/4 \), and logarithmic social welfare function \( H(u) = \ln u \).

We also set the level of public expenditures \( R = 0 \) that yields income tax receipts of about 17% of the total economy. This matches the ratio of the sum of personal income tax receipts and social insurance contributions to gross domestic product in the U.S. (Bureau of Economic

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\(^{19}\)For other estimates of the CES model for housing see Määtännönen and Terviö (2014) and Li et al. (2016).

\(^{20}\)For the U.S. housing market, the estimates of price elasticity of demand are in the range between 0.1 and 0.7 in absolute numbers and of income elasticity of demand – between 0.27 and 2.7; for European housing market, these ranges are 0.156 – 0.97 and 0.62 – 1.23, respectively. Arrazola et al. (2014) provides an excellent recent survey on elasticity estimates.

\(^{21}\)This functional form corresponds to a Cobb-Douglas production function (see, e.g., Epple et al. (2010)).

\(^{22}\)For similar estimates see also DiPasquale and Wheaton (1994) and Harter-Dreiman (2004).

Figure 1. The distribution of financial resources.

Note: Points show the shares of financial resources held by U.S. population. Financial resources are defined as net worth less equity in owner-occupied housing. Source: Wolff (2017, Table 2).

Analysis, 2017, Table 3.1. Government Current Receipts and Expenditures and Table 1.1.5. Gross Domestic Product).

Next, we set the share of profits held by agents in the economy at $\Xi = 0.85$ that approximately equals the domestic share of equity holdings in the U.S. of 86.4% (see US Treasury, 2017). This estimate should be considered as an upper bound for developed countries. In Europe, on average, only 62% of equity shares are held by domestic investors, whereas it is 50% in the United Kingdom, 60% in France, and 70% in Germany (Davydoff et al., 2013).

Lastly, we approximate the distribution of firm profits among agents by the empirical distribution of financial resources among U.S. households (see Figure 1). We take the data from Wolff (2017, Table 2) who define personal financial resources as net worth minus net equity in owner-occupied housing. Our approximation results do not change much if we consider other measures of wealth, e.g., net worth.

Figure 2 presents our main simulation results. The left diagram shows the optimal marginal tax schedules for the cases with (i) fixed price and equal profit distribution (dotted line), (ii) fixed price and empirical profit distribution (dashed line), and (iii) endogenous price and empirical profit distribution (solid line). In all cases we take housing prices be the same and equal to the equilibrium price of case (iii). Case (i) serves as a benchmark against which we

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24To obtain $\xi(n)$, we first obtain the cumulative distribution of profits in the population, $\Xi(n)$, by fitting it with the sum of two exponents $\Xi(n) = \hat{a}\exp(\hat{b}F(n)) + \hat{c}\exp(\hat{d}F(n))$, where $F(n)$ is the cumulative distribution of agent productivities, and $\hat{a}, \hat{b}, \hat{c}, \text{and} \hat{d}$ are estimated parameters. Then, we obtain function $\xi(n)$ by approximating derivative of $\Xi(n)$ and by normalizing $\int \xi(n)f(n)dn = 0.85$. 

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Figure 2. Optimal income taxation in the presence of endogenous prices.

**Note:** The left diagram presents the optimal marginal income tax for the economy with competitive housing market and (i) fixed price and equal profit distribution (dotted line); (ii) fixed price and empirical profit distribution (dashed line); and (iii) endogenous price and empirical profit distribution (solid line). The right diagram depicts the changes in tax rates for cases (ii) and (iii) against case (i).

estimate the size of the price effect. The right diagram shows the change in optimal marginal income tax (ii)–(i) and (iii)–(i) compared to the benchmark case.

The numerical simulations show that the endogenous price has an upward effect on optimal marginal income tax rates. The overall increase in marginal income tax can be attributed to two factors. The first one is the presence of foreign ownership $\Xi < 1$ that leads to the tax correction due to efficiency concerns. The second one is due to the presence of progressive distribution of firm profits $\xi'(n) > 0$ that leads to the tax correction due to equity concerns. Each factor contributes approximately three percentage points to the benchmark case, yielding a total increase of about 6 percentage points.

## 4 Oligopolistic Competition

In this section, we consider markets with various degrees of oligopolistic competition. In particular, we consider $M \geq 1$ identical firms with each firm $i$ having a convex cost function $K(X_i)$ of producing $X_i$ units of good $X$. Let us denote the inverse aggregate demand function by $p(X)$, where $X = \int x(p, \tilde{y}(n))f(n)dn$.\footnote{The assumption $x_p < 0$ ensures that the inverse aggregate demand function $p(X)$ is well defined.} We also assume that demand function is concave or not too convex such that it second derivative is small (see p. 18 for more details).

We can write firm $i$’s profit as $X_ip(X) - K(X_i)$, where the market clearing condition ensures $\sum_{i=1}^{M} X_i = X$. To model various forms of oligopolistic competition, we assume that when firm $i$ maximizes profits it forms a belief, or a conjectural variation, about the other firms response.
to a unit change in its output level,

$$\frac{d(\sum_{j \neq i} X_j)}{dX_i} = \theta, \text{ where } -1 \leq \theta \leq M - 1.$$  \hfill (21)

The first order condition for firm profit maximization problem can then be expressed by

$$p(X) - K'(X_i) + (1 + \theta)X_i p'(X) = 0.$$  \hfill (22)

The conjectural variation model was first introduced by Bowley (1924) to capture a wide variety of oligopolistic competition models. For instance, competitive equilibrium corresponds to $\theta = -1$ when firms expect the rest of industry to absorb exactly its output expansion, conjectural variation $\theta = 0$ represents the Cournot-Nash model where each firm considers the output of the other firms unchanged, and the collusive behavior of firms maximizing their joint profits leads to $\theta = M - 1$.\(^{26}\)

For subsequent analysis, it is convenient to express the market equilibrium condition in terms of product price rather than quantity. In what follows, we also limit attention to symmetric equilibria $X_i = X/M$. The market equilibrium condition then reduces to

$$J(X, X_p, p) \equiv -X_p \left( p - K' \left( \frac{X}{M} \right) \right) - (1 + \theta) \frac{X}{M} = 0,$$  \hfill (23)

where $X_p = \int x_p(p, \tilde{y}(n))f(n)dn$. We denote the total firm profits as $\Pi(p, X) = pX - MK\left( \frac{X}{M} \right)$.

Finding the optimal marginal income tax in oligopolistic markets with endogenous prices is generally a complicated problem with several effects interacting with each other. For the purpose of clarity and to illustrate a novel effect due to imperfect competition, we assume that all firm profits remain in the economy $\Xi = 1$ and that they are equally distributed among agents, i.e., $\xi(n) = 1$ for all $n$. A general analysis involving both the price effect and the non-competitive effects is considered in Appendix A.5. The public authority’s problem for oligopolistic markets can then be written as follows.

$$\max_{p, u(n), \ell(n)} \int H(u(n))f(n)dn$$

$$\begin{cases}
    u'(n) - c\ell(n)/n = 0 \quad (\mu(n), \text{ incentive compatibility}) \\
    \int [n\ell(n) - r(p, u(n), \ell(n))]f(n)dn + \Pi(p, X) \geq R \quad (\lambda, \text{ resource constraint}) \\
    J(X, X_p, p) = 0 \quad (\gamma, \text{ market equilibrium}) \\
    X - \int x(p, r(p, u(n), \ell(n)))f(n)dn = 0 \quad (\alpha_1, \text{ market demand}) \\
    X_p - \int x_p(p, r(p, u(n), \ell(n)))f(n)dn = 0 \quad (\alpha_2, \text{ market demand slope})
\end{cases}$$

---

\(^{26}\)For an excellent overview of various conjectural variational parameters see Perry (1982).
where function $r$ determines agent income $\tilde{y} = r(p,u,\ell) \equiv \frac{u+c(\ell)}{a(p)}$, and Lagrange multipliers are introduced next to the corresponding constraints. The Lagrangian can then be written as

$$
\mathcal{L} = \int [(H(u(n)) + \mu(n)(u'(n) - c_\ell(\ell(n))\ell(n))/n)
+ \lambda(n\ell(n) - r(p,u(n),\ell(n)) + \Pi(p,X) - R) + \gamma J(X,X_p,p)
+ \alpha_1(X - x(p,r(p,u(n),\ell(n)))) + \alpha_2(x_p - x(p,r(p,u(n),\ell(n))))f(n)]dn
$$

After the integration by parts and taking into account transversality condition $\mu(n) = \mu(\bar{n}) = 0$, $\Pi_p(p,X) = X$, and $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_p = -v_p/v_y = x$, the first-order conditions are

\begin{align*}
  u(n) : & \quad \left( H_u - \frac{\lambda + \alpha_1x_y + \alpha_2p_{y}x}{v_y} \right) f(n) - \mu'(n) = 0, \quad (24) \\
  \ell(n) : & \quad \left( \lambda n - \frac{c_\ell}{v_y} (\lambda + \alpha_1x_y + \alpha_2p_{y}x) \right) f(n) - \mu(n)(c_\ell + \ell c_\ell)/n = 0, \quad (25) \\
  p : & \quad \gamma J_3 - \int (\alpha_1(x_p + x_yx) + \alpha_2(x_{pp} + x_{py}x))f(n)dn = 0 \quad (26) \\
  X : & \quad \alpha_1 + \gamma J_1 + \lambda \Pi_X(p,X) = 0, \quad (27) \\
  X_p : & \quad \alpha_2 + \gamma J_2 = 0. \quad (28)
\end{align*}

where $J_1$, $J_2$, and $J_3$ are partial derivatives of $J$. We note that $J_1 = X_pK''(\frac{X}{M})\frac{1}{M} - \frac{(1+\delta)}{M} < 0$, $J_2$ equals to the negative of firm’s markup $\Pi_X(p,X) = p - K'(\frac{X}{M}) \geq 0$, and $J_3 = -X_p > 0$. By substituting (27) and (28) in (25) and (26) we obtain the following result.

**Theorem 3.** In oligopolistic markets, the optimal marginal income tax is determined by

$$
\frac{t}{1-t} = \frac{1 + E'' \mu y}{E' \lambda} - \frac{\gamma J_1 + \lambda \Pi_X}{\lambda} x_y - \frac{\gamma}{\lambda} J_2 x_{py},
$$

where

$$
\gamma = \frac{-\lambda \Pi_X(p,X) \int (x_p + x_yx)f(n)dn}{J_3 + J_1 \int (x_p + x_yx)f(n)dn + J_2 \int (x_{pp} + x_{py}x)f(n)dn}
$$

We observe that in addition to the standard Mirrleesian term there are two corrective terms that together form the non-competitive effect. The first term corresponds to the influence of a change in income distribution on optimal marginal income tax through the market demand’s level and the second term corresponds to the influence through the market demand’s slope. The second corrective term is absent in competitive markets where only the demand level determines the equilibrium price and quantity.

We now note that if demand is concave $x_{pp} < 0$ or not too convex, i.e., $\int x_{pp}f(n)dn$ is small, then Lagrange multiplier corresponding to market equilibrium is positive $\gamma \geq 0$. This follows from $\lambda > 0$, firm’s markup $\Pi_X \geq 0$, $\text{sign}(x_{py}) = \text{sign}(x_p) = -1$ for a linear indirect utility, and
the derivative of aggregate compensated demand being non-positive $\int (x_p + x_y x) f(n) dn \leq 0$. The fact $\gamma \geq 0$ now implies that both corrective terms are non-positive. In particular,

$$\frac{\gamma J_1 + \lambda \Pi_X}{\lambda} x_y = -\frac{\Pi_X (J_3 + J_2 \int (x_{pp} + x_{py} x) f(n) dn) x_y}{J_3 + J_1 \int (x_p + x_y x) f(n) dn + J_2 \int (x_{pp} + x_{py} x) f(n) dn} \leq 0$$

because $x_y > 0$ and $\Pi_X \geq 0$ and the second term is negative because $\gamma J_2 \leq 0$. Finally, we note that the absolute value of the non-competitive effect is determined by firm’s markup $\Pi_X$. In particular, the first term is proportional to firm’s markup since $\gamma \sim \Pi_X$. The second term is, however, proportional to firm’s markup squared since both $\gamma \sim \Pi_X$ and $|J_2| \sim \Pi_X$. Both terms disappear in competitive markets where the markup is zero.

Overall, in oligopolistic markets the optimal income taxes feature the non-competitive effect which works in the direction opposite to that of the price effect. Intuitively, the equilibrium of imperfectly competitive market is associated with under-production compared to constrained Pareto-efficient allocation. Hence, the public authority wants to stimulate agents to work more in order to increase market demand and obtain an outcome closer to the constrained Pareto optimum. To relate our result to previous papers, the non-competitive term is similar to a subsidy in optimal commodity taxation literature (see Myles, 1989).

In Appendix A.5, we also analyze optimal income taxation in oligopolistic markets when both non-competitive and price effects are present for any value of $\Xi \in [0,1]$ and progressive distribution of firm profits $\xi(n)$. These effects work in the opposite directions with the price effect advocating for higher marginal income taxes while the non-competitive effect for lower marginal income taxes. We illustrate both effects with numerical simulations in the following section.

**Numerical Simulations: Oligopolistic Competition**

We consider the numerical estimation of optimal income tax rates in oligopolistic markets using the U.S. housing market framework introduced in Section 3. We infer production cost function $K$ from the structure of the competitive market studied previously. In this market, the profit maximization problem yields $p - K'(\frac{X}{M_0}) = 0$, where $X$ is equal to the market supply $S(p) = sp^\varepsilon$ in market equilibrium and $M_0$ is some fixed number of firms. Thus, for the marginal cost function we obtain

$$K'(X_i) = \left(\frac{X_i M_0}{s}\right)^{\frac{1}{\varepsilon}}$$

where $s = 0.5$ and $\varepsilon = 1.75$ as used in our analysis of the competitive market and we set $M_0 = 2$. All other parameters of the model remain the same.

To illustrate both the non-competitive effect and the price effect we consider two cases: (i) equal profit distribution (non-competitive effect) and (ii) empirical profit distribution (both
Figure 3. Optimal income taxation for various degrees of conjectural variation.

Note: The figure illustrates the optimal marginal income tax schedules for various market structures $\theta \in \{0, -0.5, -1\}$, where $\theta = 0$ stands for the Cournot-Nash competition model and $\theta = -1$ for perfect competition. The left diagram depicts the schedules for the case of the equal profit distribution (only non-competitive effect), and the right diagram depicts the schedules for the case of the empirical profit distribution (both price and non-competitive effects).

In both cases, we vary conjectural variation $\theta = 0$ (Cournot-Nash), $\theta = -0.5$, and $\theta = -1$ (perfect competition) in order to see how the non-competitive effect and the overall effect change with market structure. In the case of equal distribution of firm profits, the public authority lowers income tax rates in order to stimulate labor supply and, thus, to increase aggregate market demand to offset the non-competitive effect of oligopolistic markets. The reduction in tax rates is larger for less competitive markets (larger $\theta$) as shown in the left diagram. In the case of empirical distribution of profits, we observe the interplay between the price effect and the non-competitive effect for the U.S. housing market framework. Overall, the price effect dominates the non-competitive effect. This leads to a more progressive optimal income tax schedule in less competitive markets (the right diagram).

5 Commodity and Profit Taxation

We have demonstrated in the previous sections that the optimal income tax policy has to take into account industrial structure and firm profit distribution among agents inside and outside economy. The question arises whether the price and non-competitive effects survive in the presence of other types of taxation. We study this question below.

Let us first consider commodity taxation in competitive markets. We assume that the public authority can impose a commodity tax $b$. In this case, consumer price $q$ differs from producer price $p$ with $b = q - p$. Negative commodity tax $b < 0$ is interpreted as a subsidy.
The public authority maximization problem can then be written as

\[
\max_{p, q, u(n), \ell(n)} \int H(u(n)) f(n) dn
\]

\[
s.t. \left\{ \begin{array}{l}
\int (n\ell(n) - r(q, u(n), \ell(n))) f(n) dn + \Xi \Pi(p) + (q - p) S(p) \geq R \\
S(p) = \int x(q, y(n)) f(n) dn. \\
u'(n) - \xi'(n) a(q) \Pi(p) - c_\ell \ell(n)/n = 0
\end{array} \right.
\]

After the integration by parts and taking into account the transversality condition \( \mu(\omega) = \mu(\Omega) = 0 \) the Lagrangian of the maximization equals

\[
\int \{[H(u(n)) + \lambda(n\ell(n) - r(q, u(n), \ell(n))) + \Xi \Pi(p) + (q - p) S(p) - R] + \gamma(S(p) - x(q, r(q, u(n), \ell(n))))[f(n) - \mu'(n)u(n) - \mu(n)(\xi'(n) a(q) \Pi(p) + c_\ell \ell(n)/n)\} dn.
\]

Taken into account \( r_u = 1/v_y, r_\ell = c_\ell/v_y, \) and \( r_q = -v_q/v_y = x, \) the first-order conditions are

\[
u(n): \left[ H_u - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0
\]

\[
\ell(n): \left[ \lambda n - \frac{(\lambda + \gamma x_y) c_\ell}{v_y} \right] f(n) - \mu(n)(c_\ell + c_\ell \ell(n))/n = 0
\]

\[
p: \lambda(-S(p)(1 - \Xi) + (q - p) S'(p)) + \gamma S'(p) - \int \mu(n) \xi'(n) dn a(q) \Pi'(p) = 0
\]

\[
q: \int \{\gamma(-x_q - x_q x) f(n) - \mu(n) \xi'(n)(a_q \Pi(p))\} dn = 0.
\]

Equation (30) implies that the optimal income tax formula is the same as in Theorem 2

\[
\frac{t}{1 - t} = \frac{\mu(n)v_y}{\lambda n f} \frac{1 + E^u}{E^x} + \frac{\gamma x_y}{\lambda},
\]

where \( \mu \) is determined by (29) as previously. To derive the expression for \( \gamma, \) let us denote \( H_q = \int (x_q + x_q x) f(n) dn \) and \( M \xi = \int \mu(n) \xi'(n) dn. \) Equation (32) then implies

\[
\gamma = \frac{a_q \Pi M \xi}{-H_q} < 0.
\]

By substituting the latter expression into (31) we obtain an equation that determines commodity tax \( b = q - p \)

\[
\lambda(-S(1 - \Xi) + (q - p) S(p)) = \frac{M \xi}{H_q} (a_q \Pi S_p + a \Pi_p H_q) \geq 0
\]
We obtain that commodity and income taxation are two different tax instruments that are both used by the public authority to achieve a socially desirable outcome. The commodity tax in the optimum decreases producer price leading to a lower level of profits. The decrease in profits mainly influences high productivity agents who have more profit shares (equity concerns). Commodity taxation also increases the consumer price that leads to decrease in utility for all agents (efficiency loss). However, the presence of revenue from commodity taxation relaxes the resource constraint and leads to a decrease in income tax for all agents (see (33)) (efficiency gain). A proper balance of equity and efficiency concerns then determines the optimal tax policy. Overall, we obtain the following result.

**Theorem 4. (Commodity taxation in competitive markets)** Commodity and income taxation are both used in the optimum by the public authority who has equity concerns.

We finally note that the price effect on optimal income tax vanishes if the public authority can impose 100% profit tax. Intuitively, full profit taxation implies that there is neither efficiency loss due to foreign ownership nor equity concerns due to unequal distribution of firm profits.

Let us now analyze commodity taxation in oligopolistic markets. For the purpose of exposition, we consider only the environment of Section 4 without the price effect, i.e. $\Xi = 1$ and $\xi'(n) = 0$. The complete analysis of the non-competitive effect and the price effect in oligopolistic markets is presented in the proof of Theorem 5 in Appendix A.1.

We recall that $\Pi(p, X) = pX - MK(\frac{X}{M})$ and now write the equilibrium condition as $J(X, X_q, p) \equiv -X_q \left( p - K' \left( \frac{X}{M} \right) \right) - (1 + \theta) \frac{X}{M}$, where $X_q$ is a slope of market demand for consumer price $q$. The public authority’s problem for this environment is as follows.

$$\max_{p, q, u(n), \ell(n)} \int H(u(n)) f(n) dn$$

s.t.

$$u'(n) - c_i \ell(n)/n = 0 \quad (\mu(n), \text{incentive compatibility})$$

$$\int [n\ell(n) - r(q, u(n), \ell(n))] f(n) dn + \Pi(p, X) + (q - p)X \geq R \quad (\lambda, \text{resource constraint})$$

$$J(X, X_q, p) = 0 \quad (\gamma, \text{market equilibrium})$$

$$X - \int x(q, r(q, u(n), \ell(n))) f(n) dn = 0 \quad (\alpha_1, \text{market demand})$$

$$X_q - \int x_q(q, r(q, u(n), \ell(n))) f(n) dn = 0 \quad (\alpha_2, \text{market demand slope})$$

where function $r$ determines agent income $\tilde{y} = r(q, u, \ell) = \frac{u + c(\ell)}{a(q)}$ as before, and Lagrange
multipliers introduced next to the corresponding constraints. The Lagrangian then equals

\[
\mathcal{L} = \int \left[ (H(u(n)) + \mu(n)(u'(n) - c\ell(n)/n) \\
+ \lambda(n\ell(n) - r(q,u(n),\ell(n)) + \Pi(p,X) + (q-p)X - R) + \gamma J(X,X_q,p) \\
+ \alpha_1(X - x(q,r(q,u(n),\ell(n)))) + \alpha_2(X_q - x_q(q,r(q,u(n),\ell(n)))) \right] f(n) \, dn.
\]

After the integration by parts and taking into account the transversality condition \( \mu(n) = \mu(\bar{n}) = 0 \), and \( r_u = 1/v_y, r_x = c\ell/v_y \), and \( r_q = -v_q/v_y = x \); the first-order conditions are

\[
\begin{align*}
\ell(n) : & \left( \frac{\lambda n - c\ell}{v_y} (\lambda + \alpha_1 x_y + \alpha_2 x_{qy}) \right) f(n) - \mu(n)(c\ell + \ell c\ell)/n = 0, \quad (36) \\
p : & \gamma J_3 = 0 \quad (37) \\
q : & \alpha_1 \int (x_q + x_y x) f(n) \, dn + \alpha_2 \int (x_{qy} + x_{qy} x) f(n) \, dn = 0 \quad (38) \\
X : & \alpha_1 + \gamma J_1 + \lambda (\Pi_X + q - p) = 0, \quad (39) \\
X_p : & \alpha_2 + \gamma J_2 = 0. \quad (40)
\end{align*}
\]

where \( J_1, J_2, \) and \( J_3 \) are the partial derivatives of \( J \). From equation (36) we can determine the optimal income tax as

\[
\frac{t}{1-t} = \frac{\mu(n)v_y}{\lambda n f} \left( 1 + \frac{E_u}{E^c} + \frac{\alpha_1 x_y}{\lambda} + \frac{\alpha_2 x_{qy}}{\lambda} \right).
\]

Since \( J_3 = -X_q > 0 \), we have \( \gamma = 0 \) and, as a consequence \( \alpha_1 = \alpha_2 = 0 \). Equation (39) then implies that the public authority can eliminate welfare loss due to oligopolistic competition by imposing a subsidy that perfectly corrects for the presence of firm markup

\[
b = q - p = -\Pi_X = -(p - K'(X/M)) < 0. \quad (41)
\]

This leads to the standard undistorted formula of optimal income taxation as in Mirrlees (1971). Note that when firm profits are equally distributed among agents the public authority can finance the subsidy by imposing a constant uniform tax on all agents. This result is, however, specific to the case of \( \Xi = 1 \) and \( \ell'(n) = 0 \). In general, when foreign ownership is present and firm profits are unequally distributed, the public authority cannot achieve production efficiency.

**Theorem 5. (Commodity taxation in oligopolistic markets)** In the presence of foreign ownership or unequal distribution of firm profits, both the price and non-competitive effects are present in the optimum. Only if there is no foreign ownership and the firm profits are equally
distributed (no price effect), the optimal commodity tax fully corrects the non-competitive effect.

Note that the result of Theorem 5 is different from Theorem 4 for the competitive case. Even if the public authority has no equity concerns, the presence of foreign ownership results in a combination of commodity and income taxation in the optimum.

Finally, if the public authority can impose 100% firm profit tax, one would arrive again at equation (41). In this case, the public authority maximization problem coincides with the one solved above independently of the distribution of firm profits. This result is in line with Myles (1996) who showed that a combination of ad valorem tax and commodity tax can eliminate the welfare loss due to oligopolistic competition.

Overall, commodity taxation alone cannot eliminate the effect of endogenous prices when firm profits are not fully taxed away. In the latter case, the optimal income taxation policy needs to account for the price and non-competitive effects.

6 Literature Review

The results of the present paper are connected to several strands in the literature. First, our analysis is closely related to the study of optimal income taxation in the presence of endogenous wages in labor markets. Stiglitz (1982) is one of the first to consider a setting in which workers are not perfect substitutes in production. In this case, the general equilibrium effects imply that the optimal tax policy should subsidize high-talent workers and tax low-talent workers. Extending this analysis to a setting where workers have two-dimensional skill characteristics and an occupational choice, Rothschild and Scheuer (2013) found that the ability of workers to select their occupation involves a more progressive tax schedule than in a model without occupational choice. Ales, Kurnaz, and Sleet (2015) use a related multi-task assignment model with finite one-dimensional agent types to study a change in optimal income policy in response to a technical change. In a study of the continuum of one-dimensional agent types and general constant returns to scale production functions, Sachs, Tsyvinskiy, and Werquin (2016) show that the optimal labor supply in an equilibrium is determined by a complicated integral equation. Using this equation, they analyze the incidence of a tax reform with the actual U.S. tax code as a starting point. In contrast to these papers, we do not consider general equilibrium effects. The price effect arises because of the distribution of firm profits among agents inside and outside the economy.

The literature analyzing endogenous wages in labor markets also studies the impact of externalities on the optimal income tax schedule. Rothschild and Scheuer (2016) study the corrective role of income taxation in a setting where agents can engage in a rent-seeking activity

27 See also Feldstein (1973), Allen (1982), and Stern (1982).

28 Stantcheva (2014) also studies how adverse selection in labour markets influences optimal income taxation.
with private returns being different from social returns. Lockwood et al. (2017) integrate
tax considerations into an assignment model of agents to professions in which high-paying
professions have negative externalities and low-paying professions have positive externalities.
They estimate that the welfare gains from an optimal income taxation policy targeted to
compensate these externalities are small. Rothschild and Scheuer (2014) also develop a unifying
framework to analyze externalities effects on optimal income taxation. Compared to this
literature, agents in our setting do not impose any direct externality on the other agents.
Rather agents impose pecuniary externalities. One should not correct pecuniary externalities
when agent types are perfectly observable and all firm profits remain in the economy. If agent
types are not perfectly observable and tax policy can be based only on labor income, we show
that this is no longer true. In this case, a constrained Pareto-efficient allocation cannot be
generally supported by a competitive equilibrium.

Our analysis is also closely related to the seminal production efficiency theorem of Diamond
and Mirrlees (1971) asserting that the optimal income taxation entails efficient production in
equilibrium. This seminal result holds, however, only in the absence of firm profits or when
firm profits are fully taxed. The contribution of the present paper is to analyze the optimal
income taxation in the Mirrleesian setting in the presence of firm profits for both competitive
and oligopolistic markets. The full taxation of firm profits is also an important assumption
in the commodity taxation literature. Atkinson and Stiglitz (1976) establish that commodity
taxation is unnecessary in the presence of the optimal income tax given weak separability of
utility between labor and all consumption goods (see also Mirrlees, 1976; Kaplow, 2006). In
contrast, we show that commodity taxation can play an important role on par with income
taxation when firm profits cannot be fully taxed.

Only a few papers consider models in which firm profits are not fully taxed. Stiglitz and
Dasgupta (1971) is one of the first studies to point out that production efficiency might not
be desirable if the maximum profit tax rate is limited. In this case, they show how factor
and commodity taxes may serve as a partial substitute for the tax on profits (see also Munk,
1978, 1980). Dasgupta and Stiglitz (1972) show that the government might not wish to tax
all profits away in the economy with non-identical consumers when lump-sum taxes are not
allowed. Iwamoto and Konishi (1991) derive explicitly the optimal commodity tax rule in a
setting with firm profits and many consumers. In contrast to these papers, we consider an
incomplete information environment of optimal income taxation and show that the firm profit

29 If tax policy is based on total income, pecuniary externalities need also to be corrected (see Appendix A.4).
30 See also Scheuer and Werning (2016) for an insightful connection between the seminal model of Diamond
31 Dasgupta and Stiglitz (1972) also establish that production efficiency is desirable with non-constant return
to scale production technology if government can set different percentage profit taxes to different producers;
see also Mirrlees (1972).
32 See also Atkinson and Stiglitz (1976, 2015) and Diamond and Mirrlees (1971).
distribution plays an important role. We both derive analytically and estimate numerically the influence of firm profit distribution on the optimal income tax schedule.

We also want to mention an important paper by Scheuer and Werning (2017) who consider an assignment model to study the taxation of top income earners.\footnote{See also Ales and Sleet (2016), Piketty, Saez, and Stantcheva (2014), and Shourideh (2014) for the analyses of optimal taxation of top labor and capital incomes.} They establish that the optimal income taxation schedule, as a function of agent preferences and agent skill distribution, does not directly depend on the production technology even when firm profit are not fully taxed. They do not consider, however, endogenous product prices. In particular, they assume that there is only one consumption good with a price that can be normalized to one. In contrast, we consider the prices of two goods, of which only one price can be normalized to one. The price for the other good is endogenously determined by market equilibrium condition. We showed that the optimal income tax schedule does depend on production technology when firm profits are unequally distributed.

Our paper is also related to a small body of literature analyzing commodity taxation in oligopolistic markets. Auerbach and Hines (2001) and Myles (1987, 1989) show the benefit of a corrective subsidy to offset producer markups in the presence of market power.\footnote{See also Reinhorn (2005, 2012).} Parallel to their findings, we show the benefit of a less progressive marginal income tax schedule. Similar to a corrective subsidy, a decrease in marginal income tax stimulates market demand that brings the equilibrium consumption closer to a constrained Pareto-efficiency frontier. Though our results can be seen as an extension of the previous analysis to optimal income taxation settings, in addition, we also study the distributional effect of firm profits, which is absent in the previous papers. We also establish that the non-competitive effect persists even in the presence of commodity taxation.

Our paper is also closely related to an important strand of literature focused on the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Ireland (2001), Jinkins (2016), Kanbur and Tuomala (2013), Oswald (1983).\footnote{A similar idea that individuals may seek a more equal income distribution in order to improve the terms of trade is explored by Zubrickas (2012).} These papers are typically motivated by empirical observations that people care not only about their absolute level of consumption but also how it compares with that of others. In our paper, relativity concerns arise endogenously through equilibrium product prices.

7 Conclusion

In this paper, we study how endogenous prices affect optimal income taxation policy in both competitive and oligopolistic markets. We consider a Mirrleesian framework with imperfectly observable agent productivity types and firm profits distributed among agents inside and out-
side the economy. We establish that unequal distribution of firm profits is associated with the price effect on optimal income taxation. Using the U.S. housing market, we numerically estimate that the price effect increases optimal marginal income tax by 6 percentage points at most income levels.

The presence of market power in oligopolistic markets is associated with the non-competitive effect. Using the model of the U.S. housing market, we show that the price effect dominates the non-competitive effect on income taxes. This leads to higher optimal marginal income taxes for less competitive markets. We also study how the price and non-competitive effects change in the presence of commodity and profit taxation.

Our study of the price effect is only the first step of incorporating an underlying market structure into the analysis of optimal income taxation. Housing spending accounts for only one fourth of U.S. household expenditures. Hence, the size of the price effect is potentially significantly larger than 6 percentage points if one takes into account other industries into account. In competitive markets, our main driving force for the price effect is unequal distribution of firm profits. Hence, our results should extend to a general equilibrium model with several industries. This setting can be used to analyze how income tax policy changes if firm profit sharing differs across industries. We leave this intriguing question for future research.

In addition, our analysis highlights an important interaction between income and profit taxation. This interaction cannot be analyzed within our framework because the optimal profit taxation should be 100% in our model. Countries typically do not levy 100% tax on profits. Atkinson and Stiglitz (p. 392, 2015) argue that the lack of information at the disposal of government does not allow to distinguish pure profits from the return on capital, which should be taxed at zero percent (see Chamley, 1986; Judd, 1985). Ales and Sleet (2016) and Scheuer and Werning (2017) outline other possible arguments taking into account firm entry and exit decisions. The interaction of the optimal income taxation and profit taxation is an important open question that we leave for future investigation.36 In addition, incorporating the analysis of endogenous prices into a dynamic framework is an important question that we also leave for future research.

Finally, we want to highlight that endogenous prices in product markets should be an important consideration beyond income taxation policies. The welfare assessment of subsidies, welfare benefits, pensions, minimum wages, etc., would be biased unless the public authority takes into account the price and non-competitive effects analyzed in this paper.

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36 We thank Ali Shourideh and Chris Sleet for pointing this out to us.
A Appendix

A.1 Proofs

Proof of Theorem 1. The statement follows from the argument in the main text. □

Proof of Corollary 1. The statement about normal and inferior goods follows from the argument presented in the main text. We now show that our assumption that demand for good X is either convex or concave in income implies $x_y$ is increasing (decreasing) for luxury goods (necessity goods).

Let us assume that good X is luxury. By the definition of luxury goods, its demand then increases more than proportionally as income rises, i.e., $px(p,y)/y$ is strictly increasing in $y$. From contrary, let us assume that function $x$ is concave in income. Therefore, for all $(p,y) \in \mathbb{R}^2$ and $y > 0$,

$$\frac{d}{dy} \left( \frac{x(p,y) - x(p,0)}{y - 0} \right) \leq 0.$$ 

Taken into account that $x(p,0) = 0$ we arrive at the contradiction that $px(p,y)/y$ is strictly increasing. Therefore, demand $x$ is convex in income and, hence, $x_y$ is increasing in $y$. Therefore, the optimal marginal income tax is increasing for luxury goods. A similar argument applies for necessity goods.\(^{37}\) □

Proof of Theorem 2. The individual utility maximization problem $\max_z U(p, z - T(z, n) + \xi(n)\Pi(p), z/n)$ implies that $t = 1 - c_\ell/(nv_y)$. Taking this into account, condition (16) implies

$$\frac{t}{1-t} = \left(1 + \ell c_\ell/c_\ell\right) \frac{v_y \mu(n)}{\lambda nf} + \frac{\gamma}{\lambda} x_y,$$

with multiplier $\mu(n) = \int_n^\Pi \left(\frac{\lambda + \gamma x_n}{v_y} - H_n\right) f(n)dn$ found from (15).

Next, we demonstrate that $1 + \ell c_\ell/c_\ell = (1 + E^u)/E^c$, where $E^c$ is the elasticity of compensated labor supply and $E^u$ is the elasticity of uncompensated labor supply $E^u$. The individual utility maximization condition again implies that $v_y(p, \tilde{y}) n(1-t) - c_\ell(\ell) = 0$. By denoting $\tilde{y} = w\ell + \overline{y}$, where $w = n(1-t)$ is a net wage rate and $\overline{y}$ is non-labor income, we obtain

$$v_y(p, w\ell + \overline{y}) w - c_\ell(\ell) = 0.$$ 

\(^{37}\)We note that our assumption that the demand for good X is either concave or convex in income at all income levels is generally indispensable. For example, consider function $f(y) = 0$ if $y \in [0,1]$ and $f(y) = y - 1/y$ if $y \geq 1$. Function $f$ has increasing ratio $f(y)/y$ and decreasing derivative $f_y$ at the same time.

28
Implicitly differentiating the above expression with respect to net wage we obtain
\[
\frac{\partial \ell}{\partial w} = \frac{v_{yy}w\ell + v_y}{c_{\ell \ell} - v_{yy}w^2}.
\]
Then, the elasticity of uncompensated labor supply \( E^u = \partial \ell / \partial w \) \((w/\ell)\) is equal to
\[
E^u = \frac{v_{yy}w\ell + v_y}{c_{\ell \ell} - v_{yy}w^2} \frac{w}{\ell} = \frac{v_{yy}(c_\ell / v_y)^2 + c_\ell / \ell}{c_{\ell \ell} - v_{yy}(c_\ell / v_y)^2},
\]
where we use \( w = c_\ell / v_y \). To obtain the elasticity of compensated labor supply, \( E^c \), we employ the Slutsky equation \( E^c = E^u - E^m \), where \( E^m = w(\partial \ell / \partial \overline{y}) \) is the income effect parameter:
\[
E^m = w \frac{\partial \ell}{\partial \overline{y}} = \frac{v_{yy}(c_\ell / v_y)^2}{c_{\ell \ell} - v_{yy}(c_\ell / v_y)^2}.
\]
Thus, the elasticity of compensated labor supply, \( E^c \), is given by
\[
E^c = E^u - E^m = \frac{v_{yy}(c_\ell / v_y)^2 + c_\ell / \ell}{c_{\ell \ell} - v_{yy}(c_\ell / v_y)^2} - \frac{v_{yy}(c_\ell / v_y)^2}{c_{\ell \ell} - v_{yy}(c_\ell / v_y)^2} = \frac{c_\ell / \ell}{c_{\ell \ell} - v_{yy}(c_\ell / v_y)^2}.
\]
Thus, we obtain that \( 1 + \ell c_{\ell \ell} / c_\ell = (1 + E^u) / E^c \), which completes the derivation of expression for optimal marginal income tax. Finally, the expression for Lagrange multiplier \( \gamma \) follows directly from condition (17).

Proof of Corollaries 2 and 3. The statements follow from the argument in the main text.

Proof of Theorems 3 and 4. The statements follow from the argument in the main text.

Proof of Theorem 5. Let us consider commodity taxation in oligopolistic markets with general distribution of firm profits. The public authority problem can be written as follows.

\[
\max_{p,q,a(n),\ell(n)} \int H(u(n)) f(n) dn
\]
subject to
\[
u'(n) - \xi'(n)a(q) \Pi(p,X) - c_\ell \ell(n)/n = 0 \quad \text{ (\( \mu(n) \), incentice compatibility)}
\]
\[
\int [n\ell(n) - r(q, u(n), \ell(n))] f(n) dn + \Xi \Pi(p,X) + (q - p)X \geq R \quad \text{ (\( \lambda \), resource constraint)}
\]
\[
J(X, X_q, p) = 0 \quad \text{ (\( \gamma \), market equilibrium)}
\]
\[
X - \int x(q, r(q, u(n), \ell(n))) f(n) dn = 0 \quad \text{ (\( \alpha_1 \), market demand)}
\]
\[
X_q - \int x_q(q, r(q, u(n), \ell(n))) f(n) dn = 0 \quad \text{ (\( \alpha_2 \), market demand slope)}
\]
where function $r$ determines agent income as a function of price, utility, and effort $\tilde{y} = r(q, u, \ell) = \frac{u + \xi\ell}{a(q)}$, and Lagrange multipliers introduced next to the corresponding constraints. The Lagrangian can then be written as

$$
\mathcal{L} = \int [(H(u(n)) + \mu(n)(u'(n) - \xi'(n)a(q)\Pi(p, X) - c_\ell\ell(n)/n)
+ \lambda(n\ell(n) - r(q, u(n), \ell(n))) + H(\pi(p, X) + (q - p)X - R) + \gamma J(X, X_q, p)
+ \alpha_1(X - x(q, r(q, u(n), \ell(n)))) + \alpha_2(X_q - x_q(q, r(q, u(n), \ell(n))))f(n)]dn
$$

After the integration by parts and taking into account $\mu(n) = \mu(\pi) = 0$, $\Pi_p(p, X) = X$, and $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_q = -v_q/v_y = x$, the first-order conditions are

$$
u(n) : \left( H_u - \lambda + \frac{\alpha_1 x_q + \alpha_2 x_{qq}}{v_y} \right) f(n) - \mu'(n) = 0, \quad (A.1)
$$

$$\ell(n) : \left( \lambda n - \frac{\alpha_1}{v_y} (\lambda + \alpha_1 x_q + \alpha_2 x_{qq}) \right) f(n) - \mu(n)(c_\ell + \xi\ell\ell)/n = 0, \quad (A.2)
$$

$$p : -a\Pi_p \mu(n)\ell'(n)dn + \lambda(-X(1 - \Xi)) + \gamma J_3 = 0 \quad (A.3)
$$

$$q : -a_q\pi \mu(n)\ell'(n)dn - \alpha_1 \int (x_q + x_{qq})f(n)dn - \alpha_2 \int (x_{qq} + x_{qq}x)f(n)dn = 0 \quad (A.4)
$$

$$X : \alpha_1 - a\Pi_X \mu(n)\ell'(n)dn + \gamma J_1 + \lambda(\Xi\Pi_X + q - p) = 0, \quad (A.5)
$$

$$X_p : \alpha_2 + \gamma J_2 = 0. \quad (A.6)
$$

where partial derivatives of $J$ are determined by

$$J_1 = X_q K''(X)\frac{1}{M} - \frac{(1 + \theta)}{M} < 0,
$$

$$J_2 = -\Pi_X(p, X) = -(p - K'(X/\lambda)) \leq 0,
$$

$$J_3 = -X_q > 0.
$$

We also denote $H_{qq} = \int (x_{qq} + x_{qq}x)f(n)dn$. As a reminder, $H_q = \int (x_q + x_{qq}x)f(n)dn < 0$ and $M\xi = \int \mu(n)\ell'(n)dn > 0$. We can then rewrite (A.4) and (A.6) as

$$\alpha_1 = \frac{a_q\Pi X\ell M\xi}{H_q} - \gamma J_2 \frac{H_{qq}}{H_q} = \frac{a_q\Pi X\ell M\xi}{H_q} + (a\Pi X, M\xi + \lambda X(1 - \Xi)) \frac{J_2}{J_3} \frac{H_{qq}}{H_q} \quad (A.7)
$$

$$\alpha_2 = -\gamma J_2 = (a\Pi X, M\xi + \lambda X(1 - \Xi)) \frac{J_2}{J_3} \quad (A.8)\]
We now substitute the above expression into (A.5). Taking into account (A.3) we obtain

\[
\frac{a_q \Pi M \xi}{-H_q} + (aM \xi + \lambda(1 - \Xi)) \left( \frac{XJ_2H_{qq} + XJ_1H_q}{J_3H_q} - \Pi_X \right) = -\lambda(\Pi_X + q - p) \quad (A.9)
\]

This formula determines optimal subsidy in oligopolistic markets. Note that the left-hand side of the above formula is non-positive if \(H_{qq} \leq 0\) (which holds if the demand is concave \(x_{qq} < 0\) or not too convex). In particular, the first term is non-positive because \(a_q, H_q < 0\) and \(M \xi \geq 0\). The second term is also non-positive because \(J_2 \leq 0\) and \(J_1 < 0\). Hence, the optimal subsidy is generally higher or equal to firm’s markup \(\Pi_X\) in the optimum.

Overall, equation (A.9) shows that in the absence of foreign ownership \(\Xi = 1\) and equal distribution of firm profits \(M \xi = 0\), the optimal commodity tax equals the size of firm’s markup leading to production efficiency. In the presence of foreign ownership or unequal distribution of profits, equations (A.7) and (A.8) imply that \(\alpha_1 \neq 0\) and \(\alpha_2 \neq 0\). Moreover, if \(H_{qq} < 0\), both \(\alpha_1 < 0\) and \(\alpha_2 \leq 0\), which is parallel to the result of Theorem 4 for competitive markets. \(\square\)
A.2 Supporting Equilibrium Model

In this section, we show that our model of competitive markets can be supported with a labor market and a consumer’s utility maximization problem. In particular, we consider two competitive industries: one producing the numeraire good G and the other producing good X. We label these industries as G and X respectively. We assume that agents can earn wage \( w \) for effective labor hours supplied (i.e., \( n\ell(n) \)) in both industries, the price for the numeraire good is normalized to \( p_g = 1 \), and the price for good X equals to \( p \).

Industry G has a homogeneous of degree one production technology \( F_g(L_g) \equiv L_g \), where \( L_g \) is the amount of labor used in production of good G. Since the price of numeraire good is normalized to 1, the profit maximization condition implies that \( w = p_g = 1 \), zero profits, and any level of the equilibrium labor demand \( L_g^d \) in industry G.

Industry X has a production technology with decreasing returns to scale \( F_x(L_x) \), where \( L_x \) is the amount of labor supplied. To make sure that the firm profit maximization problem has a well-defined interior solution, we assume that \( F_x \) is differentiable, is strictly concave, and satisfies Inada conditions, e.g., \( F_x(L_x) = AL_x^a \), where \( 0 < A, 0 < a < 1 \) are constants. Hence, firm profit maximization problem

\[
\max_{L_x} p \cdot F_x(L_x) - wL_x.
\]

The solution to this maximization problem leads to the equilibrium labor demand \( L_x^d(p) \) in industry X. If \( F_x(L_x) = AL_x^a \), the production function we have \( L_x^d(p) = (aAp)^{1-a} \) (taking into account \( w = 1 \)). The equilibrium market supply of good X then equals \( S_x(p) = F(L_x^d(p)) \) and firm profits equal \( \Pi(p) = \int S_x(\bar{p})d\bar{p} \).

We assume that the share of firm profits \( \Xi \) is distributed among the agents of the economy according to distribution function \( \xi(n) \) with \( \int \xi(n)f(n)dn = 1 \). The remaining share \( 1 - \Xi \) belongs to capitalists (or “foreigners”) who spend it on the consumption of numeraire good G. The government also spends all its resources \( R \) on numeraire good G.

On the demand side of the economy, we assume that agent’s preferences can be summarized by utility function \( u(x, g) - c(\ell) \), where \( (x, g) \) is the amount of good X and the numeraire G consumed by the agent. Utility \( u \) is a continuous function representing locally non-satiated preferences.

Consider an agent with productivity \( n \) who works \( \ell \) hours. Taking into account that equilibrium wage \( w = p_g = 1 \) and tax schedule \( T(n\ell) \) her income equals \( n\ell - T(n\ell) \). Hence, agent’s
maximization problem is

$$\max_{x, g, \ell} u(x, g) - c(\ell)$$

(A.10)

$$s.t. \ p \cdot x + g \leq n\ell - T(n\ell) + \xi(n)\Pi(p)$$

The solution to the above problem is labor supply $\ell^*(n, p)$ and consumption bundle $(x^*(n, p), g^*(n, p))$.

Overall, aggregate labor supply and consumer demand equal

$$L^*(p) = \int n\ell^*(n, p)f(n)dn, \quad X(p) = \int x^*(n, p)f(n)dn, \quad G(p) = \int g^*(n, p)f(n)dn.$$ 

The economy must satisfy three market clearing conditions:

$$S_x(p) = X(p)$$

(A.11)

$$S_g(p) = G(p) + (1 - \Xi)\Pi(p) + R$$

(A.12)

$$L^*(p) = L_x^d(p) + L_g^d(p),$$

(A.13)

where market clearing condition (A.12) requires that the market supply for good $G$ equals the sum of the market demand for $G$, the share of capitalists’ profits, and the government spending.

Let us show that condition (A.11) is the only one that we should consider in optimal income taxation problem. Since the constant return to scale technology for the numeraire good $G$ ensures that any level $L_g^d(p)$ satisfies the firm maximization problem (when $w = p_g = 1$), we are free to choose $L_g^d(p) = L^*(p) - L_x^d(p)$ to clear the labor market. Taking into account that $S_g(p) = L_g^d(p)$ and $\Pi(p) = pS_x(p) - L_x^d(p)$ condition (A.12) can be equivalently rewritten as

$$L_g^d(p) + L_x^d(p) = G(p) + pS_x(p) + R - \Xi\Pi(p)$$

Given conditions (A.11) and (A.13) this is equivalent to

$$\int n\ell^*(n, p)f(n)dn = G(p) + pX(p) + R - \Xi\Pi(p).$$

This condition follows from the budget constraint of agent’s maximization problem (A.10) when the government spending constraint is binding $\int T(n\ell^*(n, p))f(n)dn = R$ (as we assume). Overall, the only independent market clearing condition that we should take into account in the optimization problem is (A.11) – the market clearing condition for good $X$. 

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A.3 Competitive Markets: Non-Linear Indirect Utility

In this section, we consider the general case of non-homothetic preferences leading to agent indirect utility being non-linear in income. We also analyze the public authority maximization problem where the remaining firm profits enter with a separate social weight.

Let us again consider agent’s utility from revealing his productivity type truthfully as

\[ u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n). \]

If revealing the agent’s type truthfully is optimal then

\[ u(n) = \max_m U(p, y(m) + \xi(n)\Pi(p), z(m), n). \]  

(A.14)

The envelope theorem implies then the following first-order condition.

\[ u'(n) = U_n + U_y \xi'(n)\Pi(p). \]  

(A.15)

We note that the single-crossing condition generally does not hold for a non-linear indirect utility. Hence, we need to derive the second-order condition for truth-telling being optimal.

**Proposition A1.** Condition (A.15) ensures that truth-telling is an optimal solution of (A.14) if and only if for each productivity \( n \) schedule \( \{y(n), z(n)\} \) satisfies

\[ [(c_l z(n)/n + c_l)/n \frac{v_y}{c_l} + v_{yy} \xi'(n)\Pi(p)]y'(n) \geq 0. \]  

(A.16)

**Proof.** Let us assume that \( y(n) \) and \( z(n) \) are differentiable. The second-order condition for maximization (A.14) is

\[ u''(n) - U_{nn} - (U_{yy} \xi'(n)\Pi(p) + 2U_{yn})\xi'(n)\Pi(p) - U_{y\xi} \xi''(n)\Pi(p) \geq 0. \]  

(A.17)

Taking the derivative of (A.15) with respect to \( n \) we obtain

\[ u''(n) = U_{nn} + U_{ny} (y'(n) + \xi'(n)\Pi(p)) + U_{nz} z'(n) + \\
(2U_{yn} + U_{yy} (y'(n) + \xi'(n)\Pi(p)) + U_{yz} z'(n))\xi'(n)\Pi(p) + U_y \xi''(n)\Pi(p). \]

Hence, condition (A.17) is equivalent to

\[ U_{ny} y'(n) + U_{nz} z'(n) + (U_{yy} y'(n) + U_{yz} z'(n))\xi'(n)\Pi(p) \geq 0. \]
Given our separable utility specification this reduces to

\[ U_{ny}y'(n) + U_{nz}z'(n) + (U_{yy}y'(n) + U_{yz}z'(n))\xi'(n)\Pi(p) = \]

\[ U_{nz}z'(n) + U_{yy}y'(n)\xi'(n)\Pi(p) = \]

\[ (clz(n)/n + c)/n^2 z'(n) + vy'y(n)\xi'(n)\Pi(p) \geq 0. \]

Maximization (A.14) now implies \( vy'y(n) - clz'(n)/n = 0 \), which allows rewriting the previous inequality in the form of (A.16).

Note that the first term in (A.16) is always positive because costs function \( c(\ell) \) is increasing and convex. Hence, the second-order condition reduces to income schedule \( y(n) \) being non-decreasing if either profits are zero \( \Pi(p) = 0 \) or agent profits are equally distributed \( \xi'(n) = 0 \). When both \( \Pi(p) > 0 \) and \( \xi'(n) > 0 \) the second term in (A.16) is negative because indirect utility function is concave.

We consider a version of the maximization problem that includes the remaining firm profits with separate weight \( \omega \geq 0 \) into the objective function.

\[
\max_{p, \tilde{y}(n), \ell(n)} \int H(v(p, \tilde{y}(n)) - c(\ell(n)))f(n)dn + \omega(1 - \Xi)\Pi(p) \quad \text{subject to (3), (5), and (A.15)}.
\]

It is again convenient to change the optimization variables \( \{p, \tilde{y}(n), \ell(n)\} \) to \( \{p, u(n), \ell(n)\} \), where utility level \( u(n) = U(p, \tilde{y}(n), \ell(n)) \). From the latter expression we can invert disposable income \( \tilde{y}(n) \) and express it as a function of \( (p, u, \ell) \), i.e. \( \tilde{y} = r(p, u, \ell) \). Assuming that the second order condition (A.16) is satisfied, the maximization problem can be written as:

\[
\max_{p, u(n), \ell(n)} \int H(u(n))f(n)dn + \omega(1 - \Xi)\Pi(p) \]

s.t.

\[
\int [u(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)]f(n)dn \geq R
\]

\[
\int [S(p) - x(p, r(p, u(n), \ell(n)))f(n)dn = 0
\]

\[
u'(n) - vy(p, r(p, u(n), \ell(n)))\xi'(n)\Pi(p) - c\ell(n)/n = 0
\]

After the integration by parts and taking into account the transversality condition \( \mu(n) = \)
where \( \mu(p) = 0 \) the Lagrangian of the maximization problem can be written as

\[
\mathcal{L} = \int \left\{ [H(u(n)) + \omega(1 - \Xi)\Pi(p) + \lambda(n\ell(n) - r(p,u(n),\ell(n))) + \xi(n)\Pi(p) - R] + \gamma(S(p) - x(p,r(p,u(n),\ell(n)))) \right\} \, f(n) \, - \mu'(n)u(n) - \mu(n)(v_y(r(p,u(n),\ell(n)))\xi'(n)\Pi(p) + c_\ell\ell(n)/n) \, dn.
\]

Taken into account \( r_u = 1/v_y, r_\ell = c_\ell/v_y, \) and \( r_p = -v_p/v_y = x, \) the first-order conditions are

\[
u(n) : \left[H_u - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) - \mu(n) \frac{v_{yy}}{v_y} \xi'(n)\Pi(p) = 0 \quad (A.18)
\]

\[
\ell(n) : \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y} \right] f(n) - \mu(n) \frac{v_{yy}c_\ell}{v_y} \xi'(n)\Pi(p) - \mu(n)(c_\ell + c_\ell\ell(n))/n = 0 \quad (A.19)
\]

\[
p : \int \{[\omega(1 - \Xi)\Pi'(p) + \lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_yx)] f(n) - \mu(n)((v_{yp} + v_{yy}x)\xi'(n)\Pi(p) + v_y\xi'(n)\Pi'(p)) \} \, dn = 0.
\]

To calculate marginal income tax we consider now the individual utility maximization problem

\[
\max_{\ell} \quad v(p,n\ell - T(n\ell) + \xi(n)\Pi(p)) - c_\ell, \quad \text{where} \ T(n\ell) \ \text{is the gross income tax payment.}
\]

The first order condition with respect to \( \ell \) yields \( v_y(1 - t) = c_\ell. \)

Using the first-order condition (A.19) we can then write

\[
\frac{t}{1 - t} = \frac{nv_y}{c_\ell} - 1 = \frac{\mu(n)v_y}{\lambda n f} \left[ 1 + \frac{E^u}{E^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu(n)v_{yy}\xi'(n)\Pi(p)}{\lambda f} \right], \quad (A.21)
\]

where \( \mu \) is determined by (A.18) and multiplier \( \gamma \) is determined by (A.20). By noting that \( \Pi'(p) = S(p) = \int x(n)f(n)dn \) and \( \int \xi(n)f(n)dn = \Xi, \) we obtain the following result.

**Theorem A1.** In competitive markets with endogenous prices, the optimal marginal income tax is determined by

\[
\frac{t}{1 - t} = \frac{\mu(n)v_y}{\lambda n f} \left[ 1 + \frac{E^u}{E^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu(n)v_{yy}\xi'(n)\Pi(p)}{\lambda f} \right],
\]

where

\[
\gamma = \frac{(\lambda - \omega)S(p)(1 - \Xi) + \int (v_{yp}\Pi(p) + v_y\Pi'(p) + v_{yy}x\Pi(p))\mu(n)\xi'(n)dn}{S'(p) - \int (x_p + x_yx)f(n)dn}.
\]

Note that the optimal marginal income tax formula has three terms. The first one is the standard Mirrleesian term. The second term corresponds to the price effect of the binding market equilibrium constraint. Both terms are present in the model with linear indirect utility. The third term is new and it is responsible for the change of agent incentives due to profit.
distribution. Hence, we refer to it as an *incentive term*. When agent indirect utility is strictly concave agents with larger profit shares have smaller marginal benefit from additional income. Hence, they are less willing to exert effort and the public authority can no longer impose high marginal tax rate on them without disturbing their effort level. The third term is negative (since $v_{yy} < 0$) and make the optimal marginal income tax less progressive in contrast to the price effect (when good X is normal). A similar incentive effect would occur if agents with higher productivity levels had larger endowments that are not related to firm profit shares.

Finally, we want to note that social weight $\omega$ influences the optimal marginal income tax through multiplier $\gamma$: the larger social weight $\omega$ the smaller is the price effect. The price effect decreases with social weight $\omega$ and completely disappears when social weight $\omega$ equals to the shadow costs of raising public funds $\lambda$. 
A.4 Competitive Markets: Taxing Total Income

In this subsection, we consider the case when public authority can tax total agent income. We then denote total income as:

\[ z(n) = nl(n) + \xi(n)\Pi(p). \]

Agent’s disposable income then equals \( y(n) = z(n) - T(z(n)) \). Given that agents have different profit shares, it is now harder for them to pretend to have different productivity compared to the case when taxes are based on labor income only. Now, if agent of type \( n \) wants to pretend to be of type \( m \), he needs to work more (or less) hours:

\[ \ell = (m\ell(m) + (\xi(m) - \xi(n))\Pi(p))/n. \]

Hence, agent’s utility upon reporting type \( m \) equals:

\[ U(p, y(m), z(m), n) = v(p, y(m)) - c((z(m) - \xi(n)\Pi(p))/n). \]

We denote the agent’s utility from revealing his productivity type truthfully as:

\[ u(n) \equiv U(p, y(n), z(n), n) = v(p, y(n)) - c((z(n) - \xi(n)\Pi(p))/n). \]

If revealing agent’s type truthfully is optimal then:

\[ u(n) - U(p, y(n), z(n), n) = 0 \leq u(m) - U(p, y(n), z(n), m), \tag{A.22} \]

which leads to the following first-order condition:

\[ u'(n) = U_n = c_\ell \frac{\ell(n) + \xi'(n)\Pi(p)}{n}. \tag{A.23} \]

The latter condition coincides with the standard one when profits are zero \( \Pi(p) = 0 \) or \( \xi'(n) = 0 \). We also notice that the standard single-crossing condition is satisfied when agent total income is taxed. Hence, the second-order condition for truth-telling coincides with the one in the standard case, i.e., \( z(n) \) has to be increasing. In this case,

\[ \frac{\partial}{\partial n}\frac{\partial}{\partial m}c((z(m) - \xi(n)\Pi(p))/n) < 0 \]

Finally, the first order condition of maximizing (A.22) implies \( v_y y'(n) - c_\ell z'(n)/n = 0 \). Hence, agent income \( z(n) \) is increasing if and only if disposable agent income \( y(n) \) is increasing. Assuming that agent disposable income is increasing, the maximization problem of the public authority can be written as follows
where disposable income is determined by \( y(n) = r(p, u, \ell) \) from equation \( u(n) = U(p, y(n), \ell(n)) \).

After the integration by parts and taking into account the transversality condition \( \mu(n) = \mu(\pi) = 0 \) the Lagrangian of the maximization problem can be written as

\[
\mathcal{L} = \int \left\{ \left[ H(u(n)) + \lambda(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p) - R \right] + \right.
\]

\[
+ \left. \gamma(S(p) - x(p, r(p, u(n), \ell(n)))) \right\} f(n) - \mu' (n) u(n) - \mu(n) (c_\ell (\ell(n) + \xi'(n)\Pi(p))/n) \right\} dn.
\]

Taken into account \( r_u = 1/v_u, r_\ell = c_\ell/v_y, \) and \( r_p = -v_p/v_y = x \), the first-order conditions are

\[
u(n) : \quad H_u - \frac{\lambda + \gamma x_y}{v_y} f(n) - \mu'(n) = 0 \tag{A.24}
\]

\[
\ell(n) : \quad \lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y} f(n) - \mu(n)(c_\ell (\ell(n) + \xi'(n)\Pi(p))/n) = 0 \tag{A.25}
\]

\[
p : \quad \int \left\{ [\lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_y x)] f(n) - \mu(n)\xi'(n)\Pi'(p) \right\} dn = 0. \tag{A.26}
\]

From the individual utility maximization problem optimal marginal income is determined by \( v_y n(1-t) = c_\ell \). Using conditions (A.24)-(A.26), \( \Pi'(p) = S(p) = \int x(n) f(n) dn, \int \xi(n) f(n) dn = \Xi, \) and \( 1 + c_\ell/c_\ell = \frac{1 + E^w}{E^c} \) (see Theorem 2) we obtain the following result.

**Theorem A2.** In competitive markets with endogenous prices when agent total income is taxed, the optimal marginal income tax is determined by

\[
t \quad \frac{1}{1-t} = \frac{\mu v_y}{\lambda n f} \frac{1 + E^w}{E^c} \frac{\gamma x_y}{\lambda} \frac{\mu \xi'(p) c_\ell}{\lambda n f c_\ell}, \tag{A.27}
\]

where

\[
\gamma = \frac{\lambda S(p)(1 - \Xi) + S(p) \int \mu(n) \xi'(n) dn}{S'(p) - \int (x_p + x_y x) f(n) dn}. \tag{A.28}
\]

Given that \( \mu \geq 0 \) and \( \xi' \geq 0 \) the profit redistribution leads to more progressive taxation for top income earners. Intuitively, additional profit income makes it harder for wealthy agents to deviate. Hence, it alleviates incentive compatibility problem and allows for more progressive taxation. We also observe that the price effect is present even if all firm profits are distributed among agents \( \Xi = 1 \) (and \( \xi'(n) \neq 0 \)).
A.5 Oligopolistic Markets: Firm Profits Distribution

In this section, we analyze optimal income taxation in oligopolistic markets where a part of firm profits can leave the economy $\Xi \leq 1$ and profits are distributed according to some progressive schedule, i.e., $\xi'(n) \geq 0$ for all $n$.

As a reminder, the market equilibrium condition in oligopolistic markets can be written as

$$J(X, X_p, p) \equiv -X_p \left( p - K' \left( \frac{X}{M} \right) \right) + (1 + \theta) \frac{X}{M} = 0,$$

where $\theta$ is firm’s conjectural variation (see p. 16) and $X_p = \int x_p(p, \tilde{y}(n)) f(n) dn$. If we denote the total firm profits as $\Pi(p, X) = pX - MK(\frac{X}{M})$, the public authority’s problem can be written as follows.

$$\max_{p, u(n), \ell(n)} \int H(u(n)) f(n) dn$$

s.t. $\begin{align*}
  u'(n) - \xi'(n)a(p)\Pi(p, X) - c_\ell \ell(n)/n &= 0 \quad (\mu(n), \text{incentive compatibility}) \\
  \int [n\ell(n) - r(p, u(n), \ell(n))] f(n) dn + \Xi\Pi(p, X) &\geq R \quad (\lambda, \text{resource constraint}) \\
  J(X, X_p, p) &= 0 \quad (\gamma, \text{oligopolistic market equilibrium}) \\
  X - \int x(p, r(p, u(n), \ell(n))) f(n) dn &= 0 \quad (\alpha_1, \text{market demand}) \\
  X_p - \int x_p(p, r(p, u(n), \ell(n))) f(n) dn &= 0 \quad (\alpha_2, \text{market demand slope})
\end{align*}$

where function $r$ determines total agent income $\tilde{y} = r(p, u, \ell) = \frac{u + c(\ell)}{a(p)}$ and Largrange multipliers introduced next to their corresponding constraints. Note that the distribution of firm profits now enters both the incentive compatibility constraints as agent’s utility now depends on $\xi'(n)$ and the resource constraint as only share $\Xi$ of firm profits remains in the economy. The Lagrangian can be written as

$$\mathcal{L} = \int \left[ (H(u(n)) + \mu(n)(u'(n) - \xi'(n)a(p)\Pi(p, X) - c_\ell \ell(n)/n) \\
    + \lambda(n\ell(n) - r(p, u(n), \ell(n))) + \Xi\Pi(p, X) - R) + \gamma J(X, X_p, p) \\
    + \alpha_1(X - x(p, r(p, u(n), \ell(n)))) + \alpha_2(X_p - x_p(p, r(p, u(n), \ell(n)))) \right] f(n) dn$$

After the integration by parts and taking into account the transversality condition $\mu(n) =$
\[\mu(\overline{\pi}) = 0\] and \[r_u = 1/v_y, \quad r_\ell = c_\ell/v_y, \quad \text{and} \quad r_p = -v_p/v_y = x,\] the first-order conditions are

\[
u(n) : \left( H_u - \frac{\lambda + \alpha_1 x_y + \alpha_2 x_{py}}{v_y} \right) f(n) - \mu'(n) = 0, \tag{A.29} \]

\[
\ell(n) : \left( \lambda n - \frac{c_\ell}{v_y} (\lambda + \alpha_1 x_y + \alpha_2 x_{py}) \right) f(n) - \mu(n)(c_\ell + \ell c_\ell)/n = 0, \tag{A.30} \]

\[p : -\lambda X(1 - \Xi) + \gamma J_3 - \int (\alpha_1 (x_p + x_yx) + \alpha_2 (x_{pp} + x_{py}x)) f(n) dn \tag{A.31} \]

\[- (a_p \Pi(p, X) + a(p) \Pi_p(p, X)) \int \mu(n) \xi'(n) dn = 0, \tag{A.32} \]

\[X : \alpha_1 + \gamma J_1 + \Pi_X(p, X) \left( \lambda \Xi - a(p) \int \mu(n) \xi'(n) dn \right) = 0, \tag{A.33} \]

\[X_p : \alpha_2 + \gamma J_2 = 0. \tag{A.34} \]

where \(J_1 = X_p K''(\overline{\chi} X) H_p \frac{1}{M} - \frac{1 + \theta}{M} < 0, \) \(J_2\) equals to the negative of firm’s markup \(\Pi_X(p, X) = p - K'(\overline{\chi} X)) > 0,\) and \(J_3 = -X_p > 0.\) Condition (A.30) then implies the following result.

**Theorem A3.** In oligopolistic markets with endogenous prices, the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = \frac{1 + E^u v_y \mu}{E^c \lambda n f} + \frac{\alpha_1}{\lambda} x_y + \frac{\alpha_2}{\lambda} x_{py},
\]

Let us now discuss possible signs of the terms that enter the optimal income taxation formula. Since \(J_2 < 0\) the sign of \(\alpha_2\) coincides with the sign of \(\gamma\) (see (A.34)). However, the sign of \(\alpha_1\) is not fully determined by the sign of \(\gamma\) because of the third term in (A.33) depends on the distribution of profits in the economy. Term \(\Pi_X \left( \lambda \Xi - a(p) \int \mu(n) \xi'(n) dn \right)\) is large and positive when a share of firm profits remaining in the economy is large \((\Xi \approx 1)\) and when the distribution of profits is rather flat \((\xi'(n) \approx 0)\). In particular, for the equal distribution the third term is positive and for progressive distribution the third term can be negative.

To obtain the expression for \(\gamma\) we incorporate (A.33) and (A.34) into (A.32)

\[
\gamma = \left( \lambda X(1 - \Xi) - \lambda \Xi \Pi_X H_p + (a_p \Pi + a(p) \Pi_p + a(p) \Pi_X H_p) \int \mu(n) \xi'(n) dn \right) / J.
\]

where \(H_p = \int (x_p + x_yx) f(n) dn \leq 0\) is the derivative of the aggregate compensated demand and \(J = J_3 + \int J_1(x_p + x_yx) + J_2(x_{pp} + x_{py}x) f(n) dn.\) We note that the denominator is positive \(J > 0\) when the demand is either concave or not too convex, i.e. \(\int x_{pp} f(n) dn\) is small. The first two terms in the numerator are also positive. However, the sign of the third term cannot be clearly determined because \(a_p \Pi + a(p) \Pi_p > 0\) and \(a \Pi_X H_p < 0.\) Overall, the sign and the magnitude of the tax corrective terms depend on specific profit distribution and the share of firm profits remaining in the economy.
A.6 Numerical Simulations: Further Results

In this appendix, we provide additional numerical estimations of the size of the price effect on optimal income tax rates. Within the same framework of the U.S. economy presented earlier, we study the robustness of the price effect to different forms of housing supply and to income distribution.

In Section 3.3 we consider the competitive market with supply function $S = sp^\varepsilon$ and price elasticity $\varepsilon = 1.75$ which corresponds to the price elasticity of the average U.S. metropolitan area (Saiz (2010)). However, as also noted earlier, the price elasticity of housing supply widely differs across various countries and regions and, therefore, we reestimate the size of the price effect for the cases of (i) inelastic supply $\varepsilon = 0$ and (ii) elastic supply with $\varepsilon = 3$. The first case better describes housing supply in large U.S. coastal cities (e.g., Boston, San-Francisco) and in countries with a rigid housing planning system, e.g., the United Kingdom (see Hilbert and Schoni (2016) and Saiz and Salazar (2018)). In the second case we draw on the estimates of the price elasticity of U.S. housing supply obtained by Green et al. (2005) and Epple and Romer (1991).

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 0$</td>
<td>25.5</td>
</tr>
<tr>
<td>$\varepsilon = 1.75$</td>
<td>6.2</td>
</tr>
<tr>
<td>$\varepsilon = 3$</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 1: The average change in optimal marginal income tax (in percentage points) between endogenous and fixed price regimes for various elasticities of housing supply.

Table 1 reports changes in the optimal income tax rate for the median agent against the benchmark case of fixed prices and equal profit distribution. For the case of inelastic supply $\varepsilon = 0$, we immediately note a massive increase in the size of the price effect compared to the case of the elastic supply, $\varepsilon = 1.75$, considered in Section 3.3. Intuitively, with fixed supply any change in aggregate demand is solely translated into price change, which calls for stronger price corrective measures on the part of the public authority. In contrast, in the case of a very elastic supply of housing, $\varepsilon = 3$, we see a reduction in the size of the price effect compared to the case of price elasticity $\varepsilon = 1.75$ as changes in demand lead to smaller changes in price.

Lastly, we also reestimate the price effect for an alternative distribution of agent abilities. The lognormal distribution does not match well the upper tail of the income distribution in the United States. According to Diamond and Saez (2011), the top 1% levels of income follow the Pareto distribution with the shape parameter of 1.5. In the following estimation, we let types $n < 7.33$ follow the same lognormal distribution as before (with mean 0.368 and standard deviation 0.7) and, using a standard kernel smoother, append a Pareto tail (with shape 1.5).

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38In our simulations, we calibrate parameter $s$ of supply function $S(p) = sp^\varepsilon$ in order to match the average expenditure share of housing of 25% n the U.S.. In particular, we have $s = 0.25$ for $\varepsilon = 0$, $s = 0.5$ for $\varepsilon = 1.75$ and $\varepsilon = 3$. 
for types above 7.33. Figure 4 presents our findings for the same market structure as in the main text, in particular, the competitive market of housing with the price elasticity of supply of $\varepsilon = 1.75$. The top income tax rates are found to be higher compared to the lognormal case. The change in marginal tax rates between endogenous and fixed price economies (the price effect) became slightly smaller for top income levels. This change barely influences, however, the average change in marginal tax rate that is still around 6 percentage points as in the lognormal case.

Figure 4. Optimal income taxation and lognormal-Pareto distribution of productivities.

Note: The left diagram depicts the optimal marginal income tax rates for the economy with competitive housing market and (i) fixed price, equal profit distribution (dotted line); (ii) fixed price, empirical profit distribution (dashed line); and (iii) endogenous price, empirical profit distribution (solid line). In case (iii), the price elasticity of supply of housing is set at $\varepsilon = 1.75$. The right diagram depicts the changes in tax rates for cases (ii) and (iii) against case (i) as shown by the dashed and solid lines respectively.
References


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