Market Power and Optimal Income Taxation

Robertas Zubrickas Alexey Kushnir[1]

March 30, 2016

Abstract

When product markets are monopolistic, an income redistributive policy affects social welfare not only through changes in disposable income but also through changes in product prices. In the optimal income tax formula, we then obtain a Pigouvian term correcting for this price externality. We provide sufficient conditions when the Pigouvian term is progressive among top earners. Moreover, if the price externality is sufficiently strong, the overall optimal income tax schedule can be progressive. We also establish that the price externality becomes less important if firm profits are either taxed or socially weighted and if market competition intensifies.

Keywords: Optimal income taxation, progressive income taxation, market power, price externality, income redistribution. *JEL codes*: H21, H23, D43.

---

[1]Zubrickas: University of Bath, Department of Economics, Bath, UK, e-mail: r.zubrickas@bath.ac.uk; Kushnir: Carnegie Mellon University, Tepper School of Business, Pittsburgh, USA, email: akushnir@andrew.cmu.edu. We thank Craig Brett, Louis Kaplow, Florian Scheuer, and Ron Siegel for discussions and useful suggestions and also the audience at APET 2014 conference.

1 Introduction

Despite being universal, progressive marginal income taxes are difficult to explain theoretically. The workhorse model of optimal income taxation, developed by Mirrlees (1971), demonstrates that social welfare maximization is more consistent with regressive than progressive marginal taxes at the highest income brackets.[2] The underlying assumption of this classical model is that product markets are perfectly competitive, which implies fixed product prices. However, this assumption is more a theoretical construct than a realistic description of reality. One third of US industries are highly concentrated and about two-thirds of US publicly traded companies currently operate in more concentrated markets than 20 years ago.[3]

This paper shows that the unsatisfactory outcome of the Mirrleesian model disappears when the assumption of competitive markets is relaxed and the effect of income redistribution on product prices is accounted for. Specifically, progressive taxes become consistent with social welfare maximization in the presence of market power in product markets. With monopolistic price setting, a change in income distribution influences the utility of agents not only through the levels of income, but also indirectly through changes in product prices. If the price externality from an income change is progressive and sufficiently strong, we can obtain progressive marginal income taxes in the optimum. In general, the price externality of income redisribution has been overlooked in the previous literature, and this paper fills this gap by showing its importance for the case of optimal income taxation.

Within the framework of Mirrlees (1971) modified with non-competitive product mar-

[2]See Sadka (1976) and Seade (1977). In the Mirrleesian framework, progressive marginal taxes can arise under some special conditions such as the support of agent skills is unbounded and their distribution has a non-increasing hazard rate (Diamond 1998, Saez 2001). For more discussion, see also Tuomala (1984).

[3]... as reported by Theo Francis and Ryan Knutson, 2015, "Wave of Megadeals Tests Antitrust Limits in U.S." *The Wall Street Journal*, October 18 and The Editorial Board, 2015, "How Mergers Damage the Economy" *The New York Times*, October 31. See also The Prize in Economic Sciences (2014) on the importance of market concentration in multiple industries.

kets, we solve for the optimal income tax schedule. In the main model, we assume that one of the consumption goods available to agents is sold by a monopolistic firm. Price externality arises from the firm's profit maximization condition, which depends on the distribution of agents' disposable income. In the optimal income tax formula, we obtain an additional term – referred to as the *Pigouvian term* – the role of which is to correct for the price externality. The Pigouvian term entails effort distortions for the most productive agents, which, we argue, offers a more efficient way of reducing income inequality than does a redistributive policy with a zero marginal tax at the top income bracket optimal in the case of fixed prices.

Next, we provide a sufficient condition for a progressive Pigouvian term. The first part of this condition demands that if the public authority offers consumers a small rebate per unit of the monopolist's good, then the change in the monopolist's price made in response to the rebate is smaller than the rebate itself. The second part requires that the market demand and its elasticity be increasing and convex in income. Importantly, we show that the effect of price externality can be sufficiently strong, e.g., when the market demand for the monopolist's good is rather inelastic, to result in the overall progressive optimal marginal income tax schedule.

We illustrate the effect of price externality with a simple example. There are two types of agents, low L and high H, that are equally likely among a continuum of agents. Agents of type L have no ability to earn any income on their own, whereas agents of type H can earn either \$3 or, at some additional effort, \$6. The utility of income is determined by the consumed amount of a *non-divisible* good that is produced by a monopolistic firm at a fixed marginal cost of \$0.4 per unit. Without income taxation and redistribution, the agents of type H choose to earn \$6 (let the cost of effort be small), and the firm optimally sets the price at \$6 per unit, which results in 1 unit of consumption for agents of type H and 0 units for agents of type L. The public authority can increase the consumption of all agents by using a redistribution policy that gives each agent of type L a subsidy of \$2 paid by agents of type H. Given the new distribution of disposable income, the monopolist optimally sets the price at \$2, which leads to more consumption for both types: 2 units for H and 1 unit for L. But this is, however, not the best policy that the public authority could undertake. The same levels of consumption can be achieved if agents of type H choose to earn \$3 and transfer \$1 to agents of type L, because the firm sets the price at only \$1 per unit for this income distribution. The latter outcome is socially superior as the work effort needed to earn \$6 is saved. Hence, in the social optimum the public authority will impose a sufficiently high income tax "at the top" to prevent agents of type H from earning \$6.

We also analyze two extensions of the main model. In the first extension, firm profits receive a weighting in the social welfare function but can also be taxed to help financing public expenses. With profits taxed or socially weighted, the public authority needs to balance between the conflicting goals of increasing consumers' welfare and increasing profits, which results in a smaller role for the price externality in the optimal income taxation. In the second extension, we consider an oligopolistic market with several firms competing *a la* Cournot. Even though more competition attenuates the effect of price externality, the optimal marginal incomes tax can remain progressive with more than one firm on the market.

In the vast literature on optimal income taxation, our paper is closest to the strand of literature that studies the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Oswald (1983), Ireland (2001), Kanbur and Tuomala (2013). These papers are typically motivated by empirical observations that people care not only about their consumption, but also how it compares with that of others. The main message of this literature is that social comparisons typically (but not always) add toward progressivity of income taxation. In our paper, relativity concerns also arise because smaller income inequality improves agents' terms of trade. But unlike in the previous literature, here relativity concerns arise endogenously from the firm's profit maximization condition. A similar idea that individuals may seek a more equal income distribution in order to improve their terms of trade is also explored in Zubrickas (2012). Specifically, he applies it as an evolutionary argument to explain stronger pro-social norms observed in market-integrated traditional societies.

There is a small body of literature that analyzes optimal income taxation in the presence of externalities in labor markets. Lockwood et al. (2015) consider a model of profession choice. They show that if high-earning professions generate larger negative externalities, then progressive taxation can lead to higher social welfare by encouraging workers to move into more socially desirable professions. Rothschild and Scheuer (2016) consider a model where agents can earn their income with either traditional or rent-seeking activities. When income from some activity cannot be targeted, in their optimal tax formula they also obtain an additional externality-correcting term. They show that this term can be both smaller or larger than the Pigouvian correction term obtained when the income of each activity can be separately targeted.

The remainder of the paper is organized as follows. Section 2 introduces the model with monopolistic pricing. In Section 3, we solve for the optimal marginal income tax schedule and analyze its properties. Section 4 analyzes the extensions of the main model. Finally, Section 5 concludes.

2 Model

There is a continuum of agents in the economy. Agents differ in their productivity type n distributed according to probability density function $f(n) > 0$ with support $[\underline{n}, \overline{n}]$. For an agent with type n, the cost of earning income level z is equal to $c(z, n)$, where $c_z > 0$, $c_{zz} > 0$, $c_n < 0$, and $c_{zn} < 0$. We denote an agent's disposable income after tax $t(z)$ as $y = z - t(z)$, which is spent for the consumption of goods available in the economy.

Suppose that there are at least two goods in the economy, one of which is good X. The market for good X is monopolistic and its price, denoted by p, is determined by monopolistic price setting as described below. The prices for other goods are fixed, e.g., by assuming that the markets for the other goods are perfectly competitive. All agents have identical preferences for consumption, represented by an indirect utility function $v(p, y)$, which is concave and increasing in y. The Walrasian demand function for good X is denoted by $x(p, y)$, which is decreasing in price, $x_p < 0$. An agent's net utility after tax can be written as

$$u(p, y, z, n) = v(p, y) - c(z, n).$$

Until further notice, we assume that good X is produced by a single firm at constant marginal costs $k \geq 0$. The firm sets non-discriminatory price p to maximize its profit

$$\Pi(p) = \int \pi(p, y(n))f(n)dn,$$

where $\pi(p, y) = (p - k)x(p, y)$ is the profit accrued from an agent with income y. For convenience, we assume that the firm's profit maximization problem is well-defined for any distribution of income. In particular, we assume that there exists the unique price level p^* such that

$$\Pi'(p^*) = \int \pi_p(p^*, y(n))f(n)dn = 0, \tag{1}$$

and $\Pi''(p^*) < 0$.[4]

The social welfare function is given by the sum of agents' utilities

$$W = \int u(p^*, y(n), z(n), n)f(n)dn. \tag{2}$$

In Section 4, we consider a welfare function where firm profits also receive a social weighting.

Lastly, we say that an income schedule $\{y(n), z(n)\}_{n \in [\underline{n}, \overline{n}]}$ is feasible if it satisfies the incentive compatibility constraints

$$u(p, y(n), z(n), n) \geq u(p, y(m), z(m), n), \quad \forall n, m \in [\underline{n}, \overline{n}] \tag{3}$$

[4]Note that this assumption implicitly assumes that agents have non-homothetic preferences. See, e.g., Deaton and Muellbauer (1980) and Bertola et al. (2005) that discuss the importance of non-homothetic preferences in economic models.

and the resource constraint

$$\int [z(n) - y(n)] \, f(n)dn \geq R, \tag{4}$$

where $R \geq 0$ represents exogenous public expenditures.

3 Optimal income taxation

In this section, we formulate and solve the public authority's problem of optimal income taxation. After obtaining the solution, we analyze how the price externality of monopoly pricing influences the optimal income tax schedule. In particular, we show that the end-point results of zero tax optimal with fixed prices (Sadka 1976, Seade 1977) do not longer hold. We also demonstrate that progressive marginal taxes can arise in the optimum.

The problem of the public authority can be formulated as

$$\max \int u(p^*, y(n), z(n), n) f(n)dn \text{ subject to (1), (3), (4).} \tag{5}$$

We first note that the single-crossing condition, i.e. $c_{zn} < 0$, guarantees that the individual maximization constraint (3) is equivalent to

$$v_y y'(n) - c_z z'(n) = 0 \tag{6}$$

and $y'(n) \geq 0$ (see Lemma A1 in Appendix). In the maximization problem, we replace condition (3) with a weaker condition (6) and bear in mind that the disposable income has to be non-decreasing for the solution to be incentive compatible.

The Lagrangian of the public authority's problem is given by

$$\mathcal{L} = \int \{[v(p^*, y) - c(z, n) + \gamma \pi_p(p^*, y) + \lambda(z - y - R)]f + \mu(n)(v_y y' - c_z z')\}dn,$$

where γ, λ, and $\mu(n)$ are multipliers corresponding to constraints (1), (4), and (6). After integrating the last term by parts, the first-order conditions can be written as

$$y(n) : [v_y(p^*, y) + \gamma\pi_{py}(p^*, y) - \lambda]f - \mu'(n)v_y(p^*, y) = 0, \tag{7}$$

$$z(n) : [-c_z(z, n) + \lambda]f + \mu'(n)c_z(z, n) + \mu(n)c_{zn}(z, n) = 0, \tag{8}$$

$$p^* : \int [v_p(p^*, y)(f - \mu'(n)) + \gamma\pi_{pp}(p^*, y)f]dn = 0, \tag{9}$$

along with the transversality conditions $\mu(\underline{n}) = \mu(\overline{n}) = 0$. Condition (8), which is the same as in the standard case of fixed prices, can be used to establish that the multiplier on the individual maximization constraint needs to satisfy $\mu(n) \geq 0$ for all n (see, e.g., Tuomala 1984 for details).

Taking into account that $t = z - y$ and using individual maximization constraint (6) we deduce that the marginal tax can be calculated as $t'(z) = 1 - c_z/v_y$. Combining (7) and (8), we obtain the expression for the optimal marginal income tax.

Theorem 1. *The optimal marginal income tax in the presence of monopoly pricing is determined by*

$$t'(z) = -\frac{\mu c_{zn}}{\lambda f} - \frac{\gamma}{\lambda} s\pi_{py}, \tag{10}$$

where $s = c_z/v_y$.

The optimal tax formula has two terms. The first one, which we call the *incentive term*, captures the standard redistribution policy without price effects and it is aimed at maximizing social welfare subject to the incentive compatibility and resource constraints. The role of the second term is to correct for the price externality caused by monopoly pricing: income redistribution influences social welfare not only through the change of disposable income, but also indirectly through the change of the monopoly price. Given the externality interpretation, we call the second term the *Pigouvian term* of optimal income taxation.

3.1 End-point results

In the standard model with fixed prices the optimal tax formula has only the incentive term. The transversality condition then immediately implies the famous end-point result that the optimal marginal tax for the most productive agents is zero (Sadka 1976 and Seade 1977). Intuitively, if the marginal tax were positive, the public authority could instead impose zero tax on any income in excess of the current income of the most productive agents. Then, these agents would respond by exerting an additional effort to achieve the previously not feasible level of utility. Since the amount of tax collected would not change but the most productive agents would obtain a higher level of utility, the overall welfare would increase.

With market power in product markets, the appearance of the Pigouvian term in (10) implies that the end-point results no longer hold. The aforementioned tax relief would change the income distribution in the economy with implications for product prices. Therefore, the public authority cannot increase the utility of the most productive agents without influencing the rest of the agents when the price externality is present.

3.2 The Pigouvian term

We now analyze properties of the Pigouvian term. When the budget constraint is binding and active (as we assume), its multiplier is positive, $\lambda > 0$. In addition, the rate of substitution between gross and net levels of income is also positive, $s = c_z/v_y > 0$. Next, we provide an intuitive sufficient condition for the Lagrange multiplier γ corresponding to profit maximization condition (1) to be negative, $\gamma < 0$.

Imagine that the public authority offers consumers of good X a small unit rebate r. If the monopolist increases its price p^* in response to the rebate by an amount smaller than r, then we have $\gamma < 0$ as explained in the next proposition.

Proposition 1. *In the optimum, multiplier γ is strictly negative if in response to a small unit rebate r for good X the monopolist increases its price p^* by less than the rebate itself.*

Proof. Together with equation (7), the first-order condition (9) becomes

$$\int \frac{v_p}{v_y}(\lambda - \gamma\pi_{py}) f(n)dn = -\gamma \int \pi_{pp} f(n)dn.$$

Using Roy's identity $\frac{v_p}{v_y} = -x$, we obtain

$$\lambda \int x f(n)dn = \gamma \int (\pi_{pp} + x\pi_{py}) f(n)dn. \tag{11}$$

Since Walrasian demand $x(p^*, y)$ is positive and $\lambda > 0$, the sign of γ coincides with the sign of the right-hand side.

Now consider a small unit rebate r, which increases agent's income by $rx(p^*, y)$. Its effect on the monopolist's price is given by

$$\left.\frac{dp^*}{dr}\right|_{r=0} = \frac{\int x\pi_{py}(p^*, y(n)) f(n)dn}{-\Pi''(p^*)},$$

obtained from applying the Implicit Function Theorem to the monopolist's first-order condition (1). Since by the hypothesis $\left.\frac{dp^*}{dr}\right|_{r=0} < 1$, we have that $\int(\pi_{pp} + x\pi_{py}) f(n)dn < 0$ and, thus, $\gamma < 0$. □

Intuitively, the rebate is not entirely passed over to the monopolist if the income effect from the rebate is not too strong, i.e., $x_y x$ appearing in the term $x\pi_{py}$ of (11) is not too large. From a different perspective, Proposition 1 is equivalent to the condition that a unit rebate increases consumers' welfare. We also note that the condition of Proposition 1 is almost necessary in the following sense. If in response to a small unit rebate r the monopolist increases its price p^* by more than r then we must have $\gamma > 0$.

Though the condition on the monopolist's price response to a unit rebate is intuitive, this condition might be difficult to check. The next proposition provides a simple condition on the demand function $x(p, y)$ that is sufficient for $\gamma < 0$.

Proposition 2. *If the market demand's slope x_p is non-increasing in both price p and income y, then $\gamma < 0$.*

Proof. Using the Slutsky equation, we obtain $h_p = x_p + x x_y$, where $h(p,u)$ is the Hicksian demand. Using this expression, the integrand of the right-hand side of (11) expands as

$$\pi_{pp} + x\pi_{py} = (p^* - k)(x_{pp} + x x_{py}) + x_p + h_p.$$

The existence of the solution to the monopolist's first order condition (1) guarantees that $p^* - k > 0$. In addition, using the assumption that x_p is non-increasing in price p and income y, $x_p < 0$, and the compensated law of demand $h_p \leq 0$, we obtain that $\pi_{pp} + x\pi_{py} < 0$. The claim of the proposition then follows from (11). □

When $\gamma < 0$, the sign of the Pigouvian term and its size are determined by cross derivative π_{py}. We now show that cross derivative π_{py} captures the price externality resulting from a change in income distribution. Let us increase the level of income from $y(n')$ to $y(n') + \phi$ for all types $n' \in [n - \delta/2, n + \delta/2]$ for some n. Using the Implicit Function Theorem we obtain using equation (1)

$$\frac{dp^*}{d\phi} = \frac{\pi_{py}(p^*, y(n)) f(n)\delta}{-\Pi''(p^*)}. \tag{12}$$

Altogether, the price externality $dp^*/d\phi$ is proportional to cross derivative π_{py}, which in turn shows that the Pigouvian term is proportional to the price externality resulting from a change in income distribution.

To illustrate, consider the case of increasing price elasticity of demand $\varepsilon_y \geq 0$, where $\varepsilon(p,y) = x_p p/x$. Given that $\pi_p = x(1 + \kappa\varepsilon)$, where $\kappa = (p^* - k)/p^*$ we obtain

$$\pi_{py} = \kappa\varepsilon_y x + (1 + \kappa\varepsilon)x_y. \tag{13}$$

We then have that cross derivative π_{py} is positive at higher levels of income ($\varepsilon \geq -1/\kappa$), yielding a positive Pigouvian term, while the Pigouvian term can turn negative at lower levels of income ($\varepsilon \leq -1/\kappa$) if $\kappa\varepsilon_y x$ is small.

Next, we discuss the monotonicity properties of the Pigouvian term. For this purpose, it is convenient to express the formula of the optimal marginal income tax as

$$t'(z) = -\frac{\mu c_{zn}}{\lambda f} + \underbrace{\frac{-\gamma \pi_{py}}{\lambda - \gamma \pi_{py}}}_{\text{Part 1}} \underbrace{\left(1 + \frac{\mu c_{zn}}{\lambda f}\right)}_{\text{Part 2}}, \tag{14}$$

to derive which we used $s = 1 - t'(z)$ in (10). We can observe that the monotonicity properties depend on the cross-derivative π_{py} (Part 1 of the Pigouvian term) and on the incentive term (Part 2). The next proposition provides sufficient conditions for the Pigouvian term being progressive among top earners.

Proposition 3. *Let the condition of Proposition 2 hold. If the market demand x and its elasticity ε are non-decreasing and convex in income y, the Pigouvian term is progressive among top earners.*

Proof. By Proposition 2, we have $\gamma < 0$ and, thus, the monotonicity properties of Part 1 of the Pigouvian term in (14) coincide with those of the cross derivative π_{py}. Consider the third derivative $\pi_{pyy} = \kappa \varepsilon_{yy} x + 2\kappa \varepsilon_y x_y + (1 + \kappa\varepsilon) x_{yy}$. Note that the profit maximization condition (1) and non-decreasing elasticity ε ensure that $\epsilon \geq -1/\kappa$ for high levels of income. Where the latter condition holds, we obtain $\pi_{pyy} > 0$ because of non-decreasing and convex demand x and elasticity ε. Then, due to $y'(n) \geq 0$ in the optimum, Part 1 of the Pigouvian term has to be progressive for high values of n. Next, consider Part 2 in (14). We have $\mu c_{zn} \leq 0$ because of $\mu(n) \geq 0$ and $c_{zn} < 0$. Then, Part 2 must be non-decreasing at some non-empty interval to the left from $\overline{n}$ due to the transversality condition $\mu(\overline{n}) = 0$. Thus, the Pigouvian term must be progressive at least at the intersection where Parts 1 and 2 are non-decreasing. □

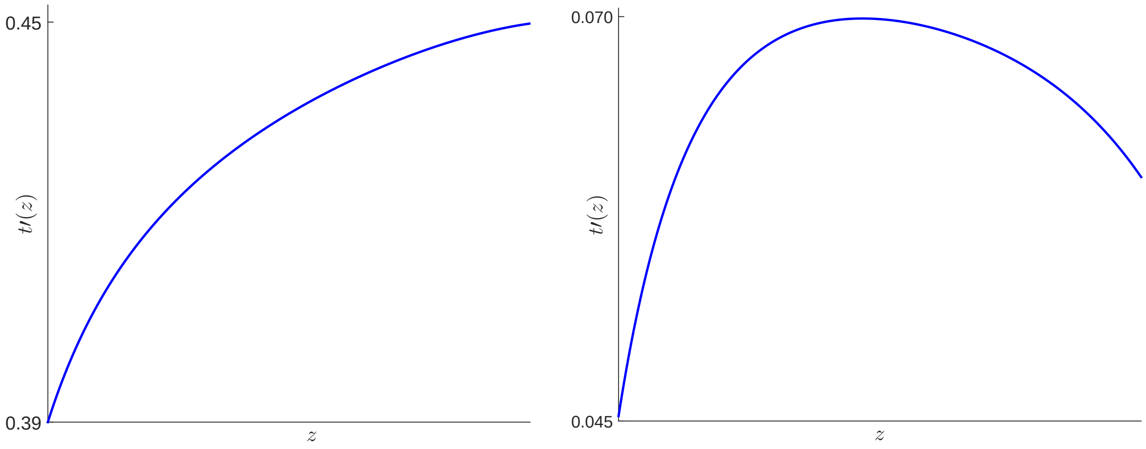
3.3 Progressive marginal taxes

In the presence of market power, the progressivity of optimal marginal tax depends on the interaction of both incentive and Pigouvian terms of the tax formula in (10). As in the model with fixed prices, the incentive term takes the inverted-U shape, which implies that the progressivity of marginal taxes depends on the progressivity of the Pigouvian term and its relative importance. The Pigouvian term shifts the inverted-U shape of the incentive term upwards in proportion to the corresponding price externality. With a progressive Pigouvian term the shift is stronger at higher income brackets than at lower ones. In particular, if price externality at higher income brackets is sufficiently strong, we obtain an overall progressive marginal tax schedule as illustrated in the following example.

Example 1. *Consider consumer preferences expressed by indirect utility function* $v(p, y) = e^{-p}(y^{0.5} + ap + a)$*, where* $a < 0$.[5] *From Roy's identity we get the demand function* $x(p, y) = 2(apy^{0.5} + y)$*. Parameter* a *determines the sensitivity of demand to price changes: the demand for good* X *is less elastic for smaller absolute values of* a*. The cost function takes the form of* $c(z, n) = 0.5z^2/n$*, productivity types are distributed over* $[10, 20]$ *according to the truncated normal distribution with the mean of* 15 *and standard deviation of* 4*. The marginal cost is set at* $k = 0.1$ *and the level of exogenous public expenditures at* $R = 1$*.*

Using computer simulations we calculate the optimal marginal tax schedules for the cases of $a = -0.5$ (inelastic demand) and $a = -5$ (elastic demand). Note that for the above specification of preferences the condition of Proposition 2 is satisfied and $\gamma < 0$ in the optimum. The form of the Pigouvian term is determined by the cross derivative equal to $\pi_{py}(p, y) = 2 + 2apy^{-0.5}$, which takes larger values when the demand for good X is less elastic. In other words, a low responsiveness of demand to price changes creates more price externality and, thus, signifies the Pigouvian term because with inelastic demand the monopolist has strong incentives to revise prices in response to income variations. With a dominant and progressive Pigouvian term (we have $\pi_{pyy} > 0$ in our example), we can

[5] This preference specification is taken, in a slightly modified form, from Hausman (1981, p. 668). The modification we made is to ensure that the monopolist's profit maximization problem is well defined. We can check that $v(p, y)$ is a valid indirect utility function which arises from consumer maximization. First of all, it is continuous and, by the normalization that the composite of goods other than X is numeraire, it is homogeneous of degree zero in prices and income. Also, it is increasing in income and $a < 0$ ensures that it is decreasing in prices. Finally, for the parameter values used in this example, we always have that the compensated law of demand is satisfied.



(a) Inelastic demand

(b) Elastic demand

Figure 1: Optimal marginal tax schedules

obtain a progressive marginal income tax schedule in the social optimum as illustrated in the case of $a = -0.5$ (Figure 1a). Intuitively, when price externality is sufficiently strong and is increasing in income the public authority finds it welfare improving to curb income inequality with progressive marginal taxes done to suppress upward pressures on prices. With elastic demand ($a = -5$), monopolistic pricing is less responsive to income variations as consumers swiftly react to price changes, thus, rendering the Pigouvian term less important. With the incentive term of the tax formula dominant, the optimal marginal tax takes form of inverted U-shape (Figure 1b).

4 Extensions

In this section, we consider two extensions of the model. First, we extend the social welfare function to account for the monopolist's profits. Second, we relax the assumption of the perfectly monopolistis market by introducing a varying degree of market competition.

4.1 Profits in the welfare function

We modify the main model to allow for the firm profits entering the public authority's welfare function with a weight $\omega \in [0, 1]$. In addition, we assume that the firm profits are subject to exogenously determined profit tax τ that is also used to finance public expenses R. Hence, the resource constraint becomes

$$\int \left[z(n) - y(n) + \tau\pi(p, y(n))\right] f(n)dn \geq R. \tag{15}$$

The Lagrangian of the public authority's problem can now be written as

$$\begin{aligned}\mathcal{L} = \int \{ & [v(p^*, y) - c(z, n) + \lambda(z - y + \tau\pi(p^*, y) - R) + \gamma\pi_p(p^*, y) \\ & + \omega(1-\tau)\pi(p^*, y)]f(n) + \mu(n)(v_y y' - c_z z')\}dn.\end{aligned} \tag{16}$$

The first-order conditions with respect $z(n)$ and p remain intact for the modified problem.The first-order condition with respect to $y(n)$ becomes

$$[v_y(p^*, y) - \lambda + (\lambda\tau + \omega(1-\tau))\pi_y(p^*, y) + \gamma\pi_{py}(p^*, y)]f(n) - \mu'(n)v_y(p^*, y) = 0 \tag{17}$$

Condition (17) together with (8) yield the optimal marginal tax

$$t'(z) = -\frac{\mu c_{zn}}{\lambda f} - \frac{s}{\lambda}\left(\gamma\pi_{py} + (\lambda\tau + \omega(1-\tau))\pi_y(p^*, y)\right). \tag{18}$$

In the model of Section 3, the authority takes into consideration only consumers' welfare, which is done at the expense of firm profits. When firm profits enter the social welfare function, or taxed, the authority has to balance the conflicting goals of improving consumers' welfare and increasing profits. When $\gamma < 0$ and good X is a normal good, i.e. x_y and, thus, $\pi_y > 0$, the price externality becomes smaller compared to the optimal tax formula (10).[6] The reduction is proportional to the marginal profit π_y weighted by $\lambda\tau + \omega(1-\tau)$. Finally, if the weight of firm profits in the welfare function is higher than the Lagrange multiplier of the resource constraint, i.e. $\omega \geq \lambda$, the additional profit taxes increase price externality, and decrease it otherwise.

[6]Propositions 1 and 2 still hold provided that marginal profit π_y does not exceed $\lambda/(\lambda\tau + \omega(1-\tau))$.

4.2 Oligopolistic market

In this subsection, we analyze how optimal marginal taxation changes with varying degree of market competition. To accomplish this goal we consider a Cournot model with N firms competing in the setting of the main model of Section 3.

For a given distribution of disposable income, the firms face a downward sloping aggregate demand curve $X(p) = \int x(p, y(n))f(n)dn$. Our assumption $x_p < 0$ guarantees that the inverse demand function, denoted as $p(X)$, is well defined. Firm i maximizes its profit by setting quantity x_i with the price determined by $p(X)$, where $X = \sum_{i=1,..,N} x_i$. Firm i's profit maximization condition can then be written as $p - k + x_i/X_p(p) = 0$. Hence, the equilibrium price level has to satisfy the following condition

$$N(p-k)X_p + X = 0. \tag{19}$$

When $N = 1$, this condition reduces to the monopolist's price maximization condition (1). Using condition (19) for determining the price of good X, we can write the Lagrangian of the public authority's problem as

$$\mathcal{L} = \int \{[v(p^*, y) - c(z, n) + \lambda(z - y - R) + \gamma(N(p-k)x_p(p^*, y) + x(p^*, y))]f(n) \\ + \mu(n)\,(v_y y' - c_z z')\}dn,$$

where multiplier γ now corresponds to condition (19). The first-order condition with respect to $z(n)$ does not change, and with respect to $y(n)$ and p^* becomes, respectively,

$$y(n) : [v_y(p^*, y) - \lambda + \gamma(N(p^*-k)x_{py}(p^*, y) + x_y(p^*, y))]f(n) - \mu'(n)v_y(p^*, y) = 0,$$
$$p^* : \int [v_p(p^*, y)(f(n) - \mu'(n)) + \gamma((N+1)x_p(p^*, y) + N(p-k)x_{pp}(p^*, y))f(n)]dn = 0.$$

The optimal marginal tax takes a form similar to that in (10)

$$t'(z) = -\frac{\mu c_{zn}}{\lambda f} - \frac{\gamma}{\lambda} s\tilde{\pi}_{py}, \tag{20}$$

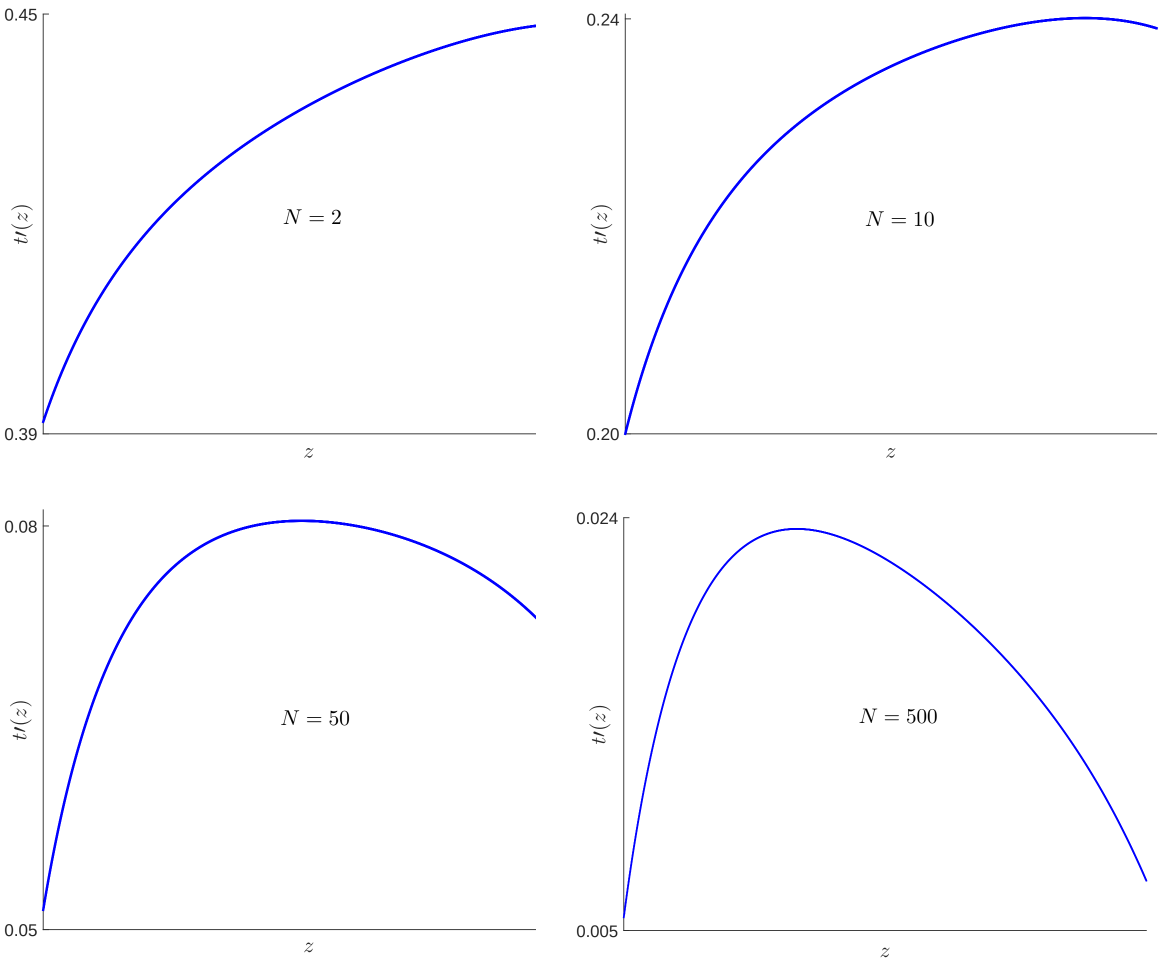


Figure 2: Optimal marginal tax schedules with N firms

where $\tilde{\pi}_{py} = N(p-k)x_{py} + x_y$. As before, the second term captures the price externality associated with a change in income.

Intuitively, the effect on the price of good X stemming from changes in income distribution will decrease with additional competition in the market. Therefore, when the number of firms increases, the optimal income tax schedule should be driven more by the incentive term in (20) than by the Pigouvian term. To obtain this effect analytically we need to calculate derivatives dp^*/dN, $d\mu/dN$, $d\gamma/dN$, and $d\lambda/dN$, which is not feasible. Instead, we revert to the previous numerical example (Example 1) to demonstrate the influence of market competition. Figure 2 illustrates that the optimal marginal tax schedule remains progressive with $N = 2$ firms as the Pigouvian term continues to play an important role. But its role starts fading away with larger N, gradually turning the tax schedule into the standard inverse of U-shape.

5 Conclusion

In this paper, we analyze the problem of optimal income taxation with non-competitive product markets. Absent perfect competition, the distribution of consumers' income matters for prices and, therefore, a redistributive policy can also have a price effect with significant welfare implications. We demonstrate that with endogenous prices the optimal income tax formula features an additional term, referred to as the *Pigouvian term*, aimed at correcting for the price externality of income redistribution brought about by the tax policy. When price externality is relatively important, as in the case of inelastic market demand for monopolistic produce, and the amount of price externality increases in disposable income, we obtain progressive marginal income taxes in the social optimum. Our main conclusion is that by allowing market power in product markets the classical Mirrleesian model of optimal income taxation can be reconciled with the practice of progressive income taxation.

Appendix

Lemma A1. *The individual maximization constraint (3) is equivalent to*

$$v_y y'(n) - c_z z'(n) = 0 \text{ and } y'(n) \geq 0 \text{ for any } n.$$

Proof. Let us denote the agent's utility from revealing his productivity type truthfully as

$$U(n) \equiv u(y(n), p^*, z(n), n) = v(y(n), p^*) - c(z(n), n). \tag{21}$$

If revealing the agent's type truthfully is optimal then

$$U(n) - u(y(n), p^*, z(n), n) = 0 \leq U(m) - u(y(n), p^*, z(n), m), \tag{22}$$

which leads to the following first-order condition

$$U'(n) = u_n(y(n), p^*, z(n), n). \tag{23}$$

Let us assume that $y(n)$ and $z(n)$ are differentiable. Taking the derivative of (21) over n we notice that condition (23) is equivalent to

$$v_y y'(n) - c_z z'(n) = 0. \tag{24}$$

The second-order condition for maximization (22) is

$$U''(n) - u_{nn}(y(n), p^*, z(n), n) \geq 0. \tag{25}$$

Taking the derivative of (23) over n we notice that condition (25) is equivalent to $u_{yn} y'(n) + u_{zn} z'(n) \geq 0$. The first-order condition (24) then allows us to rewrite this inequality as

$$0 \leq u_{yn} y'(n) + u_{zn} z'(n) = -c_{zn} \frac{v_y}{c_z} y'(n).$$

Given our assumptions on the agent's utility this leads to necessary condition $y'(n) \geq 0$.

We now show that if $\{y(n), z(n)\}_{n \in [\underline{n}, \overline{n}]}$ satisfies condition (24) and $y'(n) \geq 0$ then it satisfies the individual maximization constraint. In contradiction, suppose that there exist n_1 and n_2 such that

$$u(y(n_1), p^*, z(n_1), n_2) > u(y(n_2), p^*, z(n_2), n_2)$$

If $n_1 < n_2$ (a similar argument applies when $n_1 > n_2$) there exists $n_3 \in (n_1, n_2)$ such that

$$\frac{\partial}{\partial n} u(y(n), p^*, z(n), n_2)|_{n=n_3} < 0$$

This derivative is equal to

$$\begin{aligned}\frac{\partial}{\partial n}u(y(n), p^*, z(n), n_2)|_{n=n_3} &= v_y(y(n_3), p^*)y'(n_3) - c_z(z(n_3), n_2)z'(n_3)\\ &= (c_z(z(n_3), n_3) - c_z(z(n_3), n_2))\frac{v_y}{c_z}y'(n_3) < 0,\end{aligned}$$

where we used condition (24) to derive the equality. The latter inequality cannot be satisfied because $c_{zn} < 0$ and $y'(n) > 0$. Hence, we arrive at a contradiction. $\square$

References

Bertola, G., Foellmi, R., Zweimüller, J., 2005. *Income Distribution in Macroeconomic Models*. Princeton University Press.

Boskin, M.J., Sheshinski, E., 1978. Optimal Redistributive Taxation When Individual Welfare Depends Upon Relative Income. *Quarterly Journal of Economics* 92, 589–601.

Deaton, A., Muellbauer, J., 1980. *Economics and Consumer Behavior*. Cambridge University Press: Cambridge, UK.

Hausman, J.A., 1981. Exact Consumer's Surplus and Deadweight Loss. *American Economic Review* 71, 662–676.

Ireland, N.J., 2001. Optimal income tax in the presence of status effects. *Journal of Public Economics* 81, 193–212.

Kanbur, R., Tuomala, M., 2013. Relativity, Inequality, and Optimal Nonlinear Income Taxation. *International Economic Review* 54, 1199–1217.

Lockwood, B. B., Charles G. Nathanson, C. G., and Weyl, E.G., 2015. *Taxation and the allocation of talent*, Mimeo, Harvard University and University of Chicago.

Mirrlees, J.A., 1971. An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies* 38, 175–208.

Diamond, P., 1998. Optimal Income Taxation: An Example with a U-shaped Pattern of Optimal Marginal Tax Rates. *American Economic Review* 88, 83–95.

Mirrlees, J.A., 1976. Optimal tax theory: A synthesis. *Journal of Public Economics* 6, 327–358.

Oswald, A.J., 1983. Altruism, jealousy and the theory of optimal non-linear taxation. *Journal of Public Economics* 20, 77–87.

Rothschild, C., Scheuer, F., 2016. Optimal Taxation with Rent-Seeking. *Review of Economic Studies*, forthcoming.

Sadka, E., 1976. On Income Distribution, Incentive Effects and Optimal Income Taxation. *Review of Economic Studies* 43, 261–267.

Saez, E., 2001. Using Elasticities to Derive Optimal Income Tax Rates. *Review of Economic Studies* 68, 205–229.

Seade, J.K., 1977. On the shape of optimal tax schedules. *Journal of Public Economics* 7, 203–235.

The Prize in Economic Sciences 2014. *Nobelprize.org.* Nobel Media AB 2014. `http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2014/advanced-economicsciences2014.pdf`

Tuomala, M., 1984. On the optimal income taxation: Some further numerical results. *Journal of Public Economics* 23, 351–366.

Zubrickas, R., 2012. How exposure to markets can favor inequity-averse preferences. *Journal of Economic Behavior and Organization* 84, 174–181.