Abstract

We study an optimal income taxation problem in a Mirrleesian setting with endogenous product prices and positive firm profits. When firm profit are progressively distributed among agents, we show that the public authority favors lower equilibrium prices in competitive markets that leads to more progressive taxation in the optimum. Using a calibrated model of the U.S. housing market, we quantify the price effect and show that it adds 5 percentage points on average to marginal income tax rates. In oligopolistic markets, market power creates an additional non-competitive effect that exerts downward pressure on optimal tax rates. Using the U.S. housing model, we show that, as the market structure varies, changes in the price effect and the non-competitive effect almost cancel each other out leading to a stable prediction of optimal income tax policy across various markets. We also analyze how commodity and profit taxation influence the price and non-competitive effects.

Keywords: Optimal income taxation, endogenous price, competitive market, oligopolistic market, housing market.

JEL Classification: H21, H23, D43.

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1 Introduction

The design of income taxes is among the most important economic problems. At the same time, the income taxation literature commonly assumes that product markets are perfectly competitive. While this assumption is convenient for analysis it is far from an accurate description of most markets. According to recent studies (Azar et al., 2017; De Loecker et al., 2018), one third of U.S. industries are highly concentrated and over 75% of them are operating in markets that are more concentrated than 20 years ago. Similar market structures are observed in Europe, where the top five food retailers typically have a 43% to 69% share of the market (European Commission, 2015).

What are the implications for optimal income taxation if markets are not perfectly competitive? What is the size of optimal income tax change due to the presence of endogenous prices? Our goal in this paper is to answer these questions and, thus, to bridge a gap in the public economics literature by incorporating endogenous prices into the analysis of income taxation.

We consider the standard Mirrlees (1971) framework with a continuum of agents who differ in their productivity types. Agents earn labor income and care about the consumption of two goods: the numeraire good and the “main” good. The numeraire good is produced with a constant returns to scale technology and has a perfectly elastic supply function, whereas the main good is produced with a decreasing returns to scale technology and has a strictly increasing supply function. As a consequence, the price of the numeraire good is constant and the price of the main good is endogenously determined by the market equilibrium condition. Also, firms producing the numeraire good realize zero profits and firms producing the main good realize positive profits. Motivated by empirical evidence, we assume that firm profits are progressively distributed among agents in the economy with more productive agents receiving larger profit shares (see Saez and Zucman, 2016).

The public authority problem is to design the optimal income tax schedule that maximizes a concave objective of agent utilities subject to three constraints: the resource constraint, which demands that the public authority raises a fixed level of public funds; the incentive compatibility constraint, which requires agents to reveal their productivity types; and the market equilibrium condition, which determines the price level.

When agent productivity is imperfectly observable and the public authority has a limited set of tax instruments, not every Pareto efficient allocation can be supported with competitive prices. To illustrate, let us consider an example in which all agents have the same productivity

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2 Atkinson (2012, p. 775) offers an extensive discussion of how the existing taxation literature mostly fails to take the underlying market structure and endogenous prices into account. Two recent exceptions are Kaplow (2018) and Costa and Maestri (2018). We provide an extensive discussion of these and other relevant papers in literature review section.
type but different profit shares. If the agents have linear utility in income, the difference in profit shares does not create any difference in the agents’ optimal labor supply. Then, all agents have the same labor income in equilibrium and are subject to the same income tax. Hence, their income cannot be freely redistributed. Overall, a tax policy based on labor income alone does not allow for full discretion in redistribution, which leads to the failure of the second welfare theorem.\(^3\)

The failure of the second welfare theorem leads to a binding market equilibrium condition in the optimum. The public authority, motivated by equity concerns, uses then the price level as an additional redistributing tool: a decrease in price level benefits low-productivity agents as they can afford to consume more products, but hurts high-productivity agents as their utility is mainly influenced by the decrease in firm profits.\(^4\) The public authority, however, cannot control the price level directly. Instead, it imposes a more progressive income tax schedule that decreases agent labor supply and lowers agent income. The lower agent income decreases agent demand for products leading to a smaller product prices in equilibrium. In addition, if we assume that a share of firm profits belongs to agents outside the economy, efficiency concerns start to play an important role in the choice of income tax policy.\(^5\) In this case, motivated by both efficiency and equity concerns, the public authority favors even lower price levels and an even more progressive income taxation in the optimum.

To understand whether the price effect can significantly alter the optimal income taxation policy, we estimate it using the U.S. housing market. This market is particularly suitable for illustrating our theoretical results because housing costs comprise the largest share of overall household expenditures.\(^6\) The housing sector is also associated with large firm profits, which distribution is one of the drivers of the price effect on optimal income taxation. Using a calibrated model of the housing market (based on Miles and Sefton (2018) and Saiz (2010)), we found that the price effect increases optimal marginal income taxes by 5 percentage points on average in competitive markets.

We also consider markets with various forms of oligopolistic competition. In oligopolistic markets, the presence of market power leads to under-production in equilibrium. As a coun-

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\(^3\)The second welfare theorem also fails if tax policy is based on total income and if firm profits can be separately taxed (see p. 11).

\(^4\)This argument is in line with recent empirical evidence that connects individual earnings growth with real stock returns. Guvenen et al. (2017) show that individual earnings growth is especially sensitive to a change in real stock returns for high- and low-income levels. This relationship for high-income levels supports our reasoning that a decrease in firm profits hurts more high-income agents than moderate- and low-income agents. For low-income levels, the sensitivity can be explained by a high correlation between real stock returns and GDP growth. That is, labor earnings are higher in booms and lower in recessions.

\(^5\)A substantial share of equities is held by foreigners: the share of foreign equity holdings amounts to 13.6% in the U.S. (U.S. Treasury, 2017) and, on average, 38% in European countries (Davydoff et al., 2013).

\(^6\)See Consumer Expenditure Survey, 2017, Table 1203. Income Before Taxes: Annual Expenditure Means, Shares, Standard Errors, and Coefficients of Variation and Bureau of Economic Analysis, 2016, Table 2.3.5U. Personal Consumption Expenditures by Major Type of Product and by Major Function that report approximately 25% as the share of housing costs (including utilities) in U.S. household expenditure. Housing costs account for a similar average share of household expenditure in Europe (Eurostat, 2016).
term measure, the public authority seeks to stimulate labor income and, thus, aggregate demand by decreasing marginal income taxes. This non-competitive effect works in the direction opposite to that of the price effect. Using the U.S. housing model, we show that the change in the price effect due to a change in market structure is almost compensated by the change in the non-competitive. Hence, the optimal income taxation schedule barely varies across market structures, the result that was not previously pointed out in the literature.\footnote{To avoid any confusion, we want to highlight that for a large number of firms the oligopolistic model does not converge to the standard Mirrleesian setting. Instead, the oligopolistic model converges to our competitive model with upward sloping supply curve, where the price effect on optimal income taxation is present.}

Further, we investigate how commodity and profit taxation influence the price and non-competitive effects on income taxation. For both competitive and oligopolistic markets, we show that the price effect—or a correction to the optimal income tax schedule due to progressive distribution of firm profits—is needed if the optimal commodity taxation is at the discretion of the public authority. This result is similar in spirit to Naito (1999) who showed that the seminal result of “uniform commodity tax under nonlinear income taxation” by Atkinson and Stiglitz (1976) no longer holds if marginal costs of production are non-constant. At the same time, the commodity taxation fully eliminates the non-competitive effect in oligopolistic markets by providing firms with a subsidy to correct inefficiencies associated with the presence of market power. Specifically, if firm profits are equally distributed among agents (no price effect) the combination of optimal commodity and income taxes brings the firm production to the efficient level.\footnote{See Myles (1996) for a related result.} We also show that the price effect on optimal income taxation persists in the presence of profit taxation in an extension of our model where firm can entry and exit the market.

We want to highlight here that one should not regard our analysis as income tax design should correct for market inefficiencies. For this purpose the government might have better instruments such as commodity and profit taxation. Instead, we want to emphasize that the optimal income tax design should take into account market inefficiencies and market structure. This claims pertains even to settings where other tax instruments such as commodity and profit taxation are at the discretion of the public authority.

Next, we relate our findings to the three tests on policy relevance proposed by Diamond and Saez (2011). First, our results are based on an economic mechanism whereby the second welfare theorem does not hold in incomplete-information markets when only a limited set of tax instruments is available. Second, we show that the price effect is empirically relevant and of the first order by estimating it as equal to around 5 percentage points based on the U.S. housing market. As we show in the appendix, our results are also robust to various market structures, model specifications, and simulation assumptions. Third, our tax policy recommendation favors more progressive income tax rates, which is socially acceptable on equity grounds. Lastly, we note that the price and non-competitive effects are relevant to any policy with implications for
income redistribution such as the design of the minimum wage, basic income, welfare benefits, and pensions.

We postpone a detailed literature review until Section 7 and, here, only briefly outline our main contributions in relation to previous research. Compared to the growing literature on optimal income taxation with endogenous wages in labor markets (e.g., Rothschild and Scheuer, 2013; Sachs, Tsywnisky, and Werquin, 2016), we consider endogenous prices in product markets. The price effect is driven by the distribution of firm profits among agents inside and outside the economy rather than general equilibrium effects analyzed in this literature.

In relation to studies focused on optimal income taxation with externalities (e.g., Rothschild and Scheuer, 2016; Lockwood et al., 2017; Rothschild and Scheuer, 2014) agents in our paper do not impose any direct externality. Instead, in our model, agents impose only pecuniary externalities. The welfare theorems imply that pecuniary externalities do not limit the set of feasible outcomes in complete information markets. If agent productivity is not perfectly observable and taxes are based on labor income, we show that a constrained Pareto-efficient allocation is not generally supported by a competitive equilibrium. This leads to a binding market equilibrium constraint and the price effect on optimal income taxation.

Finally, the literature analyzing taxation in the presence of imperfect competition is rather thin. Kaplow (2018) considers income taxation for various market structure. He studies income taxation policies in economies with exogenously given firm markups. Hence, he does not consider the influence of income taxation policy on equilibrium product prices, which is the main subject of our analysis. Costa and Maestri (2018) study analytically optimal income taxation in non-competitive labor markets. They assume that firm profits are spread uniformly across agents thus abstracting from firm profit distribution effect, the main subject of our analysis. There are also a few important papers on commodity taxation in oligopolistic markets (Auerbach and Hines, 2001; Myles 1987; Reinhorn, 2005). In contrast to these papers, we consider an incomplete information setting and highlight the influence of profit distribution among agents on the optimal tax schedule.

The remainder of the paper is organized as follows. In Section 2, we introduce the model. In Section 3, we consider competitive markets and analyze the properties of the optimal income tax schedule. In Section 4, we calibrate our model and estimate the size of the price effect. In Section 5, we present our analysis for oligopolistic markets. We investigate the price and non-competitive effect in the presence of commodity taxation in Section 6. In Section 7, we provide a detailed literature review, and in Section 8, we present our conclusion. The omitted proofs are postponed to Appendix A.1. Appendices A.2-A.6 comprise extensions of our main model including the analysis of optimal profit taxation and provide additional simulation results.
2 Model

There is a continuum of agents indexed by productivity type \( n \) and distributed according to the probability density function \( f(n) > 0 \) with support \([n_1, n]\). Agent \( n \)'s labor income is given by \( z = n\ell \), where \( \ell \) is the number of hours worked. The labor cost is represented by an increasing and convex function \( c(\ell) \). The labor income is taxed according to schedule \( T(z) \). After tax, the agent’s disposable income is equal to \( y = z - T(z) \).

In the economy, there are two goods: a numeraire good and good X (referred to as the main good in the introduction). The numeraire good is produced with a constant returns to scale technology that results in a fixed price normalized to 1 and zero firm profits. Good X is produced with a decreasing returns to scale technology that yields positive profits. We denote \( p \) and \( \Pi(p) \) as the price and the profit, respectively, of firms producing good X.\(^9\)

We assume that firm profits are distributed in the form of dividends with agent \( n \) receiving share \( \xi(n) \geq 0 \). Motivated by empirical evidence, we consider a progressive distribution of firm profits when agents with higher labor income possess a larger share of firm profits \( \xi'(n) \geq 0 \) (see Saez and Zucman, 2016). Although we mainly analyze the case when all profits are distributed inside the economy \( \int \xi(n)f(n)dn = 1 \), we also consider an extension when some firm profits are held by foreigners, as observed in both U.S. and European data (see U.S. Treasury, 2017; Davydoff et al., 2013).

For clarity, we do not consider explicitly profit taxation in the main theoretical analysis. To make our calibration exercise plausible, we assume that firm profits are taxed at a fixed rate of 15% in our simulations. This corresponds to the U.S. tax rate on qualified dividends at most income levels. In Appendix A.5, we also investigate the interaction between optimal profit and labor income taxation in a model with endogenous firm entry and exit.\(^10\)

Overall, an agent’s income consists of labor income and dividends (profit shares) \( \tilde{y}(n) = y(n) + \xi(n)\Pi(p) \). Agents’ preferences for consumption are represented by an indirect utility function \( v(p, \tilde{y}) \). When firm profits are unequally distributed, agent indirect utility \( v(p, \tilde{y}) \) can violate the single-crossing property (Mirrlees, 1976). To avoid this problem, we consider homothetic preferences in the main text. This case also corresponds to our numerical simulations in Section 4, where we consider the constant elasticity of substitution (CES) preferences.\(^11\)

Homothetic preferences imply linear indirect utility

\[ v(p, y) = a(p)y, \]

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\(^9\)We do not model explicitly why firms that produce the numeraire good do not switch to producing the more profitable good X. A lack of technology and/or patents may function as a barrier to entry to an industry. The high degree of profit and performance differences even among similar firms is a well-documented phenomenon (see, for example, Syverson (2011) for a recent survey).

\(^10\)See also Scheuer (2014) for a related analysis of optimal profit taxation under endogenous firm formation.

\(^11\)Recent empirical studies found small income effects on individual labor supply (see a discussion in Sorensen, 2009), which also justifies the use of homothetic preferences in our analysis.
where \( a(p) \) is some strictly decreasing function. Appendix A.3 provides a detailed analysis of general non-homothetic preferences. Overall, the agent’s net utility is defined as the difference between indirect utility from consumption and labor cost

\[
U(p, \tilde{y}, \ell) = v(p, \tilde{y}) - c(\ell). \tag{1}
\]

The social welfare function is given by

\[
W = \int W(U(p, \tilde{y}(n), \ell(n)))f(n)dn, \tag{2}
\]

where \( W \) is the social value of utility. Assuming that the public authority has equity concerns, we let \( W \) be a differentiable, increasing, and strictly concave function. The public authority wants to maximize social welfare function \( W \) subject to three constraints. The first is the resource constraint:

\[
\int T(z(n))f(n)dn = \int (n\ell(n) - \tilde{y}(n) + \xi(n)\Pi(p))f(n)dn \geq R \tag{3}
\]

that ensures the public authority covers its own expenses \( R \geq 0 \), which are spent solely on the numeraire good. The second is the incentive compatibility constraint:

\[
U(p, z(n) - T(z(n)) + \xi(n)\Pi(p), \ell(n)) \geq U(p, z(m) - T(z(m)) + \xi(n)\Pi(p), z(m)/n), \tag{4}
\]

for all \( n, m \in [n, \bar{n}] \), which ensures that an agent with productivity \( n \) does not want to seek the labor income of an agent with productivity \( m \).

The third is a market equilibrium condition that determines price \( p \). This condition varies across market structures. In competitive markets (Section 3), where we assume that firms are price-takers, the market equilibrium condition requires the market supply to be equal to the market demand for good X. In oligopolistic markets (Section 5), where each firm takes into account its influence on the level of product price, the market equilibrium is determined by the firm profit maximization condition.

Overall, the main difference between our framework and the one of Mirrlees (1971) is that one of the product prices is endogenously determined. In addition, in our model firms obtain positive profits that are progressively distributed among agents. The effect of endogenous prices and profit distribution on optimal income taxation is the main subject of our analysis.

Finally, we want to comment on our modelling choice. We utilize the Mirrleesian framework rather than a more recent framework by Saez (2001) who uses an empirically relevant income distribution to analyze the optimal income taxation. We follow this route because agent income distribution is endogenous to prices, and we are interested in the comparative statics of different price regimes. In Section 3, we relate our results to the ones of Saez (2001) in more detail.
3 Competitive Market

In this section, we analyze optimal income taxation in competitive markets, in which the price of good X is determined by the market equilibrium condition

\[ S(p) = \int x(p, \tilde{y}(n)) f(n) dn. \]  \hspace{1cm} (5)

On the left-hand side we have market supply \( S(p) \), and on the right-hand side the market demand for good X, where \( x(p, \tilde{y}) \) is the Walrasian demand of agent with disposable income \( \tilde{y} \).

We consider an increasing supply function \( S'(p) > 0 \) and zero fixed costs so that the total firm profits coincide with producer surplus: \( \Pi(p) = \int_0^p S(\tilde{p}) d\tilde{p} \). The Walrasian demand function can be determined using Roy’s identity as \( x(p, \tilde{y}) = -v_p(p, \tilde{y})/v_y(p, \tilde{y}) \). We assume that good X is normal \( x_y > 0 \), and that the demand for good X satisfies the law of demand \( x_p < 0 \). In Appendix A.2, we show how our model can be supported with a labor market. We also explain there why condition (5) clears both product and labor markets if the government expenditures are spent solely on the numeraire good.

Before we study the public authority’s maximization problem, let us simplify incentive compatibility constraints (4). Denote an agent’s utility from revealing his/her productivity type truthfully as

\[ u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n). \]

If the truthful revelation is optimal, then

\[ u(n) = \max_m (v(p, y(m) + \xi(n)\Pi(p)) - c(z(m)/n)). \]  \hspace{1cm} (6)

Using the envelope theorem, we obtain the following first-order condition:

\[ u'(n) = c_\xi z(n)/n^2 + \xi'(n)a(p)\Pi(p). \]  \hspace{1cm} (7)

Note that the presence of firm profits does not change the incentive compatibility condition when agent indirect utility is linear because firm profit shares can be taken out of the maximization problem (6). Hence, the second-order condition that ensures truth-telling to be optimal is that income schedule \( y(n) \) is non-decreasing as in the standard case (see Mirrlees, 1976). At the same time, profit distribution \( \xi(n) \) does influence the level of agent utility \( u(n) \). This will be an important factor since the public authority has equity concerns (\( W \) is strictly concave).

The public authority problem is then to find the combination of price \( p \), income schedule
we express the Lagrangian of the maximization problem as

\[
\max_{p, \tilde{y}(n), \ell(n)} \int W(v(p, \tilde{y}(n)) - c(\ell(n))) f(n) dn \quad \text{subject to (3), (5), and (7)}.
\]

It is convenient to change variables \( \{p, \tilde{y}(n), \ell(n)\} \) to \( \{p, u(n), \ell(n)\} \), where utility level is defined by \( u(n) = a(p) \tilde{y}(n) - c(\ell(n)) \). From the latter expression, we can invert the disposable income \( \tilde{y} \) and express it as \( \tilde{y} = r(p, u, \ell) \equiv \frac{x + c(\ell)}{a(p)} \). The maximization problem can then be written as

\[
\max_{p, u(n), \ell(n)} \int W(u(n)) f(n) dn
\]

s.t.

\[
\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)] f(n) dn \geq R
\]

\[
S(p) - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0
\]

\[
u'(n) - \xi'(n)a(p)\Pi(p) - c\ell(n)/n = 0.
\]

Let \( \lambda, \gamma, \) and \( \mu(n) \) be multipliers corresponding to constraints (9), (10), and (11), respectively. After integration by parts and taking into account the transversality condition \( \mu(n) = \mu(\bar{n}) = 0 \), we express the Lagrangian of the maximization problem as

\[
\int \{[W(u(n)) + \lambda(n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p) - R) +
\gamma(S(p) - x(p, r(p, u(n), \ell(n))))] f(n) - \mu'(n)u(n) - \mu(n)(\xi'(n)a(p)\Pi(p) + c\ell(n)/n)\} dn.
\]

Now that \( r_u = 1/v_y, r_\ell = c_\ell/v_y, \) and \( r_p = -v_p/v_y = x \). Taking into account \( \Pi'(p) = S(p) = \int x(p, \tilde{y}) f(n) dn \) and \( \int \xi(n) f(n) dn = 1 \), the first-order conditions can be written as

\[
u(n) : \left[ W_u - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0
\]

\[
\ell(n) : \left[ \lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y} \right] f(n) - \mu(n)(c_\ell + c_\ell\ell(n))/n = 0
\]

\[
p : \int \{[\gamma(S'(p) - x_p - x_\ell x)] f(n) - \mu(n)\xi'(n)(a_p\Pi(p) + a\Pi'(p))\} dn = 0.
\]

The optimal marginal income tax \( t(z) = T'(z) \) is then determined by agent utility maximization problem \( \max_z [U(p, z - T(z) + \xi(n)\Pi(p), z/n)] \) that results in \( t(z) = 1 - c_\ell/(nv_y) \). The expression for \( t(z) \) then follows from (12) and (13) using that \( 1 + \ell c_\ell/c_\ell = (1 + \zeta)/\zeta \), where \( \zeta \) is the elasticity of the (un-)compensated labor supply.\(^{13}\)

\(^{12}\)We maximize over \( p \) because the price is implicitly determined by equation (5). If we had an explicit price function, we could introduce it into indirect utility and maximize over \( (\tilde{y}(n), \ell(n)) \) alone.

\(^{13}\)Note that the compensated elasticity of labor supply coincides with the uncompensated one in the absence of income effects (linear indirect utility).
Theorem 1. In competitive markets, the optimal marginal income tax is determined by
\[
\frac{t}{1-t} = A(n)B(n) + \frac{\gamma}{\lambda} x_y(1 + A(n)), \quad \text{where}
\]
(15)
\[
A(n) = 1 + \frac{\xi}{\zeta} \frac{1 - F(n)}{n f(n)},
\]
(16)
\[
B(n) = \frac{1}{1 - F(n)} \int_{n}^{\pi} \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m) dm,
\]
(17)
\[
\gamma = \frac{(a_p \Pi(p) + a \Pi'(p)) \int W_u(1 - \xi(m)) f(m) dm}{S'(p) - \int (x_p + x_g x) f(m) dm}.
\]
(18)

The optimal marginal income tax formula (15) contains in addition to the standard Mirrleesian term \(A(n)B(n)\) a new term \(\gamma x_y(1 + A(n))/\lambda\).\(^{14}\) The new term is associated with the presence of market equilibrium condition (10) that is responsible for the price effect on the optimal income tax schedule. To explain the mechanism underlying the price effect, let us consider how a small decrease in product price influences agent welfare. The price decrease benefits low-productivity agents because they can afford to consume more products. At the same time, the price decrease hurts high-productivity agents whose utility is mostly influenced by the resultant decrease in firm profit shares. Overall, the public authority uses the equilibrium price level on par with income taxation to achieve its redistributive objectives. The public authority, however, cannot enforce the price level directly. Instead, it uses an additional marginal income tax to decrease agent labor supply, which leads to a lower agent income. The lower agent income decreases the market demand for good \(X\) leading to a decrease of price \(p\) in equilibrium.\(^{15}\)

Let us now show that the second term in equation (15) is positive suggesting a more progressive optimal income tax schedule in the presence of endogenous prices.\(^{16}\) We first note that the marginal value of public funds is positive \(\lambda > 0\) and \(x_y > 0\) as good \(X\) is normal. To establish that \(\gamma > 0\), we note that the denominator in (18) is positive because market supply has an upward slope \(S'(p) > 0\) and the slope of the compensated demand is negative \(h_p = x_p + x_g x \leq 0\) (where \(h\) is the Hicksian (compensated) demand function). To explain the other terms in equation (18) note that agent utility equals \(a(p) \Pi(y(n)) + x(n) \Pi(p))\). Hence, \(a_p \Pi(p) + a \Pi'(p)\) corresponds to a change in aggregate agent utility due to a price change.

\(^{14}\)See Saez (2001) for the derivation of the standard Mirrleesian term. The above formula is derived using the standard Lagrange optimization method used by Mirrlees (1971). It could have been alternatively obtained using the perturbation method developed by Saez (2001).

\(^{15}\)A decrease in labor supply can also potentially influence the supply curve for good \(X\). This channel is absent in our setting because any change in labor supply is fully absorbed by the sector producing the numeraire good (see Appendix A.2 for details).

\(^{16}\)The positive second term provides only suggestive evidence because marginal welfare weights \(W_u\) and the marginal value of public funds \(\lambda\) are endogenous to the optimum. We will use the calibration exercise in Section 4 to confirm this suggestion using simulations.
associated with the presence of dividends. To show that this term is positive, note that

$$ \Pi'(p) = S(p) = \int x(p, \tilde{y}) f(n) dn = -\frac{a_p(p)}{a(p)} \int \tilde{y}(n) f(n) dn \geq -\frac{a_p(p)}{a(p)} p S(p) \geq -\frac{a_p(p)}{a(p)} \Pi(p), $$

where the first equation follows from $\Pi(p) = \int_0^p S(\tilde{p}) d\tilde{p}$, the second from the market equilibrium condition, and the third from Roy’s identity; the first inequality follows from an agent’s individual budget constraint, and the second one from $p S(p) \geq \int_0^p S(\tilde{p}) d\tilde{p}$. Finally, term $\int W_u(u(\tilde{m}))(1 - \xi(\tilde{m})) f(\tilde{m}) dm$ is positive because more productive agents have higher profit shares $\xi(\tilde{m})$ (as follows from $\xi'(\tilde{m}) > 0$), but lower marginal social weights $W_u$ (as follows from $W_{uu} < 0$ and $u'(\tilde{m}) > 0$).

The presence of the price effect could also be explained by the failure of the second welfare theorem in markets with incomplete information and a limited set of tax instruments. Tax schedules that depend only on labor income could restrict the set of feasible outcomes that the public authority can achieve, as we discussed in the introduction. At the same time, the market equilibrium condition is not binding $\gamma = 0$ if firm profits are zero $\Pi(p) = 0$. In this case, for any constrained Pareto optimal profile of agent utilities and labor input $\{u(n), \ell(n)\}$, one can find an income tax schedule that supports this profile with equilibrium price $p$. The same conclusion holds if the firm profits are uniformly distributed $\xi(n) = 1$, in which case the presence of firm profits can simply be eliminated with a uniform lump sum tax. In these two cases, the second welfare theorem holds and the equilibrium price level has no influence on the optimal marginal income tax schedule.

Since the price effect on the optimal income taxation is a consequence of the failure of the second welfare theorem, we explore the limits of this result in more detail. In Appendix A.4, we show that if we assume that income tax is based on total agent income (labor income plus dividends) instead of labor income alone, the market equilibrium condition remains binding. In this case, it becomes more difficult for high-income agents to deviate because they need to mimic not only smaller labor income of low-productivity agents but also a smaller income from profit shares. This alleviates agent incentive compatibility problem and leads to a more progressive taxation in the optimum. In Appendix A.5, we also consider the optimal profit taxation in an extension of our model where firms have idiosyncratic fixed costs and can freely enter and exit the market. We show that the optimal profit tax is below 100% and, as a consequence, firms earn positive profits in the optimum. The presence of firm profits leads again to the failure of the second welfare theorem and to the price effect on optimal income taxation. Overall, Theorem 1 and its extensions (Theorems A1, A2 and A3) establish that

---

17 If agents had non-labor assets $e(n)$ instead of firm profit shares $\xi(n)\Pi(p)$, the price effect on optimal income taxation would be negative as $(a(p)e(n))' p < 0$ and $\gamma < 0$. This result is in line with Abraham et al. (2016) who obtain a less regressive labor income taxation in two-period model where the government has a limited ability to tax non-labor assets.

18 The second welfare theorem would be restored if the public authority could separately tax away all dividends.
the optimal income taxation should take into account the market structure in economies with incomplete information and progressive distribution of firm profits.

We now consider an extension of our model to allow for foreign ownership of firm profits.\textsuperscript{19} This extension is motivated by empirical evidence that a substantial share of equities is held by foreigners in many countries: the share of foreign equity holdings amounts to 13.6\% in the U.S. (U.S. Treasury, 2017) and, on average, 38\% in European countries (Davydoff et al., 2013).

If we assume that \( \int \xi(n)f(n)dn = \Xi < 1 \) we obtain the following extension of our main result.

\textbf{Theorem 2.} In competitive markets with foreign ownership \( 1 - \Xi > 0 \), the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = A(n)B(n) + \frac{\gamma}{\lambda} x_y (1 + A(n)), \quad \text{where}
\]

\[
A(n) = \frac{1 + \zeta - F(n)}{n f(n)},
\]

\[
B(n) = \frac{1}{1 - F(n)} \int_{\pi}^{\pi} \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m)dm,
\]

\[
\gamma = \frac{\lambda S(p)(1 - \Xi) + (a_p \Pi(p) + a \Pi'(p)) \int W_u(\Xi - \xi(m))f(m)dm}{S'(p) - \int (x_p + x_y x) f(m)dm}.
\]

The additional term in (22) suggests a more progressive taxation compared to the case when no profits leave the economy. To interpret, note that agent labor supply contributes to firm profits that belong in part to foreigners. Hence, competitive equilibrium results in production inefficiency. To correct for this inefficiency, the public authority imposes an additional income tax and thereby bringing the competitive equilibrium outcome closer to the Pareto efficiency. Overall, Theorem 2 establishes that in competitive markets with foreign ownership the public authority uses the equilibrium price level on par with income taxation to achieve both efficiency and equity objectives.

We now analyze the optimal tax rate for top income earners. Saez (2001) was first to derive an asymptotic tax rate formula for top income earners. He argued that the upper tail of U.S. income distribution is well described by a Pareto distribution, which also corresponds to an upper Pareto tail of agent productivity. Moreover, Pareto parameter \( a \) of income distribution corresponds to Pareto parameter \( \sigma(1 + \zeta) \) of agent productivity distribution for high produc-

\textsuperscript{19} We assume that foreigners spend their profit income on the numeraire good to keep the same market equilibrium condition. The foreigners also have a zero weight in welfare function.
tivity agents (see p. 218 in Saez (2001)). Following Saez (2001), we denote the asymptotic ratio of social marginal utility to the marginal value of public funds for the government for the top income earners as \( \bar{g} = \frac{W_u v_y}{\lambda} \). Then, term \( A(n) \) reduces to \( 1/(\zeta a) \) and term \( B(n) \) reduces to \( 1 - \bar{g} \). Overall, the top tax rate can be written as

\[
t_{\text{top}} = \frac{1 - \bar{g} + \frac{\gamma}{\lambda} x_y (1 + \zeta a)}{\zeta a + 1 - \bar{g} + \frac{\gamma}{\lambda} x_y (1 + \zeta a)}.
\]

We observe that the optimal tax rate in a competitive economy with endogenous prices coincides with the standard one if the public authority puts a smaller social weight on top income earners \( \bar{g} \rightarrow g \rightarrow \frac{\gamma}{\lambda} x_y (1 + \zeta a) \). This correction, however, is not solely determined by the characteristics of the upper tail but rather depends on the parameters of the whole population as suggested by equation (18). This reflects that product price \( p \) influences the utility level of all agents not only the top ones. In the next section, we estimate the correction term when we do our calibration exercise.

### 4 Numerical Simulations

In this subsection, we provide numerical simulations to estimate the size of the price effect on optimal income taxes. To accomplish this goal, we consider the U.S. housing market that suits particularly well to quantify our results. First of all, housing is the largest consumption item that accounts for about 25% of total household expenditure. In addition, housing is a non-competitive sector with large firm profits whose distribution is the main driver of the price effect on optimal income tax schedule.

To incorporate housing demand, we consider a model estimated by Miles and Sefton (2018) with a constant elasticity of substitution (CES) utility function:

\[
u(x_o, x) = (\epsilon x_o^{1-1/\rho} + (1 - \epsilon) x^{1-1/\rho})^{\rho/\rho - 1},
\]

where \( x \) denotes the consumption of housing, \( x_o \) denotes the consumption of the other goods, \( \epsilon \) is the weight on the other goods (relative to housing consumption) in utility, and \( \rho \) is the elasticity of substitution between housing and the other goods. Following Miles and Sefton (2018), we take \( \epsilon = 0.85 \) and \( \rho = 0.6 \). This utility specification results in the absolute

\[20\] More precisely, Pareto parameter should be \( a(1 + \zeta u) \), where \( \zeta u \) is uncompensated labor elasticity of income. For homothetic preferences, the compensated and uncompensated elasticities coincide.

\[21\] See Consumer Expenditure Survey, 2017, Table 1203. Income Before Taxes: Annual Expenditure Means, Shares, Standard Errors, and Coefficients of Variation and Bureau of Economic Analysis, 2016, Table 2.3.5U. Personal Consumption Expenditures by Major Type of Product and by Major Function. Similar numbers are also observed in the European Union, for which Eurostat (2016) reports that on average housing accounts for 24.4% of household expenditure.

\[22\] For other estimates of the CES model for housing, see Määttänen and Terviö (2014) and Li et al. (2016).
value of the price elasticity of demand of 0.7 and the unit income elasticity of demand in our simulations, which are consistent with elasticity values estimated in the literature.\textsuperscript{23}

We model the supply of housing using the standard constant price elasticity function \( S(p) = sp^\varepsilon \), where \( s \) is a scale parameter and \( \varepsilon \) is the price elasticity of supply.\textsuperscript{24} The estimates of the price elasticity of supply \( \varepsilon \) vary significantly across countries and even across cities. In particular, Saiz (2010) shows that \( \varepsilon \) highly depends on geographical and regulatory constraints within U.S. metropolitan areas. Drawing on his estimates for the average U.S. metropolitan area, we take \( \varepsilon = 1.75 \).\textsuperscript{25} We calibrate scale parameter \( s \) to match the average share of housing expenditure of 25%, which renders \( s = 18.5 \). In Appendix A.6, we present the results for inelastic supply \( \varepsilon = 0.01 \) that better describe housing supply in large U.S. coastal cities (e.g., Boston and San-Francisco) and in countries with a rigid housing planning system (e.g., the UK).\textsuperscript{26} We also present results for the price elasticity of supply \( \varepsilon = 3 \) that are closer to the estimates obtained by Epple and Romer (1991).

Following Saez et al. (2012), we consider cost function \( c(\ell) = \ell^4/4 \) with a constant labor supply elasticity of \( \zeta = 1/3 \). We take the distribution of agent productivities from Mankiw et al. (2009), who proxy it with the distribution of hourly wages in the U.S. Specifically, this distribution consists of several parts. There is a 5% mass of agents with \( n = 0 \) which matches the percentage of total employees on public disability insurance in the U.S. The productivities of agents between the 5\textsuperscript{th} and 95\textsuperscript{th} percentiles follow the lognormal distribution with mean 2.757 and standard deviation 0.5611. The remaining top 5% levels of productivity follow the Pareto distribution with the shape parameter of 2.\textsuperscript{27} We used a standard kernel smoother to merge the log-normal and Pareto parts of the distribution together. We set public expenditures at \( R = 0 \) and consider logarithmic social welfare function \( W(u) = \ln u \).

Next, we set the share of profits held by agents in the economy at \( \Xi = 0.85 \), which approximately equals the 86.4% domestic share of equity holdings in the U.S. (see U.S. Treasury, 2017). This estimate should be considered an upper bound for developed countries. In Europe, on average, only 62% of equity shares are held by domestic investors, whereas the corresponding figure is 50% in the UK, 60% in France, and 70% in Germany (Davydoff et al., 2013).

Lastly, we approximate the distribution of firm profits among agents by the empirical distribution of dividends across U.S. households in 2012, as presented in Table 1.\textsuperscript{28} To make our

\begin{itemize}
\item \textsuperscript{23}For the U.S. housing market, the estimates of price elasticity of demand are in the range 0.3 to 1 in absolute numbers and the estimates of income elasticity of demand are in the range of 0.4 to 1 (see Albouy et al., 2016). For the European housing market, the corresponding estimates are in the range of 0 to 1.43 and 0.74 to 1.23, respectively (see Arrazola et al., 2015).
\item \textsuperscript{24}This functional form corresponds to the Cobb-Douglas production function (see, e.g., Epple et al. (2010)).
\item \textsuperscript{25}For similar estimates, see also DiPasquale and Wheaton (1994) and Harter-Dreiman (2004).
\item \textsuperscript{26}See Hilbert and Schöni (2016). For a discussion of the U.K. housing crisis in the popular press, see “How to solve Britain’s Housing Crisis,” The Economist, 2017, August 5.
\item \textsuperscript{27}This shape parameter corresponds to \( \pi(1 + \zeta) \) derived by Saez (2001) from income distribution, where \( \pi = 1.5 \) is the shape parameter of the Pareto tail of the U.S. income distribution (see Diamond and Saez, 2011).
\item \textsuperscript{28}The cumulative distribution of profits is approximated well by functional form \( \Xi(n) = \exp(\hat{b}(-\log F(n))^{1/2}) \).
\end{itemize}
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<tr>
<th>Bottom</th>
<th>Top 90%</th>
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<th>31.2%</th>
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Table 1: The Distribution of Dividends in the U.S., 2012

Note: The table shows the percentage shares of dividends held by the U.S. population. Source: Saez and Zucman (2016, Table B23).

estimates more realistic, we slightly depart from our theoretical analysis and assume a flat profit tax at 15% (as in U.S. for most income levels). The proceeds from the profit tax go to financing government expenditures and they are included in the resource constraint.  

Figure 1 presents our main simulation results. The left-hand diagram shows the optimal marginal tax schedules for cases with (i) fixed price and equal profit distribution (dotted line), (ii) endogenous price and uniform profit distribution (dashed line), and (iii) endogenous price and empirical profit distribution (solid line). In case (i), we take the price of housing equal to the equilibrium price of case (iii). Case (i) corresponds to the standard setting of Mirrlees (1971) and serves as a benchmark against which we estimate the size of the price effect. The right-hand diagram shows the change in optimal marginal income tax (ii)–(i) and (iii)–(i) compared to the benchmark case.

We note that the estimated optimal income tax schedules are of a shape similar to those obtained by previous studies. The high tax rates at the bottom correspond to the phasing-out of the guaranteed income level (Mankiw et al. 2009; Saez, 2001). Due to the Pareto tail, at top income levels the optimal marginal income taxes flatten out and are equal to approximately 64% for the main case (iii) and 58% for the benchmark case (i). Similarly to Mankiw et al. (2009), the minimum of the optimal marginal income tax schedule is achieved around labor income of $60, 000.

Our numerical simulations show that the endogenous price has an upward effect on optimal marginal income tax rates, adding 4.9 percentage points on average (see the solid line in the right diagram of Figure 1). The overall increase in marginal income tax can be attributed to two factors: the presence of progressive distribution of firm profits $\xi(n) > 0$, which leads to a tax correction due to equity concerns, and the presence of foreign ownership $\Xi < 1$, which leads to a tax correction based on efficiency concerns. The factor of progressive firm profit

where $F(n)$ is the cumulative distribution of agent productivities and $\hat{b} = -18.1$; the coefficient of determination equals $R^2 = 0.95$. We obtain function $\xi(n)$ as the derivative of $\Xi(n)$ and by normalizing $\int \xi(n)f(n)dn = 0.85$. Our estimation results of the price effect do not significantly change if we consider instead the distribution of capital income (Saez and Zucman, 2016, Table B21) or the distribution of wealth and financial resources (Wolff, 2017) to approximate the distribution of profit shares.

Supplementary materials “OIT_simulation_documentation.pdf” present how the first order condition change in the presence of the profit tax.

The high optimal tax rate at the bottom disappears if one considers a labor participation (see Saez, 2002).
distribution adds 4.3 percentage points to the benchmark case compared to 0.6 percentage point added by foreign ownership. In Appendix A.6, the size of price effect is robust to other assumptions about the distribution of productivity types. We show that the total price effect remains around 5% if we remove the Pareto tail and consider only the lognormal distribution of productivities as in Mankiw et al. (2009).

We also observe that the price effect is increasing up to labor income around $300,000 and afterwards decreases. The decrease is associated with the presence of Pareto tail, which we establish by comparing our results with the simulations using lognormal distribution of agent productivities (see Appendix A.6). The asymptotic tax rate (see equation (23)) still remains rather high around 6%, which is above the average price effect for the whole population.

Finally, we note that the estimate of the price effect due to foreign ownership is rather small. This can be explained by the fact that the share of foreign ownership in U.S. is less than 15%. With a larger share of foreign ownership, like 50% in the case of the UK, the effect of foreign ownership on income tax rates would become larger. On contrary, the degree of inequality of dividends in U.S. is rather large, and we should expect the price effect due to progressive profit distribution to be smaller for UK and other European countries.
5 Oligopolistic Competition

In this section, we consider markets with various degrees of oligopolistic competition. Since our simulations show that the effect of foreign ownership on optimal income taxation is small, we assume that all firm profits remain in the economy \( \int \xi(n)f(n)dn = 1 \). We consider \( M \geq 1 \) identical firms with each firm \( i \) having a convex cost function \( K(X_i) \) of producing \( X_i \) units of good \( X \). Let us denote the inverse aggregate demand function by \( p(X) \), where \( X = \int x(p, \tilde{y}(n))f(n)dn \).\(^{31}\)

We can write firm \( i \)'s profit as \( X_i p(X) - K(X_i) \), where the market clearing condition ensures \( \sum_{i=1}^{M} X_i = X \). To model various forms of oligopolistic competition, we assume that when firm \( i \) maximizes profits it forms a belief, or a conjectural variation, about the other firms’ responses to a unit change in its output level,

\[
\frac{d(\sum_{j \neq i} X_j)}{dX_i} = \theta, \quad \text{where} \quad -1 \leq \theta \leq M - 1. \tag{25}
\]

The conjectural variation model was introduced by Bowley (1924) to capture a wide variety of oligopolistic competition models. For instance, the competitive equilibrium corresponds to \( \theta = -1 \) when firms expect the rest of the industry to absorb exactly its output expansion, the conjectural variation \( \theta = 0 \) represents the Cournot-Nash model where each firm expects the output of the other firms in its industry to remain unchanged, and the collusive behavior of firms maximizing their joint profits leads to \( \theta = M - 1 \).\(^{32}\)

The first-order condition for the profit maximization problem can then be expressed by

\[
p(X) - K'(X_i) + (1 + \theta)X_i p'(X) = 0. \tag{26}
\]

We assume that the firm maximization problem is well-behaved and has a unique maximum.\(^{33}\) For the rest of our analysis, it is convenient to consider product price being independent variable. We also limit our attention to symmetric equilibria \( X_i = X/M \). After the change of variable, the market equilibrium condition then reads as

\[
p - K'(X/M) + \frac{(1 + \theta)}{M} X_p = 0, \tag{27}
\]

where \( X_p = \int x_p(p, \tilde{y}(n))f(n)dn \). We note that the assumption of homothetic preferences implies that market demand \( X \) is a linear function of aggregate income. Therefore, the price elasticity of market demand is a function of price only \( \varepsilon_d(p) = X_p p/X \). Then the equilibrium

\[^{31}\]The assumption \( x_p < 0 \) ensures that the inverse aggregate demand function \( p(X) \) is well defined.

\[^{32}\]For an excellent overview of various conjectural variational parameters, see Perry (1982).

\[^{33}\]For an insightful discussion of problems associated with the firm objective function being non-concave, see Guesnerie and Laffont (1978).
condition can be transformed into a condition for the equilibrium markup \( \alpha(p) \)

\[
p - K'(X/M) = -\frac{(1 + \theta)}{\varepsilon_d(p)} \equiv \alpha(p)
\]  

(28)

To relate the oligopolistic case to our previous analysis, we rewrite the last equation in terms of the quantity produced by all firms in equilibrium

\[
\tilde{S}(p) \equiv M(K')^{-1} (p - \alpha(p)).
\]  

(29)

To be able to write formula (29), we assume \( p - \alpha(p) \) is an increasing function, which also implies \( \tilde{S}'(p) > 0 \). This property can be guaranteed in equilibrium, for example, if \( \varepsilon'_d(p) \leq 0 \). This assumption is trivially satisfied for CES preferences as \( \varepsilon'_d(p) = 0 \).

Lastly, we note that the total firm profit function equals \( \Pi(p) = p\tilde{S}(p) - MK(\tilde{S}(p)/M) \).

The public authority’s problem for oligopolistic market can then be written as follows:

\[
\begin{align*}
\max_{p,u(\ell(n)),\ell(n)} & \int W(u(n))f(n)dn \\
\text{s.t.} & \int [n\ell(n) - r(p, u(n), \ell(n))]f(n)dn + \Pi(p) \geq R \\
& \tilde{S}(p) - \int x(p, r(p, u(n), \ell(n)))f(n)dn = 0 \\
& u'(n) - \xi'(n)a(p)\Pi(p) - c_\ell \ell(n)/n = 0
\end{align*}
\]

where \( r(p, u, \ell) \equiv \frac{u + \ell(n)}{a(p)} \). Let us denote the slope of the compensated market demand as \( H'(p) = \int (x_p + x_y x)f(m) \). Taking into account that \( \Pi'(p) = \tilde{S}(p) + \pi(p)\tilde{S}'(p) \) and having the same notation for Lagrange multipliers as before we obtain the following result.

**Theorem 3.** In oligopolistic markets, the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = A(n)B(n) + \frac{\gamma}{\lambda} x_y(1 + A(n)), \quad \text{where}
\]

\[
A(n) = \frac{1 + \zeta}{\zeta} \frac{1 - F(n)}{nf(n)},
\]

(31)

\[
B(n) = \frac{1}{1 - F(n)} \int \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m)dm,
\]

(32)

\[
\gamma = -\lambda \alpha(p) \tilde{S}'(p) + (a_p \Pi(p) + a\Pi'(p)) \int W_u(1 - \xi(m))f(m)dm \\
\tilde{S}'(p) - H'(p).
\]

(33)

If equilibrium price satisfying (28) exists, we have \( 1 - \alpha'(p) = 1 - \varepsilon_d(p)\frac{\alpha_d(p)}{\varepsilon_d(p)} \) that is positive if \( \varepsilon_d(p) \leq 0 \).
We observe that expression (30) contains an additional term
\[-\alpha(p) \frac{\tilde{S}'(p)x_y(1 + A(n))}{S'(p) - H'(p)} \leq 0.\]
This term corresponds to the non-competitive effect of the market equilibrium on optimal income taxation and it is proportional to firm markup. To explain its presence, note that the equilibrium of an imperfectly competitive market is associated with under-production in relation to a constrained Pareto optimum. To obtain an outcome closer to the constrained Pareto optimum, the public authority wants to stimulate market demand. To accomplish this objective, the public authority increases agent income by lowering income tax rates in the optimum. This result is closely related to the previous studies in the commodity taxation literature as the non-competitive effect is similar to a subsidy in non-competitive markets (see Myles, 1989).

The non-competitive effect and the price effect work in the opposite direction. The non-competitive effect advocates for lower marginal income taxes while the price effect advocates for higher marginal income taxes. We illustrate the interplay of the two effects with numerical simulations in the following subsection.

**Numerical Simulations: Oligopolistic Competition**

We consider the numerical estimation of optimal income tax rates in oligopolistic markets using the U.S. housing market framework introduced in Section 3. We infer production cost function \( K(\cdot) \) from the specification of the competitive market in the previous section. The profit maximization problem in competitive market yields \( p - K'(\frac{X}{M_0}) = 0 \), where \( X \) is equal to the market supply \( S(p) = sp^\varepsilon \) in market equilibrium and \( M_0 \) is some fixed number of firms. Thus, for the marginal cost function, we obtain \( K'(X_i) = (\frac{X_i M_0}{s})^{\frac{1}{\varepsilon}} \) and, hence,
\[
\tilde{S}(p) = s(p - \alpha(p))^{\varepsilon}
\]
where \( s = 18.5 \) and \( \varepsilon = 1.75 \) as used in our analysis of the competitive market. Further, we set \( M_0 = 2 \). All other parameters of the model remain the same.

To illustrate both the non-competitive effect and the price effect, we consider two cases: (i) equal profit distribution (only non-competitive effect) and (ii) empirical profit distribution (both price and non-competitive effects), illustrated in Figure 2. In both cases, we vary the conjectural variation \( \theta = 0 \) (Cournot-Nash), \( \theta = -0.5 \), and \( \theta = -1 \) (perfect competition) in order to see how the non-competitive effect and the overall effect change with market structure. In the case of equal profit distribution (the left diagram of Figure 2), only non-competitive effect is present. In less competitive markets (larger \( \theta \)) the public authority lowers income tax rates
in order to offset the increase in production inefficiency. In the case of empirical distribution of profits (the right diagram of Figure 2), the public authority has in addition larger redistributive concerns in less competitive markets leading to a larger price effect and higher income tax rates. Both the non-competitive effect and the price effect increase in the size of market power due to larger production inefficiency and larger firm profits. However, these two effects work in the opposite direction and almost cancel each other. As a result, the optimal income taxation policy remains stable for various market structures—the result that has not be highlighted in the previous literature.

6 Commodity Taxation

In the previous sections, we demonstrate that the optimal income tax policy has to take into account market structure and the distribution of firm profits. The question then arises as to whether the price and non-competitive effects survive in the presence of other types of taxation. We study the question of optimal commodity taxation in this section. The question of optimal profit taxation we postpone until Appendix A.5 where we consider an extension of our main model allowing for endogenous firm exit and entry.

We use the framework from Section 5 which allows us to consider simultaneously the cases of competitive markets \((\alpha(p) = 0)\) and oligopolistic markets \((\alpha(p) = -\frac{1+\theta}{M} \frac{p}{\pi(p)} > 0)\). Assume that the public authority can impose a commodity tax \(b\). Then, the consumer price, denoted by \(q\), differs from producer price \(p\) by \(b = q - p\). Negative commodity tax \(b < 0\) is interpreted
as a subsidy. We also write \( H'(q) = \int (x_q + x_y) f(n)dn < 0 \) for the derivative of the aggregate compensated demand and \( M\xi = \int W_u(1 - \xi(m)) f(m)dm > 0 \) for the term responsible for redistributive concerns of the public authority.

The public authority maximization problem can then be written as

\[
\max_{p,q,u} \int W(u(n)) f(n)dn
\]

s.t.

\[
\begin{align*}
\int (n\ell(n) - r(q,u(n),\ell(n))) f(n)dn + \Pi(p) + (q - p)\tilde{S}(p) & \geq R \\
\tilde{S}(p) - \int x(q,\bar{y}(n)) f(n)dn & = 0. \\
u'(n) - \xi'(n)a(q)\Pi(p) - c_\ell\ell(n)/n & = 0
\end{align*}
\]

Following the same steps as in the proof of Theorems 1 - 3, we obtain the following result for the optimal income taxation in the presence of optimal commodity taxation.

**Theorem 4. (Commodity taxation)** In competitive and oligopolistic markets with the optimal commodity taxation, the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = A(n)B(n) + \frac{\gamma}{\lambda}x_y(1 + A(n)), \text{ where}
\]

\[
A(n) = 1 + \frac{\zeta F(n)}{n f(n)}, \quad B(n) = \frac{1}{1 - F(n)} \int \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m)dm,
\]

\[
\gamma = \frac{a_q(q)\Pi(p)M\xi}{-H'(q)} < 0,
\]

and the commodity tax by

\[
b = -\alpha(p) + \frac{M\xi}{\lambda} \left( \frac{a_q(q)\Pi(p)}{H'(q)} + \frac{a(q)\Pi'(p)}{S'(p)} \right) > 0.
\]

When profits are positive \( \Pi(p) > 0 \) and progressively distributed \( \xi'(n) > 0 \), commodity tax and a correction to income taxation are both present in the optimum. This result is similar in spirit to Naito (1999) who showed that the seminal result of “uniform commodity tax under nonlinear income taxation” by Atkinson and Stiglitz (1976) no longer holds if marginal costs of production are non-constant. For competitive markets (\( \alpha(p) = 0 \)), commodity tax \( b \) is positive in the optimum and consists of the two terms that correspond to consumer and producer tax burden. The commodity taxation results in a reduction of producer surplus but mainly to the disadvantage of high-productivity agents who own larger profit shares (equity concern).
The commodity taxation reduces also consumer surplus, but uses the resultant tax revenue to compensate agents by lowering income tax rates \((\gamma < 0)\). Hence, the working of the price effect is quite different in the presence of commodity taxation. The public authority uses the revenue from commodity taxation in order to reduce the tax burden on agents to bring the production level closer to the optimum that was initially distorted by commodity taxation.

For oligopolistic markets \((\alpha(p) > 0)\), the public authority in addition reduces the optimal commodity tax by the amount of firm markup \(\alpha(p)\). This reduction, as we discussed in Section 5, is aimed to correct for the market inefficiency associated with the presence of market power. The optimal commodity tax does not disappear even when firm profits are equitably distributed, \(\xi'(u) = 0\). In this case, the optimal tax is a subsidy \(b = -\alpha(p)\) that exactly brings the economy to the efficient level of production. This result is similar to the one by Myles (1996) who showed that a combination of ad valorem tax and commodity tax can eliminate the welfare loss that arises from oligopolistic competition.

7 Literature Review

The results of the present paper are connected to several strands in the literature. First, our analysis is closely related to the study of optimal income taxation in the presence of endogenous wages in labor markets. Stiglitz (1982) was one of the first to consider a setting in which workers are not perfect substitutes in production. In this case, the general equilibrium effects imply that the optimal tax policy should subsidize high-talent workers and tax low-talent workers. Extending this analysis to a setting where workers have two-dimensional skill characteristics and an occupational choice, Rothschild and Scheuer (2013) found that the ability of workers to select their occupation involves a more progressive tax schedule than does a model without occupational choice. Ales, Kurnaz, and Sleet (2015) use a related multi-task assignment model with finite one-dimensional agent types to study a change in optimal income policy in response to a technical change. In a study of the continuum of one-dimensional agent types and general constant returns to scale production functions, Sachs, Tsyvinskiy, and Werquin (2016) show that the optimal labor supply in an equilibrium is determined by a complicated integral equation. Using this equation, they analyze the incidence of a tax reform with the actual U.S. tax code as a starting point. In contrast to these papers, the price effect in our model arises because of the distribution of firm profits among agents.

The literature in which endogenous wages in labor markets also studies the impact of externalities on the optimal income tax schedule.\(^{35}\) Rothschild and Scheuer (2016) study the corrective role of income taxation in a setting where agents can engage in rent-seeking activity.

\(^{35}\)Stantcheva (2014) also studies how adverse selection in labor markets influences optimal income taxation. Stantcheva (2017) provides a joint analysis of optimal income tax and optimal human capital policies.
whereby private returns are different from social returns. Lockwood et al. (2017) integrate tax considerations into an assignment model of agents to professions in which high-paying professions have negative externalities and low-paying professions have positive externalities. They estimate that the welfare gains from an optimal income taxation policy targeted to compensate for these externalities are small. See also Rothschild and Scheuer (2014) for a unifying framework to analyze the effects of externalities on optimal income taxation.

Compared to this literature, agents do not impose direct externalities in our model. Instead, agents impose pecuniary externalities. Pecuniary externalities do not influence the set of feasible outcomes when agent types are perfectly observable - the consequence of the second welfare theorem. When agent types are not perfectly observable and tax policy can be based only on labor income (or total income) the second welfare theorem fails. In this case, a constrained Pareto-efficient allocation cannot be generally supported with competitive product prices. A similar effect of pecuniary externalities arises in the analysis of the optimal taxation of profits and labor income by Scheuer (2014).

Our analysis is also closely related to the seminal production efficiency theorem of Diamond and Mirrlees (1971), who assert that optimal income taxation entails efficient production in equilibrium. This seminal result holds, however, only in the absence of firm profits or when firm profits are fully taxed. The contribution of the present paper is to analyze optimal income taxation in the Mirrleesian setting in the presence of progressively distributed firm profits.

Only a few papers consider models in which firm profits are not fully taxed. Stiglitz and Dasgupta (1971) were among the first to show that production efficiency may not be desirable when the maximum profit tax rate is limited. Dasgupta and Stiglitz (1972) show that the government might not wish to tax all profits away in the economy with non-identical consumers when lump-sum taxes are not allowed. Guesnerie and Laffont (1978) and Iwamoto and Konishi (1991) analyze the optimal commodity tax rule in a setting with firm profits and many consumers. In contrast to these papers, we consider an incomplete information environment of optimal income taxation. We both derive analytically and estimate numerically the influence of firm profit distribution on the optimal income tax schedule. We also establish that the government does not want to tax all profits when firms have idiosyncratic costs and can freely enter and exit the market.

Further, the absence of firm profits is an important assumption in the commodity taxation literature. Under this assumption, Atkinson and Stiglitz (1976) establish that commodity taxation is unnecessary in the presence of optimal income tax given weak separability of utility between labor and all consumption goods (see also Mirrlees, 1976). Naito (1999) shows that this

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36 See also Scheuer and Werning (2016) for an insightful connection between the seminal model of Diamond and Mirrlees (1971) on commodity taxation and Mirrlees (1971) on income taxation.

37 Dasgupta and Stiglitz (1972) also establish that production efficiency is desirable if the government can set different percentage profit taxes for different producers; see also Mirrlees (1972).

38 See also Atkinson and Stiglitz (1976, 2015), Diamond and Mirrlees (1971), and also Munk (1978, 1980).
result depends on the assumption of marginal cost of production being constant. Similarly, we show that once this assumption is abandoned and firm profits are progressively distributed, the commodity taxation plays an important role on par with income taxation for both competitive and oligopolistic markets.

We also want to mention an important paper by Scheuer and Werning (2017) who consider an assignment model to study the taxation of top income earners. They establish that the conditions for the efficiency of a marginal tax rate schedule do not depend on the production technology even when firm profits are not fully taxed. However, for a given social objective function the optimal marginal tax policy does depend on the production technology and the distribution of firm profits. Scheuer and Werning (2017) also do not consider endogenous product prices, which is the main subject of our analysis.

As mentioned in the introduction, there is almost no literature relating optimal income taxation to various market structures. One exception is a recent work by Kaplow (2018) who studies the influence of market power on income taxation. One major difference between his approach and ours is that the former analyzes income taxation in an economy with exogenously given firm markups. Hence, Kaplow (2018) does not consider how income taxation policy influences equilibrium prices and firm markups, which is the main subject of our analysis. The other exception is Costa and Maestri (2018) who study optimal income taxation in non-competitive labor markets. They assume that firms’ ownership is spread uniformly across agents thus abstracting from firm profit distribution effect on optimal income taxation, the main subject of our analysis.

Our paper is also related to a small body of literature focused on analyzing commodity taxation in oligopolistic markets. Auerbach and Hines (2001) and Myles (1987, 1989) show the benefit of a corrective subsidy to offset producer markups in the presence of market power (see also Reinhorn, 2005, 2012). Similar to a corrective subsidy, a decrease in marginal income tax stimulates market demand that brings the equilibrium consumption closer to a constrained Pareto-efficiency frontier. Though our results can be seen as an extension of the previous analysis to optimal income taxation settings, we also study the distributional effect of firm profits, a consideration that is absent from previous papers. We also establish that the price effect influence optimal income tax schedule even in the presence of optimal commodity taxation.

Our paper is also closely related to an important strand of literature focused on the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Ireland (2001), Jinkins (2016), Kanbur and Tuomala (2013), Oswald (1983). These papers are typically motivated by empirical observations showing that people care not only about their

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39 See also Ales and Sleet (2016), Kleven et al. (2013), Piketty, Saez, and Stantcheva (2014), and Shourideh (2014) for analyses of optimal taxation of top labor income and capital income.

40 A similar idea according to which individuals may seek a more equal income distribution in order to improve the terms of trade is explored by Zubrickas (2012).
absolute level of consumption but also how it compares with that of others. In our paper, relativity concerns arise endogenously through equilibrium product prices.

8 Conclusion

In this paper, we considered how endogenous prices affect optimal income taxation policy in both competitive and oligopolistic markets. We focused on a Mirrleesian framework with imperfectly observable agent productivity types and firm profits distributed among agents inside and outside the economy. We established that unequal distribution of firm profits is associated with the price effect on optimal income taxation. Using the U.S. housing market, we numerically estimated that the price effect increases optimal marginal income tax by more than 5 percentage points.

The presence of market power in oligopolistic markets is associated with the non-competitive effect. Using the model of the U.S. housing market, we showed that, as the market structure varies, changes in the price effect and the non-competitive effect almost cancel each out. Hence, the optimal income taxation policy remains robust across various market structures. We also studied the price and non-competitive effects in the presence of commodity and profit taxation.

Our study of the price effect is only the first step in incorporating an underlying market structure into the analysis of optimal income taxation. Spending on housing accounts for only one fourth of U.S. household expenditure. Hence, the size of the price effect may be significantly larger than 5 percentage points when other industries are taken into account. The exact size of the effect would depend, however, on the level and the distribution of firm profits in various industries and how the price effects interact across industries. Hence, to estimate the overall price effect on optimal income taxation one would need to develop and carefully calibrate a general equilibrium model. We leave this intriguing question for future research.

Many sectors including healthcare can be better described with dynamic models (Grossman, 1972). Hence, combining the literature on dynamic optimal income taxation (e.g., Albanesi and Sleet, 2006) and the insights of the present paper to analyze the interaction between market structure and income taxation in a dynamic model will be an important exercise.

Finally, we want to highlight that endogenous prices in product markets should be an important consideration beyond income taxation policies. The welfare assessment of subsidies, welfare benefits, pensions, and the minimum wage, etc., would be biased unless the public authority takes into account the price and non-competitive effects analyzed in this paper.
A Appendix

A.1 Proofs

Proof of Theorem 1. Taking $t(z) = 1 - c_\ell/(nv_y)$ into account, condition (13) leads to

$$\frac{t}{1-t} = \left(1 + \frac{\ell c_\ell}{c_\ell}\right) \frac{v_y \mu(n)}{\lambda n f} + \frac{\gamma}{\lambda} x_y. \tag{A.1}$$

We demonstrate now that $1 + \ell c_\ell/c_\ell = (1 + \zeta)/\zeta$, where $\zeta$ is the elasticity of the uncompensated labor supply. The individual utility maximization condition also implies $v_y(p, \tilde{y}) n(1 - t) - c_\ell(t) = 0$. Let us denote a net wage rate as $w = (1 - t)n$. Implicitly differentiating the last expression with respect to net wage and taking into account that indirect utility $v$ is linear in income we obtain $\partial t/\partial w = v_y/c_\ell$. We finally arrive at $\zeta = \partial t/\partial w = c_\ell/(\ell c_\ell)$, where we used that $v_y w - c_\ell = 0$. The expression above then follows.

The formula for Lagrange multiplier $\mu(n)$ follows from equation (12) together with transversality condition $\mu(n) = \mu(\bar{\pi}) = 0$

$$\mu(n) = \int_n^{\bar{\pi}} [(\lambda + \gamma x_y)/v_y - W_u(u(m))] f(m) dm. \tag{A.2}$$

Taking into account that terms $x_y$ and $v_y$ are constant when agent indirect utility is linear, we substitute (A.2) and $1 + \ell c_\ell/c_\ell = (1 + \zeta)/\zeta$ into (A.1) to obtain the formula for the optimal marginal income tax

$$\frac{t}{1-t} = \frac{1 + \zeta}{\zeta} \frac{1 - F(n)}{n f(n)} \left(1 - \frac{W_u v_y}{\lambda} \right) \int_n^{\bar{\pi}} \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m) dm + \frac{\gamma}{\lambda} x_y. \tag{A.3}$$

To obtain the expression for Lagrange multiplier $\gamma$, we again exploit that terms $x_y$ and $v_y$ are constant. Hence, equation (12) together with the transversality condition imply that $\int W_u f(n) dn = (\lambda + \gamma x_y)/v_y$ and $\int \mu(n) \xi'(n) dn = - \int \mu'(n) \xi(n) dn = \int W_u (1 - \xi(n)) f(n) dn$. Taking the latter expression into account the expression for $\gamma$ follows from equation (14).

Proof of Theorem 2. The public authority optimization problem for competitive markets with foreign ownership is the same as without foreign ownership (8)–(11). The first order conditions with respect $u(n)$ and $\ell(n)$ also remain unchanged. The first order condition with respect to price $p$ is slightly different

$$p : \int \{[\lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_y x)] f(n) - \mu(n) \xi'(n)(a_p \Pi(p) + a \Pi'(p))\} dn = 0.$$

Taking $\Pi'(p) = S(p) = \int x(p, \tilde{y}) f(n) dn$ and $\int \xi(n) f(n) dn = 1$ into account, the statement of the theorem then follows from the same steps as in the proof of Theorem 1.
Proof of Theorem 3. We obtain the following first order conditions for the public authority maximization problem for the oligopolistic market.

\[ u(n) : \left[ W_u - \frac{\lambda + \gamma x y}{v_y} \right] f(n) - \mu'(n) = 0 \]

\[ \ell(n) : \left[ \lambda n - \frac{(\lambda + \gamma x y) c_\ell}{v_y} \right] f(n) - \frac{\mu(n)(c_\ell + c_\ell \ell(n))}{n} = 0 \]

\[ p : \lambda(-X + \Pi'(p)) + \gamma(S'(p) - H'(p)) - (a_p \Pi + a \Pi'(p)) \int \mu(n) \xi'(n) dn = 0 \]

The result then follows from the same steps as in the proof of Theorem 1 and taking market clearing condition \( X = \tilde{S}(p) \) and \( \Pi'(p) = \tilde{S}(p) + \alpha(p) \tilde{S}'(p) \) into account.

Proof of Theorem 4. Having the same notation for Lagrange multipliers as before, we obtain the following first order conditions for the public authority maximization problem.

\[ u(n) : \left[ W_u - \frac{\lambda + \gamma x y}{v_y} \right] f(n) - \mu'(n) = 0 \quad (A.4) \]

\[ \ell(n) : \left[ \lambda n - \frac{(\lambda + \gamma x y) c_\ell}{v_y} \right] f(n) - \frac{\mu(n)(c_\ell + c_\ell \ell(n))}{n} = 0 \quad (A.5) \]

\[ p : \lambda(q - p)S'(p) + \gamma S'(p) - a \Pi'(p) \int \mu(n) \xi'(n) dn = 0 \quad (A.6) \]

\[ q : \int \{ \gamma(-x_q - x y x)f(n) - a_q \Pi(p) \mu(n) \xi'(n) \} dn = 0. \quad (A.7) \]

The formula for the optimal income tax then follows from (A.4) and (A.5) following the same steps as for Theorem 1. The expressions for \( b = q - p \) and \( \gamma \) follow from (A.6) and (A.7).
A.2 Supporting Equilibrium Model

In this section, we show that our model of competitive markets can be supported with a labor market and a consumer’s utility maximization problem. In particular, we consider two competitive industries: one producing the numeraire good G and the other producing good X. We label these industries as G and X respectively. We assume that agents can earn wage $w$ for effective labor hours supplied (i.e., $n\ell(n)$) in both industries, the price for the numeraire good is normalized to $p_g = 1$, and the price for good X is equal to $p$.

Industry G has a homogeneous of degree one production technology $F_g(L_g) \equiv L_g$, where $L_g$ is the amount of labor used to produce good G. As the price of the numeraire good is normalized to 1, the profit maximization condition implies that $w = p_g = 1$, zero profits, and any level of the equilibrium labor demand $L^d_g$ in industry G.

Industry X has a production technology with decreasing returns to scale $F_x(L_x)$, where $L_x$ is the amount of labor supplied. To ensure that the firm profit maximization problem has a well-defined interior solution, we assume that $F_x$ is differentiable and strictly concave, and that it satisfies the Inada conditions, e.g., $F_x(L_x) = AL_x^a$, where $0 < A$, $0 < a < 1$ are constants.

Hence, the firm profit maximization problem

$$\max_{L_x} p \cdot F_x(L_x) - wL_x.$$ 

The solution to this maximization problem leads to the equilibrium labor demand $L^d_x(p)$ in industry X. If the production function is $F_x(L_x) = AL_x^a$, we have $L^d_x(p) = (aAp)^{1/a}$ (taking into account $w = 1$). The equilibrium market supply of good X then equals $S_x(p) = F(L^d_x(p))$ and firm profits equal $\Pi(p) = \int_0^p S_x(\tilde{p})d\tilde{p}$. We assume that firm profits are distributed among the agents in the economy according to some distribution function $\xi(n)$ with $\int \xi(n)f(n)dn = 1$.\footnote{For the economy with foreign ownership, we assume that only share $\Xi$ of profits is distributed among the agents in the economy. The remaining share $1 - \Xi$ belongs to “foreigners” who spend it on the consumption of numeraire good G.}

On the demand side of the economy, we assume that an agent’s preferences can be summarized by utility function $u(x, g) - c(\ell)$, where $(x, g)$ is the amount of good X and the numeraire good G consumed by the agent. Utility $u$ is a continuous function representing locally non-satiated preferences. We also assume that the government spends all its resources $R$ on numeraire good G.

Consider an agent with productivity $n$ who works $\ell$ hours. With equilibrium wage $w = p_g = 1$ and tax schedule $T(n\ell)$ taken into account, her income equals $n\ell - T(n\ell)$. Hence, the
agent’s maximization problem is

\[
\max_{x, g, \ell} u(x, g) - c(\ell)
\]

\[
s.t. \ p \cdot x + g \leq n\ell - T(n\ell) + \xi(n)\Pi(p)
\]

The solution to the above problem is labor supply \(\ell^*(n, p)\) and consumption bundle \((x^*(n, p), g^*(n, p))\).

Overall, aggregate labor supply and consumer demand equal

\[
L_s(p) = \int n\ell^*(n, p)f(n)dn, \quad X(p) = \int x^*(n, p)f(n)dn, \quad G(p) = \int g^*(n, p)f(n)dn.
\]

The economy must satisfy three market clearing conditions:

\[
S_x(p) = X(p) \quad (A.9)
\]
\[
S_g(p) = G(p) + R \quad (A.10)
\]
\[
L^s(p) = L^d_x(p) + L^d_g(p), \quad (A.11)
\]

where market clearing condition (A.10) requires market supply for good G to equal the sum of the market demand for G and the extent of government spending.\(^{42}\)

Let us show that condition (A.9) is the only one that we should consider in the optimal income taxation problem. As the constant return to scale technology for the numeraire good ensures that any level \(L^d_g(p)\) satisfies the firm maximization problem (when \(w = p_g = 1\)), we are free to choose \(L^d_g(p) = L^s(p) - L^d_x(p)\) to clear the labor market. Taking into account that \(S_g(p) = L^d_g(p)\) and \(\Pi(p) = pS_x(p) - L^d_x(p)\), we can rewrite condition (A.10) equivalently as

\[
L^d_g(p) + L^d_x(p) = G(p) + pS_x(p) + R
\]

Given conditions (A.9) and (A.11) this is equivalent to

\[
\int n\ell^*(n, p)f(n)dn = G(p) + pX(p) + R.
\]

This condition follows from the budget constraint of the agent’s maximization problem (A.8) when the government spending constraint is binding \(\int T(n\ell^*(n, p))f(n)dn = R\) (as we assume).

Overall, the only independent market clearing condition that we should take into account in the optimization problem is (A.9) – the market clearing condition for good X.

\(^{42}\)For the economy with foreign ownership, the market clearing condition for good G is \(S_g(p) = G(p) + (1 - \Xi)\Pi + R\). The following derivations should be modified accordingly.
A.3 Competitive Markets: Non-Linear Indirect Utility

In this section, we consider the general case of non-homothetic preferences. Let us again denote an agent’s utility from revealing his productivity type truthfully as

\[ u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n). \]

If revealing the agent’s type truthfully is optimal then

\[ u(n) = \max_m U(p, y(m) + \xi(n)\Pi(p), z(m), n). \]  \hspace{1cm} (A.12)

The envelope theorem implies the following first-order condition

\[ u'(n) = U_{n} + U_{y} \xi'(n)\Pi(p). \]  \hspace{1cm} (A.13)

We note that the single-crossing condition does not generally hold when agent indirect utility is non-linear. Hence, we need to derive the second-order condition when truth-telling is optimal.

**Proposition A1.** Condition (A.13) ensures that truth-telling is an optimal solution of (A.12) if and only if for each productivity \( n \) schedule \( \{y(n), z(n)\} \) satisfies

\[ [(c_{l}z(n)/n + c_{l})/n\frac{v_{y}}{c_{l}} + v_{yy}\xi'(n)\Pi(p)]y'(n) \geq 0. \]  \hspace{1cm} (A.14)

**Proof.** Let us assume that \( y(n) \) and \( z(n) \) are differentiable. The second-order condition for maximization (A.12) is then

\[ u''(n) - U_{nn} - (U_{y}\xi'(n)\Pi(p) + 2U_{yn})\xi'(n)\Pi(p) - U_{y}\xi''(n)\Pi(p) \geq 0. \]  \hspace{1cm} (A.15)

Taking the derivative of (A.13) with respect to \( n \) we obtain

\[ u''(n) = U_{nn} + U_{ny}\xi'(n)\Pi(p) + U_{nz}z'(n) + (U_{yn} + U_{yy}\xi'(n)\Pi(p)) + U_{yz}z'(n)\xi'(n)\Pi(p) + U_{y}\xi''(n)\Pi(p). \]

Hence, condition (A.15) is equivalent to

\[ U_{ny}\xi'(n) + U_{nz}z'(n) + (U_{yy}\xi'(n) + U_{yz}z'(n))\xi'(n)\Pi(p) \geq 0. \]

Given our separable utransility specification, this reduces to

\[ (c_{l}z(n)/n + c_{l})/n^{2}z'(n) + v_{yy}\xi'(n)\Pi(p) \geq 0. \]
Maximization condition (A.12) implies $v_y y'(n) - c_\ell z'(n)/n = 0$, which allows rewriting the previous inequality in the form of (A.14).

Note that the first term in (A.14) is always positive because the cost function $c(\ell)$ is increasing and convex. Hence, the second-order condition reduces to income schedule $y(n)$ being non-decreasing if either profits are zero $\Pi(p) = 0$ or agent profits are equally distributed $\xi'(n) = 0$. When both $\Pi(p) > 0$ and $\xi'(n) > 0$ the second term in (A.14) is negative because the indirect utility function is concave.

Assuming the second-order condition (A.14) is satisfied, the public authority maximization problem can be written as

$$\max_{p, u(n), \ell(n)} \int W(u(n)) f(n) dn$$

s.t.

$$\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)] f(n) dn \geq R$$

$$S(p) - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0$$

$$u'(n) - v_y(p, r(p, u(n), \ell(n)))\xi'(n)\Pi(p) - c_\ell\ell(n)/n = 0$$

Having the same notation for Lagrange multipliers as before, the first-order conditions equal

$$u(n) : \left[ W_u - \frac{\lambda + \gamma_x y}{v_y} \right] f(n) - \mu'(n) - \mu(n) \frac{v_y y'}{v_y} \xi'(n)\Pi(p) = 0 \quad (A.16)$$

$$\ell(n) : \left[ \lambda n - \frac{(\lambda + \gamma_x y)\ell}{v_y} \right] f(n) - \mu(n) \frac{v_y y'}{v_y} \xi'(n)\Pi(p) - \mu(n)(c_\ell + c_\ell\ell(n))/n = 0 \quad (A.17)$$

$$p : \int \{ \gamma(S'(p) - x_p - x_y y) f(n) - \mu(n)((v_{yp} + v_{yy} y)\xi'(n)\Pi(p) + v_y \xi'(n)\Pi'(p)) \} dn = 0. \quad (A.18)$$

To calculate the optimal marginal income tax, we consider the individual utility maximization problem $\max_{\ell} (v(p, n\ell - T(n\ell) + \xi(n)\Pi(p)) - c(\ell))$ as before. The first-order condition with respect to $\ell$ yields $t(z) = 1 - c_\ell/(v_y n)$. Using equation (A.17) we can then write

$$\frac{t}{1 - t} = \frac{\mu(n)v_y}{\lambda n f} \frac{1 + \zeta^u}{\zeta^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu(n)v_{yy} \xi'(n)\Pi(p)}{\lambda f}, \quad (A.19)$$

where $\zeta^u$ is the elasticity of the uncompensated labor supply, $\zeta^c$ is the elasticity of the compensated labor supply, and we exploited that $1 + \ell c_{\ell\ell}/c_\ell = (1 + \zeta^u)/\zeta^c$.

Lagrange multiplier $\mu(n)$ is determined by first-order linear differentiation equation (A.16)

$$\mu'(n) + C(n)\mu(n) = D(n).$$
where \( C(n) = \frac{v^{(n)}_y}{v^{(n)}_y} \zeta(n) \Pi(p) \) and \( D(n) = \left( W_u^{(n)} - \frac{\lambda + \gamma x_y^{(n)}}{v^{(m)}_y} \right) f(n) \) and superscript \( (n) \) means that functions are evaluated at productivity \( n \). Taking into account the transversality condition \( \mu(n) = \mu(\bar{n}) = 0 \) its solution equals

\[
\mu(n) = - \int_n^\pi E_{mn} D(m) dm = \int_n^\pi E_{mn} \left( \frac{\lambda + \gamma x_y^{(m)}}{v^{(m)}_y} - W_u^{(m)} \right) f(m) dm
\]

where \( E_{mn} = \exp(\int_n^m C(m') dm') = \exp(\Pi(p) \int_n^m v^{(m')}_y \zeta(m') dm') \).

Lagrange multiplier \( \gamma \) is then determined by equation (A.18). Let us denote \( G(n, p) = (v^{(n)}_{yy} + v^{(n)}_{yy} x^{(n)}) \Pi(p) + v^{(n)}_y \Pi'(p) \) and \( H'(p) = \int (x^{(n)}_p + x^{(n)}_y x^{(n)}) f(n) dn \). We then have

\[
\gamma(S'(p) - H'(p)) = \int_n^\pi G(n, p) \xi'(n) \left( \int_n^\pi E_{mn} \left( \frac{\lambda + \gamma x_y^{(m)}}{v^{(m)}_y} - W_u^{(m)} \right) f(m) dm \right) dm
\]

Therefore,

\[
\gamma = \frac{\int_n^\pi G(n, p) \xi'(n) \int_n^\pi \lambda E_{mn} / v^{(m)}_y (1 - \frac{W_u^{(m)} v^{(m)}_y}{\lambda}) f(m) dm dn}{S'(p) - H'(p) - \int_n^\pi G(n, p) \xi'(n) \int_n^\pi E_{mn} x^{(m)}_y / v^{(m)}_y f(m) dm dn}
\]

Substituting the above expressions in (A.19), we obtain the following result.

**Theorem A1.** In competitive markets with endogenous prices, the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = A(n) B(n) + \frac{\gamma}{\lambda} \left( x^{(n)}_y + A(n) v^{(n)}_y \int_n^\pi x^{(m)}_y E_{mn} / v^{(m)}_y f(m) dm \right) \frac{1 - F(n)}{n f(n)}
\]

\[
A(n) = \left( \frac{1 + \zeta^{(n)}_y}{\zeta^{(n)}_y} + n \frac{v^{(n)}_{yy}}{v^{(n)}_y} \xi'(n) \Pi(p) \right) \frac{1 - F(n)}{n f(n)},
\]

\[
B(n) = \frac{v^{(n)}_y}{1 - F(n)} \int_n^\pi E_{mn} / v^{(m)}_y \left( 1 - \frac{W_u^{(m)} v^{(m)}_y}{\lambda} \right) f(m) dm,
\]

\[
\gamma = \frac{\int_n^\pi \xi'(n) G(n, p) \int_n^\pi \lambda E_{mn} / v^{(m)}_y (1 - \frac{W_u^{(m)} v^{(m)}_y}{\lambda}) f(m) dm dn}{S'(p) - H'(p) - \int_n^\pi \xi'(n) G(n, p) \int_n^\pi E_{mn} x^{(m)}_y / v^{(m)}_y f(m) dm dn}
\]

Note that the optimal marginal tax formula reduces to the one of Theorem 1 when indirect utility is linear in income, i.e., \( v(p, y) = a(p)y \). In this case, \( x_y \) and \( v_y \) does not depend on productivity, \( E_{mn} = 1 \), and \( G(n, p) = a_p(p) \Pi(p) + a(p) \Pi'(p) \).
A.4 Competitive Markets: Taxing Total Income

In this subsection, we consider the case when public authority can tax total agent income. We denote total income as
\[ z(n) = n\ell(n) + \xi(n)\Pi(p). \]
The agent’s disposable income then equals
\[ y(n) = z(n) - T(z(n)). \]
Given that agents have different profit shares, it is now harder for agents to pretend to have different productivity levels compared to the case when taxes are based on labor income only. Now, if an agent of type \( n \) wants to pretend to be of type \( m \), he must work more (or less) hours \( \ell = (m\ell(m) + (\xi(m) - \xi(n))\Pi(p))/n \). Hence, the agent’s utility from reporting type \( m \) is equal to
\[ U(p, y(m), z(m), n) = v(p, y(m)) - c((z(m) - \xi(n)\Pi(p))/n). \]
We denote the agent’s utility from revealing his productivity type truthfully as
\[ u(n) \equiv U(p, y(n), z(n), n) = v(p, y(n)) - c((z(n) - \xi(n)\Pi(p))/n). \]
If revealing the agent’s type truthfully is optimal, then
\[ u(n) - U(p, y(n), z(n), n) = 0 \leq u(m) - U(p, y(n), z(n), m), \tag{A.20} \]
which leads to the following first-order condition
\[ u'(n) = U'_n = c\ell(n) + \xi'(n)\Pi(p)/n. \tag{A.21} \]
The latter condition coincides with the standard one when profits are zero \( \Pi(p) = 0 \) or \( \xi'(n) = 0 \). We also notice that the standard single-crossing condition is satisfied when agent total income is taxed. Hence, the second-order condition for truth-telling coincides with the one in the standard case, i.e., \( z(n) \) must be increasing. In this case,
\[ \frac{\partial}{\partial n}\frac{\partial}{\partial m}c((z(m) - \xi(n)\Pi(p))/n) < 0 \]
Finally, the first-order condition of maximizing (A.20) implies \( v_y y'(n) - c\ell z'(n)/n = 0 \). Hence, agent income \( z(n) \) is increasing if and only if disposable agent income \( y(n) \) is increasing. On the assumption that agent disposable income is increasing, the maximization problem of the public authority can be written as follows
\[
\max_{p,u(n),\ell(n)} \int W(u(n)) f(n) dn
\]
\[
\begin{cases}
\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)] f(n) dn \geq R \\
S(p) - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0 \\
u'(n) - c\ell(\ell(n) + \xi'(n)\Pi(p))/n = 0,
\end{cases}
\]

where disposable income is determined by \( y(n) = r(p, u, \ell) \) from \( u(n) = U(p, y(n), \ell(n)) \). Having the same notation for Lagrange multipliers as before, the first-order conditions equal

\[
\begin{align*}
u(n) & : \left[ W_u - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0 \\
\ell(n) & : \left[ \lambda n - \frac{(\lambda + \gamma x_y)c\ell}{v_y} \right] f(n) - \mu(n)(c\ell + c\ell\ell(n) + \xi'(n)\Pi(p))/n = 0 \\
p & : \int \{\gamma(S'(p) - x_p - x_yx) f(n) - \mu(n)\xi'(n)c\ell(\ell(n))\Pi(p)\} dn = 0.
\end{align*}
\]

From the individual utility maximization problem, optimal marginal income is determined by \( t(z) = 1 - c\ell/(v_y n) \). Using the first-order conditions, \( \Pi'(p) = S(p) = \int x(n) f(n) dn, \int \xi(n) f(n) dn = 1, \) and \( 1 + \ell c\ell/c\ell = 1 + z \), we obtain the following result, where and superscript \( (m) \) means that functions are evaluated at productivity \( m \).

**Theorem A2.** In competitive markets with endogenous prices when agent total income is taxed, the optimal marginal income tax is determined by

\[
\frac{t}{1-t} = A(n)B(n) + \frac{\gamma}{\lambda} x_y(1 + A(n)), \text{ where}
\]

\[
A(n) = \left( \frac{1 + \zeta + \xi'(n)\Pi(p)/\ell(n)}{\zeta} \right) \frac{1 - F(n)}{nf(n)}.
\]

\[
B(n) = \frac{1}{1 - F(n)} \int_n^\pi \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m) dm,
\]

\[
\gamma = \frac{\Pi'(p) \int_n^\pi \left[ (1 - F(n)) \int_m^\pi W_u^{(m)} f(m) dm - \int_m^{\pi} W_u^{(m)} f(m) dm \right] \xi'(n)c\ell(n)/n] dn}{S'(p) - H'(p)}
\]

Compared to Theorem 1, term \( A(n) \) is now influenced by the distribution of firm profits and the equilibrium labor supply. For \( \xi'(n) > 0 \) and \( \Pi(p) > 0 \), Lagrange multiplier \( \gamma \) is still positive. Intuitively, when tax is based on total income, it becomes more difficult for high income earners to deviate as they also realize additional profits. Hence, total income tax alleviates the incentive compatibility problem that also leads to a more progressive taxation.
A.5 Firm Profit Taxation

To analyze firm profit taxation, we consider a competitive model with firms having a possibility to enter and exit the market. There is a unit continuum of firms $[0, 1]$ indexed by $i$. Each firm has the same marginal costs that result in the same individual firm supply curve $S(p)$. Therefore, if firm $i$ enters the market, it supplies $S(p)$ amount of good $X$ receiving accounting profit equal to $\pi(p) = \int_0^p S(\tilde{p}) d\tilde{p}$.

In addition, each firm has firm-specific fixed costs $k_i$ that are distributed according to some function $G$. We think about fixed costs as firm opportunity costs. These costs can be interpreted as earning possibilities in another country or manager’s earning potential in the labor market. These costs can also be frictions and risks associated with starting a new firm that cannot be captured by accounting costs. Overall, fixed costs enter firm economic profit and influence their entry and exit decisions, but they can not accounted for in profit taxation.

With idiosyncratic fixed costs $k_i$ taken into account and if the public authority sets profit tax schedule $T_{\pi}(\pi)$, firm $i$ enters the market if and only if $\pi(p) - T_{\pi}(\pi(p)) - k_i \geq 0$.

Let us denote the fixed costs of the marginal firm that enters the market as $k^* = \pi(p) - T_{\pi}(\pi(p))$. The market supply curve then given by $G(k^*)S(p)$ and the total firm profits after taxes equals $\Pi(k^*) = G(k^*)(\pi(p) - T_{\pi}(\pi(p))) = G(k^*)k^*$. Assuming that the total firm shares sum up to one $\int \xi(n)f(n)dn = 1$, the sum of labor income and profit taxes equal

$$\int [n\ell(n)\; - \; \hat{g}(\ell(n)) + \xi(n)(\pi(p) - k^*)]f(n)dn =$$

$$= \int [n\ell(n)\; - \; \hat{g}(\ell(n)) + G(k^*)\pi(p)]f(n)dn$$

The public authority needs to solve the following maximization problem.

$$\max_{p, k^*, a(n), \ell(n)} \int W(u(n)) f(n)dn$$

s.t.

$$\int [n\ell(n) - r(p, u(n), \ell(n)) + G(k^*)\pi(p)]f(n)dn \geq R$$

$$G(k^*)S(p) - \int x(p, r(p, u(n), \ell(n)))f(n)dn = 0$$

$$u'(n) - \xi'(n)a(p)\Pi(k^*) - c\ell(n)/n = 0$$

43The supply curve is determined by the short-run firm-profit maximization condition with price equal marginal costs in the optimum.

44For convenience, we normalize the accounting fixed costs to zero.
The above maximization problem is similar to the one of Theorem 1. The main difference is that now the public authority can control both the market supply through profit taxation or $k^\ast$ and the market demand through agent income tax $T(z)$. If the public authority puts 100%-profit tax, no firm enters the market. This profit tax scheme cannot be optimal. Having the same notation for Lagrange multipliers as before, the first-order conditions equal

\begin{align*}
\ell(n) & : \left[ \lambda n - \frac{(\lambda + \gamma x_y) c_{\ell}}{v_y} \right] f(n) - \mu(n)(c_{\ell} + c_{\ell}\ell(n))/n = 0 \\
p & : \int \gamma(G(k^\ast)S'(p) - x_p - x_yx)f(n)dn - \int \mu(n)\xi'(n)a_p\Pi(k^\ast)dn = 0 \\
k^\ast & : \lambda G'(k^\ast)\pi(p) + \gamma G'(k^\ast)S(p) - \int \xi'(n)a_p\Pi'(k^\ast)dn = 0
\end{align*}

Using these first order conditions we obtain the following result.

**Theorem A3.** *(Profit taxation in competitive markets)* In competitive markets with the optimal profit taxation, the optimal marginal income tax is determined by

\begin{align*}
\frac{t}{1-t} &= A(n)B(n) + \frac{\gamma x_y}{\lambda}(1 + A(n)), \text{ where} \\
A(n) &= \frac{1 + \zeta 1 - F(n)}{nf(n)}, \\
B(n) &= \frac{1}{1 - F(n)} \int_n^m \left[ 1 - \frac{W_u v_y}{\lambda} \right] f(m)dm, \\
\gamma &= \frac{a_p(p)\Pi(k^\ast)\int W_u(1 - \xi(m))f(m)dm}{G(k^\ast)S'(p) - H'(p)}.
\end{align*}

Note that the expression for $\gamma$ is similar to the one for competitive economy with optimal commodity taxation (38). The only difference is the consumer price $q$ in Section 6 is determined by a shift in the market demand only. Hence, we had $-H'(q)$ in the denominator of (38). The change in price in profit taxation case is determined by both the price elasticity of supply and the compensated price elasticity of demand. Hence, we have $G(k^\ast)S'(p) - H'(p)$ in the denominator of $\gamma$.

We also note that $a_p(p) < 0$, $G(k^\ast)S'(p) - H'(q) > 0$, and $\int W_u(1 - \xi(m))f(m)dm > 0$. Therefore, multiplier $\gamma$ is negative in the case of profit taxation. The public authority can now control the level of firm profits directly through profit tax. The profit tax revenue now relaxes the budget constraint. Hence, the public authority can now decrease optimal marginal income tax to stimulate demand in equilibrium.
A.6 Numerical Simulations: Further Results

In this appendix, we provide additional numerical estimations of the size of the price effect on optimal income tax rates. Within the same framework of U.S. housing market, we study the robustness of the price effect to different forms of housing supply and to income distribution.

In Section 4 we consider the competitive market with supply function $S = sp^\varepsilon$ and price elasticity $\varepsilon = 1.75$ which corresponds to the price elasticity of the average U.S. metropolitan area (Saiz, 2010). However, as also noted earlier, the price elasticity of housing supply widely differs across various countries and regions and, therefore, we reestimate the size of the price effect for the cases of (i) inelastic supply $\varepsilon = 0.01$ and (ii) elastic supply with $\varepsilon = 3$. The first case better describes housing supply in large U.S. coastal cities (e.g., Boston, San-Francisco) and in countries with a rigid housing planning system, e.g., the UK (see Hilbert and Schoni (2016) and Saiz and Salazar (2018)). In the second case we draw on the estimates of the price elasticity of U.S. housing supply obtained by Epple and Romer (1991).

\[
\begin{array}{c|c|c|c}
\varepsilon & \Delta t \\
\hline
0.01 & 18.9\% & 4.9\% & 3.3\% \\
\end{array}
\]

Table 2: The average change in optimal marginal income tax (in percentage points) between endogenous and fixed price regimes for various elasticities of housing supply.

Table 2 reports the average changes in the optimal income tax rate against the benchmark case of fixed prices and equal profit distribution.\(^{45}\) For the case of inelastic supply $\varepsilon = 0.01$, we immediately note a massive increase in the size of the price effect compared to the case of the elastic supply, $\varepsilon = 1.75$, considered in Section 4. Intuitively, with fixed supply any change in aggregate demand is solely translated into price change, which calls for stronger price corrective measures on the part of the public authority. In contrast, in the case of an elastic supply of housing, $\varepsilon = 3$, we see a reduction in the size of the price effect compared to the case of price elasticity $\varepsilon = 1.75$ as changes in demand lead to smaller changes in price.

Lastly, we also reestimate the price effect only for the lognormal distribution of agent productivities with the parameter values of mean $m = 2.757$ and standard deviation $\sigma = 0.5611$. We adjust parameter $s = 15$ of supply function $S(p) = sp^\varepsilon$, $\varepsilon = 1.75$, in order to match the average housing expenditure share of 25%. Figure 3 presents our findings. Compared to the lognormal-Pareto case, we see that the removal of the Pareto tail leads to a decline of marginal income taxes at high income levels. At the same time, the change in marginal tax rates between endogenous and fixed price economies becomes larger for top income levels. Altogether, for this distribution specification the average change in marginal income tax rate is equal to 4.8%.

\(^{45}\)In our simulations, we calibrate parameter $s$ of supply function $S(p) = sp^\varepsilon$ in order to match the average expenditure share of housing of 25%. In particular, we have $s = 15.5$ for $\varepsilon = 0.01$, $s = 18.5$ for $\varepsilon = 1.75$, and $s = 21.50$ for $\varepsilon = 3$. 

37
Figure 3. Optimal income taxation with lognormal distribution of productivities.

Note: The left-hand diagram depicts the optimal marginal income tax rates for the economy with competitive housing market and (i) fixed price, equal profit distribution (dotted line); (ii) endogenous price, uniform profit distribution (dashed line); and (iii) endogenous price, empirical profit distribution (solid line). The right-hand diagram depicts the changes in tax rates for cases (ii) and (iii) in comparison with benchmark case (i). In case (i), we take the price of housing equal to the equilibrium price of case (iii).
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