No-Go Theorems for Time Machines in Classical General Relativity*

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Abstract

A time machine is a device that creates closed timelike curves. If this is taken to imply that every (suitable) maximal extension of a future domain of dependence contains such curves (i.e. if the so-called 'potency condition' [5] holds), then a recent theorem due to Krasnikov [11] amounts to the prohibition of a very general class of such devices. However, this theorem leaves open the possibility of an *incremental time machine*, a device that increases the probability of the emergence of closed timelike curves.

1 Introduction

Philosophers have long been disturbed by the prospect of travelling back in time and have fiercely debated the consequences such travel might have. Consequently, they have attempted to resolve the paradoxes arising from time travel (TT), in particular the infamous grandfather paradox has caught

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their attention. The grandfather paradox states that a time traveller, let us call him Tim, undertakes the assassination of his grandfather before Grandfather gets the opportunity to beget his father, thereby precluding his own (Tim's) birth, and consequently preventing himself from travelling back in time to execute his mean intention.[1] Some philosophers have concluded from the paradoxes engendered by journeys back in time that TT is not possible on logical or conceptual grounds.[2]

These conclusions, however, are no longer warranted once we turn to classical general relativity (CGR). CGR predicts the possibility of spacetimes containing closed timelike curves (CTCs).[3] Timelike curves represent permitted paths for material particles, such as the world line of our time traveller. CTCs are continuous, smooth, timelike, everywhere future-directed curves that intersect themselves, thus forming a causal loop. The potential existence or emergence of CTCs in relativistic spacetimes should provide sufficient ground for taking the conceptual difficulties involved seriously, at least to the extent we take CGR seriously. The relevance of a conceptual analysis of CGR for the philosophy of space and time sufficiently motivates an enquiry into the strange realm of CTCs.

The purpose of this paper is to explore recent results pertaining to the possibility of time machines (TMs). A TM is a device that creates CTCs in a region to the future of its operation, where otherwise none would have existed. Thus, spacetimes that allow for TMs stand in contrast to spacetimes with 'naturally' occurring CTCs, i.e. physically possible worlds where the

[1] For an extended and now classical recounting of the paradox, see [12], 75ff.

[2] See for instance [13], 132-135.

[3] Examples of spacetimes with CTCs include, but are not limited to, Gödel, Kerr-Newman, Taub-NUT, Misner, van Stockum, and Oszváth spacetimes. For details, cf. [2], 168f, [3], 278f; see [7], 165-179, for general properties of these spacetimes.

time traveller can take advantage of the pre-existing causally malign structure of the spacetime to travel along a CTC without the need to produce one. The possibility of TMs in the context of CGR has first been conjectured in the seminal [14]. In section 2, I review what general properties of a spacetime need to be satisfied such that an arbitrarily advanced civilisation could operate a TM.

In section 3 I present a recent result by Krasnikov [10, 11] and discuss its implications for some necessary conditions for the possibility of a TM. In fact, Krasnikov's Theorem, as I call it, suggests that the 'potency condition' (PC) defined by Earman and Smeenk [5] must be violated in a very general class of spacetimes. This will lead me to reconsider the PC in section 4 and to conclude that Krasnikov's Theorem does not preclude a weaker version of the PC to hold. Such a weaker PC entails that the operator does not have the complete control over her TM in that not all admissible evolutions of the spacetime contain CTCs. Rather, if a mitigated PC holds, she could only increase the probability of the emergence of CTCs by operating her TM.

In what follows, I will confine myself to results found in classical GR and disregard no-go results in semi-classical quantum gravity and quantum field theory on curved spacetime. For an overview of the prospect of conclusive results in these areas, consult Earman and Smeenk's [5].

2 Time Machines and the Potency Condition

We need to specify some general features a spacetime necessarily has to display in order to qualify as a universe which allows for the operation of TMs. Unfortunately, it is impossible to list conditions on some finite spacetime region that are sufficient to causally determine the emergence of CTCs. It will soon become clear why this is the case. However, there are necessary conditions that have to be fulfilled for the operation of a proper TM in relativistic spacetimes.

First, the spacetime $(\mathcal{M}, g_{\mu\nu})$ with a connected C^{∞} Hausdorff manifold $\mathcal{M}$ without boundary and a metric $g_{\mu\nu}$ of Lorentz signature must permit a global spacelike hypersurface Σ that divides $\mathcal{M}$ into a 'before' and an 'after'. Given the failure of absolute simultaneity, of course, these temporal relations should not be taken to imply that all points in Σ are simultaneous, but rather that the hypersurface Σ intersects all future-directed timelike curves (henceforth 'chronological curves') into a past and a future section. An assignment of past and future sections of those chronological curves that intersect Σ is *unique* iff the curves intersect Σ only once. A spacelike hypersurface Σ is then called a *partial Cauchy surface* if no causal (i.e. future-directed and non-spacelike) curve intersects Σ more than in one point. To insist on Σ to be a partial Cauchy surface then is tantamount to presupposing that $\mathcal{M}$ is causally benign up to, and including, 'time' Σ. In what follows, I assume Σ to be a partial Cauchy surface. If this were not the case, then why should anyone bother to construct a TM?

A TM requires more than a spacelike hypersurface. It involves initial data on Σ such that its operation within some finite and compact region K—the TM region—of the future domain of dependence $D^{+}(\Sigma)$, i.e. the set of all points $p \in \mathcal{M}$ such that every past-inextendible non-spacelike curve through p intersects Σ,[4] brings about the existence of a region where CTCs

[4]One can think of the future domain of dependence of Σ as the region of spacetime through which all causal influences emanating from Σ have to pass through. The past domain of dependence $D^{-}(\Sigma)$ is defined analogously as the set of all points $p \in \mathcal{M}$ such that every future-inextendible non-spacelike curve through p intersects Σ exactly once.

occur, the chronology-violating domain V. If any causal signals are supposed to reach V from K at all, then $V \subset J^+(K)$, where $J^+(\mathcal{U})$ is the *causal future of* $\mathcal{U}$, defined as the set of all points in $\mathcal{M}$ which can be reached from $\mathcal{U}$ by a causal curve in $\mathcal{M}$. ($J^-(\mathcal{U})$, the causal past of $\mathcal{U}$, is defined analogously) For an illustration, see Fig. 1. Similarly, $I^\pm(\mathcal{U})$ denotes the *chronological future (past) of* $\mathcal{U}$ defined as the set of all points in $\mathcal{M}$ which can be reached from $\mathcal{U}$ by a chronological curve in $\mathcal{M}$ (set of all points in $\mathcal{M}$ from which $\mathcal{U}$ can be reached by a chronological curve in $\mathcal{M}$).

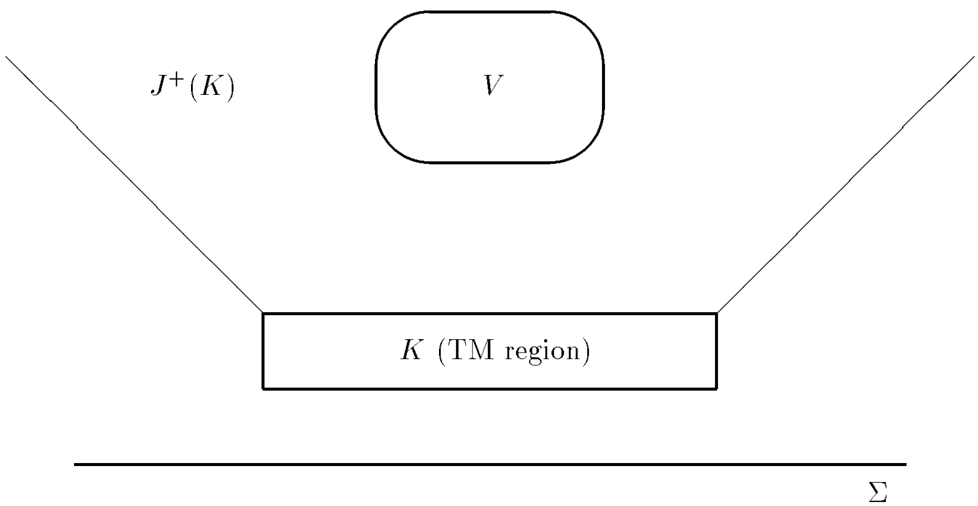

Figure 1: Basic structure of a TM spacetime. (Adopted without authorisation from [5])

A sufficient condition for the operation of a TM would be the time traveller's ability to bring about the emergence of CTCs in a causally deterministic way, i.e. such that *all* causal curves that reach V intersect Σ. Clearly,

The domain of dependence $D(\Sigma)$ is defined as $D^+(\Sigma) \cup D^-(\Sigma)$.

the above requirement that $V \subset J^+(K)$ does not preclude the possibility that causal influences from outside $J^-(V) \cap \Sigma$ penetrate V. Causal influences on V emanating from outside Σ are avoided iff $V \subset D^+(\Sigma)$, which cannot be the case since V contains CTCs and Σ is a partial Cauchy surface. Therefore, there exist causal curves that reach V and yet do not intersect Σ. Although it may thus become hopeless to define *sufficient* criteria for the operation of a TM, we are also in dire need to find more stringent necessary conditions.

The possibility of a creation of CTCs as an unavoidable result of the operation of a TM in K in terms of causal determinism is hence precluded. However, there must be some sense in which the operation of a TM brings about the emergence of CTCs, for otherwise the device could not rightfully be called a TM. The task at hand, then, is to find the strongest sense in which the appearance of CTCs can be due to the operation of a TM in K and to define a necessary condition that has to be met in order for the scenario to qualify as a case of a TM in this sense.

Earman and Smeenk [5] have suggested such a necessary criterion, viz. what they dub the

Potency Condition (PC). *In a TM scenario, every smooth, maximal, hole-free extension of* $D^+(\Sigma)$ *contains CTCs. These extensions must be solutions of the Einstein field equations (EFEs) and satisfy energy conditions.*

The more requirements an admissible extension of the domain of dependence must meet, the weaker the PC becomes since the number of relevant extensions decreases. The stronger the PC is, the more effectively will its failure provide universal safety for historians as it encompasses the larger number of admissible extensions. The predicament, then, originates from having to steer between the Scylla of the demands of proving a violation of PC in mathematically possible, but physically implausible worlds and the Charybdis of insufficient no-go theorems that leave open loopholes of physically possible TM scenarios. When discussing Krasnikov's results in section 3, I will return to the question of how strong PC can be made.

No-go theorems for particular limited cases have been proved. The most important of these results[5] was Hawking's [6]

Chronology Protection Theorem (CPT). *Consider a spacetime* ($\mathcal{M}$, $g_{\mu\nu}$) *which is a solution of the EFEs for an energy-momentum tensor* $T^{\mu\nu}$ *and which satisfies the weak energy condition (WEC).*[6] *(a) If the future Cauchy horizon* $H^+(\Sigma)$ *of a partial Cauchy surface* $\Sigma \subset \mathcal{M}$ *is compact, then so is* Σ*. Furthermore, (b) if* $H^+(\Sigma)$ *is compactly generated, then* Σ *cannot be noncompact. Finally, (c) whether or not* Σ *is compact, matter-energy cannot cross* $H^+(\Sigma)$*.*[7]

It follows, from (a) by *modus tollens*, that if Σ is noncompact, then $H^+(\Sigma)$ is noncompact. Hawking's CP Theorem amounts to a no-go result for TM just in case it is a necessary condition on a TM scenario that the Cauchy horizon be compactly generated. Although if the compact generation of the Cauchy horizon implies the violation of strong causality on the horizon and thus probably that CTCs emerge in all maximal extensions that satisfy the qualifications of PC, CPT does not cover the case of non-

[5]For a valuable survey of the results obtained up to 1995, see [4].

[6]The weak energy condition requires that the energy-momentum tensor at each $p \in \mathcal{M}$ obeys the inequality $T_{\mu\nu}V^\mu V^\nu \geq 0$ for any timelike vector $V^\mu \in T_p$ where T_p is the tangent space at p.

[7]The *future Cauchy horizon* $H^+(\mathcal{U})$ is defined as the future boundary of $D^+(\mathcal{U})$, that is $\overline{D^+(\mathcal{U})} - I^-(D^+(\mathcal{U}))$. $H^+(\mathcal{U})$ is *compactly generated* iff every past-directed null geodesic generator of $H^+(\mathcal{U})$ enters and remains within a compact set $\mathcal{C}$. If a future Cauchy horizon is compactly generated, then its generators cannot originate from singularities or infinities.

compactly generated Cauchy horizons. Krasnikov [9] has argued that there is no principled reason why TMs involving noncompactly generated horizons can be dismissed. In particular, he argues that the salvation of causal determinism does not constitute a justification to rule out noncompactly generated Cauchy horizons since, *per definitionem*, the TM region V lies beyond the Cauchy horizon. Therefore, for none of the TMs can the evolution be completely controlled by setting the initial data on Σ or some portion thereof.

Despite its undisputed merits, consequently, Hawking's CP Theorem does not provide an all-encompassing chronology protection, notwithstanding Hawking's own conjecture that the laws of physics do not allow the appearance of CTCs.[8]

3 Krasnikov's No-Go Theorem

The evolution of a spacetime in CGR is fundamentally non-unique. Into whatever extension $\mathcal{U}' \supsetneq \mathcal{U}$ the initial spacetime $\mathcal{U}$ evolves, there is always another spacetime $\mathcal{U}''$, in fact, infinitely many, which also represents a possible evolution of $\mathcal{U}$. The question of no-go theorems can thus be reformulated as the search for general classes of spacetimes for which all maximal extensions contain CTCs, or at least all maximal extension of a particular kind. Krasnikov [10, 11] has recently proved a theorem suggesting that whatever happens in the causal region of a spacetime $\mathcal{M}$, it can always evolve ac-

[8]Hawking takes the fact that we are not swamped in tourists from the future as 'strong experimental evidence in favour of the conjecture'. ([6], 610) However, this argument is flawed because this is exactly what we would expect assuming we are still in the past of Σ. But this region is *defined* by the non-occurrence of CTCs. Although probably not expected to be taken seriously, Hawking's remark at the end of his seminal paper is therefore rather puzzling.

cording to a maximal extension where $V = \emptyset$, thereby violating PC and disallowing TMs.

Krasnikov ([10], 15) considers adding a new postulate to the theory in order to mitigate the threat of unpredictability in CGR. If we stipulate that 'a spacetime must remain globally hyperbolic for as long as possible', then the emergence of extensions including singularities and infinities could be avoided for as long as we consider only maximal Cauchy developments. However, Krasnikov dismisses such a restriction as a spacetime has infinitely many extensions beyond a Cauchy horizon and none of these extensions can be globally hyperbolic. Moreover, I argue, imposing such an additional postulate would be tantamount to begging the question. Adding this postulate, despite its physical plausibility, would certainly require independent justification exactly of the sort that seems impossible to obtain in CGR. The way the problem was initially set up, CTCs can only evolve beyond the future Cauchy horizon of the specified spacelike hypersurface Σ anyway. Consequently, such a postulate does not point to a way to violate the PC and does not affect the question whether or not there are ways to set initial data on Σ such that all possible evolutions would lead to a formation of CTCs in the region beyond $H^+(\Sigma)$. Such an additional postulate of global hyperbolicity would therefore not help at all against the threat of causality-violating TMs.

That the operation of a TM is indeed impossible suggests a theorem published by Krasnikov in November 2001 [11]:

Krasnikov's Theorem (KT). *Any spacetime $\mathcal{U}$ has a maximal extension $\mathcal{M}_{max}$ such that all closed causal curves in $\mathcal{M}_{max}$ (if they exist there) are confined to the chronological past of $\mathcal{U}$.*

Although of a topological nature, KT purports to establish the physically relevant consequence that TMs, if understood as devices that must necessarily satisfy PC, are impossible to operate. That there are important caveats that should not be dismissed light-heartedly will be argued shortly.

KT entails that any set of initial data is allowed on Σ without fear of engendering paradox. Therefore, in any causal spacetime, the laws of motion, which, for their validity inside a region $\mathcal{U}$, do not depend on anything outside of $\mathcal{U}$, are compatible with any initial data in this causal region.

Let me illustrate KT with an example. Consider the causal part of the Misner spacetime,[9] i.e. a spacetime with topology $S^1 \times \mathbb{R}^1$ and the metric $g_{\mu\nu}$ given by

$$ds^2 = -t^{-1}dt^2 + td\psi^2$$

where $0 \leq \psi \leq 2\pi$. The singularity $t = 0$ can be removed in two 'natural' ways ([7], 171) permitting its (maximal) extension into the entire Misner spacetime–a cylinder with closed timelike curves in the extension of the causal region (resulting from the 'tipping over' of the lightcones at the future Cauchy horizon at $t = 0$). We identify the causal part of the Misner spacetime with the spacetime $\mathcal{U}$ in the above formulation of KT. KT then tells us that there are ways to extend the causal region $\mathcal{U}$ to an inextendible spacetime not containing CTCs. In fact, there are infinitely many ways to do so. Earman and Smeenk propose one possible construction by cutting out strips of finite length along $\psi = \psi_0$ rays and fit in appropriate conformal factors such as to guarantee that $\mathcal{M}_{\max}$ is maximal.[10] The resulting spacetime $\mathcal{M}_{\max}$ is a maximal extension of $\mathcal{U}$ not harbouring any CTCs.

[9] See [7], 171-174.

[10] Cf. [5], 11 and their corresponding Fig. 4b.

The proof of the theorem ensues along the following lines. A causally maximal extension $\mathcal{M}_{\text{max}}$ to the past of any spacetime $\mathcal{U}$ is a maximal element of the set $I^-_{\mathcal{U}'}(\mathcal{U})$, where $\mathcal{U}'$ are all possible extensions of $\mathcal{U}$. $\mathcal{M}_{\text{max}}$ is a *causally convex subset* of any of its extensions, i.e. no past-directed causal curve is extendible beyond $\mathcal{M}_{\text{max}}$. In fact, $\mathcal{M}_{\text{max}}$ is *locally causally convex*, which captures causal convexity without depending on how the spacetime is embedded into a larger space. Krasnikov then shows that an extendible locally causally convex spacetime can always be extended so that no new CTCs emerge. In fact, it can be extended so that its extension is locally causally convex as well while still precluding the appearance of new CTCs. The theorem then follows from the Zorn Lemma.

Allow a few remarks. First, KT does not rule out the existence of CTCs. There may well be CTCs to the past of the partial Cauchy surface indicating the future boundary of $\mathcal{U}$ and they will not disappear regardless of how $\mathcal{U}$ evolves. Thus, KT does not preclude just any CTCs. This is no reason for concern though, because we presupposed the causally benign character of the past side of Σ in the characterisation of what constitutes a TM in any interesting sense. Applying KT to this scenario, take the spacetime $\mathcal{U}$ to be the domain of dependence $D(\Sigma)$. The assumption of a causally benign past of Σ taken together with KT then seems to imply that all TM scenarios considered can turn out to be universally causally virtuous, i.e. containing no CTCs at all.

But this conclusion is only warranted if the chronological past of $D(\Sigma)$, except the region to the future of Σ, is a subset of the chronological past of Σ. For otherwise, the possibility arises that although there are no CTCs in the past of Σ, as was stipulated in the definition of the TM scenario, there may well be CTCs in the past of $D(\Sigma)$. An obvious way to avoid the need to explicitly establish $I^-(D(\Sigma)) \subseteq I^-(\Sigma)$ would be to claim that as long as there are no CTCs in $I^-(\Sigma)$, then the time traveller has every reason to attempt the construction of a TM and the fact whether or not CTCs transpire in $I^-(D(\Sigma))$ is simply irrelevant for her. But this is somewhat unsatisfactory, particularly because the relation between $I^-(D(\Sigma))$ and $I^-(\Sigma)$ follows immediately from their definitions. First, for any set S, from $S \subset I^-(S)$ and $S \subset D(S)$ we obtain

$$S \subset I^-(D(S)). \tag{1}$$

Also, since for any $p \in I^-(S)$, there exists a past-directed timelike curve from S to p and thus $I^-(p) \subset I^-(S)$, we have more generally:

$$\text{if } S_1 \subset I^-(S_2), \text{then } I^-(S_1) \subset I^-(S_2). \tag{2}$$

For $S = \Sigma$, (1) and (2) yield

$$I^-(\Sigma) \subset I^-(D(\Sigma)). \tag{3}$$

From (3) we can conclude that if $I^-(\Sigma)$ contains CTCs, so does $I^-(D(\Sigma))$. But it is still possible that $I^-(D(\Sigma))$ hosts CTCs while $I^-(\Sigma)$ does not. In order to preclude this, we must show that $I^-(D(\Sigma)) - I^-(\Sigma)$ does not contain CTCs. Since every $p \in D^-(\Sigma)$ can be reached by a timelike curve starting from a point in $D^+(\Sigma)$, it is clear that $I^-(D^-(\Sigma)) \subset I^-(D^+(\Sigma))$ and therefore

$$I^-(D(\Sigma)) = I^-(D^+(\Sigma)) \cup I^-(D^-(\Sigma)) = I^-(D^+(\Sigma)). \tag{4}$$

Since every past-inextendible causal curve (and *a fortiori* every past-inextendible chronological curve) from any point in $D^+(\Sigma)$ intersects Σ, $I^-(D^+(\Sigma))$ and $I^-(\Sigma)$ are identical to the past side of Σ and finally

$$I^-(D(\Sigma)) - I^-(\Sigma) = D^+(\Sigma) - \Sigma. \tag{5}$$

But since we have presupposed that Σ is a partial Cauchy surface, $I^-(D(\Sigma))$ cannot contain CTCs. Together with Krasnikov's Theorem, this entails that there are always CTC-free extensions of $D(\Sigma)$ in TM scenarios.

Second, while it does not outlaw TT altogether, KT prohibits the TM operator to bring about an inevitable appearance of CTCs to the future of Σ. There may well pop up, at some point into the future of Σ, CTCs. These CTCs emerge, as Krasnikov puts it, 'spontaneously', as opposed to the 'artificially crafted' CTCs with a causal agency that produces them. Typically, a causal spacetime can still evolve into one of infinitely many extensions containing CTCs. The problem is that this is true in spite of the locally causal character of the dynamical laws, which are presumed to hold in any neighbourhood $\mathcal{N}(p) \subset \mathcal{M}$. Arntzenius and Maudlin [1] have argued that TT generically results not in contradictions in V nor in constraints on the initial data on Σ, but rather in an underdetermination of what happens beyond the future Cauchy horizon by the initial data.[11] The task at hand, it seems, should be to try and find natural laws or additional conditions that restrict the class of admissible extensions beyond the future Cauchy horizon and whose validity can be independently confirmed. It is ironic, then, that KT achieves the opposite of limiting the class of admissible extensions: rather than confining the range of possibilities, it shows that no setting of initial data on Σ can necessitate the emergence of CTCs and that therefore, the spacetime can always evolve into a broader class of extensions than the TM operator hoped it would.

Third, the possibility that the time traveller can comply with PC is saved

[11]Krasnikov [8] has shown how the typical paradox of TT caused by self-interaction is only apparent in that the existence of such trajectories does not imply any harm to local physics and is thus not paradoxical by itself. For the discussion of similar scenarios, see Earman [3], 297-303.

despite KT if it turns out that there are cases in which all CTC-free maximal extensions of $D(\Sigma)$ violate one or several of the further requirements of PC. The TM operator could maintain her hope on the basis that the spacetimes Krasnikov considers are only minimally specified in that they have to be manifolds with Lorentz metrics. In other words, how can the historian ascertain that if $D(\Sigma)$ is hole-free, satisfies the EFEs and energy conditions, at least one of its evolutions *sans* CTCs does as well? There are no problems with respect to hole-freeness. The concept of *locally causal convexity* of spacetimes central to Krasnikov's proof provides a tool for assessing whether or not a spacetime is hole-free.[12] A locally causally convex spacetime is always hole-free, but not vice versa. Since KT shows that any locally causally convex spacetime $\mathcal{U}$ possesses a CTC-free evolution to a maximal extension $\mathcal{M}_{\max}$ that is also locally causally convex, the danger of appearing holes in the spacetime fabric is barred.

Also with respect to EFEs and energy conditions, Krasnikov's result seems to dash the hopes of our TM operator. In the version of his paper submitted to a journal, Krasnikov added the following remark not found in the paper he put in the archive: 'We can change the definition of a spacetime by including any *local* requirement on the metric and [...] the theorem will remain valid.'[13] In other words, if we impose local requirements on the metric by specifying the properties of the stress-energy tensor, then for any spacetime $\mathcal{U}$ with a metric that satisfies these requirements, there is a maximal extension such that all CTCs are confined to the chronological past of $\mathcal{U}$ and this maximal extension also satisfies the same local conditions

[12] For a technical definition of locally causal convexity, see [11], p. 8, and related definitions.

[13] Serguei Krasnikov, personal communication.

on its metric. In particular, if $\mathcal{U}$ satisfies EFEs and energy conditions, then so will at least one of its CTCs-free maximal extensions.

An generalisation of KT such as to include any local restrictions on the metric would be warranted if the two spacetimes $(\mathcal{U}, g_{\mu\nu})$ and $(\mathcal{M}_{\text{max}}, g'_{\mu\nu})$ are isometric, that is if there is a diffeomorphism $\varphi : \mathcal{U} \to \mathcal{M}_{\text{max}}$ which carries the metric $g_{\mu\nu}$ into the metric $g'_{\mu\nu}$. However, I see no compelling evidence in Krasnikov's paper that for every $\mathcal{U}$ there can always be found a $\mathcal{M}_{\text{max}}$ such that this is the case. But let me put these worries aside for the present occasion and turn to the question of what we would be left with in case such a generalisation of KT holds.

4 A Mitigated Potency Condition?

What retreat strategies are open to a would-be TM operator in the aftermath of KT? Now that KT has effectively dismantled her hopes to necessitate the appearance of CTCs, the TM operator finds herself in dire need of realigning the conceptual support for her business. First, it turned out to be impossible to uniquely determine an extension containing CTCs by setting the initial data on Σ appropriately, because any initial data uniquely determine (up to a diffeomorphism) an extension only within the domain of dependence $D(\Sigma)$ and the causality violation region V lies without this domain. Next, the prospect of restricting by suitable manipulation of the initial data on Σ the range of admissible extensions to those which contain CTCs was shattered by KT. But, as was stated in the previous section, nothing so far precludes the emergence of spontaneous CTCs.

In order to stem against the spontaneous appearance of CTCs in future extensions into which a causal spacetime with suitable physical properties can evolve, one would have to appeal to—and prove—robust no-go theorems. Of course, such an enterprise had to specify at the outset what it means for spacetimes to possess suitable physical properties. The less suitable physical features such a spacetime has to display, the more robust the no-go theorems turns out to be. However, it is clear that at least some physically motivated requirements have to be met, for otherwise there are always trivial extensions that contain CTCs. Hence, although the end of the ladder is not yet reached, KT makes a step ahead in that it becomes increasingly difficult to find ways in which to satisfy the original version of PC.

An obvious attempt to save the operator from bankruptcy is to reformulate PC into a somewhat less rigid criterion which has to be met in order for her craft to qualify as a TM. Define an *incremental time machine* (ITM) as a device that creates a tendency of a spacetime to evolve towards an extension with CTCs. This vague definition is most fruitfully cashed out with reference to increasing the probability that the domain of dependence $D(\Sigma)$ evolves such that the causality-violating region V to the future of the Cauchy horizon $H^{+}(\Sigma)$ is non-empty. The PC would then have to be restated accordingly as a

Mitigated Potency Condition (MPC). *The operation of an ITM must increase the probability that any suitable extension of* $D(\Sigma)$ *contains CTCs. Admissible extensions must be smooth, maximal, hole-free, and satisfy EFEs and WEC in order to be suitable.*

By construction, KT does not have any implications for MPC over and above the acknowledgment that the operation of an ITM cannot increase the probability that CTCs emerge in the extension to 1. Intuitively, it should be possible to operate an ITM, if only for the reason that the choice of initial data on Σ should affect in some way how history unfolds.

If we attempt to specify MPC in a more precise manner, however, we run into several obstacles. How are we supposed to measure the probability of whether or not there will be CTCs in the future as a result of the operator's manipulations? A naïve way of conceiving such an assessment would be to determine the range of admissible evolutions before the operations of the ITM, then assign a (normalised) probability to each one of the possible outcomes and add up the probabilities of those evolutions that develop CTCs. The same procedure would then be applied again after the operation of the ITM and finally the calculated values for the two probabilities would be compared.

The increase in probability of the appearance of CTCs could then be due to the following mechanisms. Either the operation of an ITM disallows certain non-CTCs containing extensions such that those extension where $V \neq \emptyset$ feature more prominently in the range of admissible extensions after the ITM operation. Or the ITM could leave the range of extensions untouched, but reassign the probabilities of the evolutions such that, on average, the probabilities of those extensions harbouring CTCs raise whereas those of the causally benign extensions drop.

The first problem, then, is to establish the range of admissible extensions. The main difficulty in this respect stems from the fact that the EFEs are non-linear and there are no known techniques to find the analytic solution. This non-linearity is intrinsic rather than extrinsic such as in the case of the formation of a non-linear system with other fields like the electromagnetic field, the scalar field, etc., by mutual interaction. The gravitational field is self-interacting, i.e. it is non-linear even in the absence of other fields. The reason for this is that it defines the spacetime over which it propagates. The underlying spacetime can no longer be considered as a fixed background on which a field evolves. Rather, the joint system of a field *cum* spacetime must co-evolve. But as long as the most general solution of the EFEs is unknown, it is unclear how the range of admissible evolution could be established.

The second difficulty concerns the assignment of probabilities to all admissible extensions. One way to circumvent this problem is to equate the probabilities of each admissible evolution. Apart from the fact that such an assumption requires an independent justification on its own, it would be illegitimate in case the operation of an ITM reassigns the probabilities of a stable set of admissible extensions, thus upsetting the preset (equal) probabilities. Either this suggests the impracticability of this approach, or a principled way of determining the probabilities of extensions is needed.

Unfortunately, there is not much in the literature as to how to assign measures over the space of maximal extensions. Most of the attempts to define such measures are concerned with causally well-behaved spacetimes. It is unclear how difficult it would be to apply these results to causally pathological yet physically possible spacetimes.[14]

Thus, there is no natural way of define and justify a measure on the relevant sets of maximal extensions. Furthermore, it is far from clear how the probabilities prior and posterior to the operation of an ITM can be assessed, even if a measure on the set of extensions is given. In order for these probabilities to be calculable, the mechanism which is unleashed by the operation of the ITM and which conveys the causal signal into the future extension of $D(\Sigma)$ must be known. If we do not want to open Pandora's box of counterfactual discourse, this mechanism must provide a calculable

[14] I wish to thank Chris Smeenk for references and helpful comments on this topic.

alteration of the relevant probabilities. Unfortunately, unlike in the case of the original PC where knowledge of the mechanism of the TM was not necessary, the notions of ITM and MPC remain rather vague as long as we are ignorant of how an ITM can be realised in terms of calculable physical processes and how these processes interact with the spacetime structure. I see no obvious way out of the quandary.

5 Conclusions

The fact that CGR allows for solutions to the EFEs that host CTCs has renewed interest among physicists and philosophers of science in the possibility of time travel and of time machines. In the present paper, I discussed the possibility of the operation of a device that produces CTCs in the light of recent results in CGR.

According to Earman and Smeenk's potency condition, such a device can be dubbed a TM if it brings about the appearance of CTCs in all its future extensions by manipulation of the initial data on a (global) partial Cauchy surface that delimits the causally benign past of the time travelling age. They have argued that previous results such as Hawking's Chronology Protection Theorem do not effectively preclude the possibility of the operation of a TM in the sense of the PC. A recent theorem due to Krasnikov, however, shows that for a very general class of scenarios, the emergence of CTCs can be circumvented. He proved that any spacetime can be maximally extended such that all CTCs, if they transpire at all, are confined to the chronological past of the spacetime. Moreover, he argued that any locally causally convex spacetime has locally causally convex extensions, thus satisfying the requirement of the PC to the effect that all admissible extensions need to be hole-free. However, it does quite not provide, as far as I can see, a definite proof that if the initial spacetime is a solution of the EFEs and satisfies energy conditions, there always exists a maximal extension *sans* CTCs that does so too.

If this additional requirement could be found to obtain, then there cannot be a TM in the sense of the PC. However, the time traveller can retreat to a mitigated PC according to which the operation of an incremental TM increases the probability that any physically suitable extension of the domain of dependence of Σ contains CTCs. The assessment of MPC, however, faces difficulties regarding the definition (and justification) of a measure on the space of extensions as well as the challenge to find a intelligible, i.e. calculable, physical mechanisms that would amount to an ITM. But without having overcome these obstacles, there is little hope of a fruitful investigation of a probabilistic TM.

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