1. Biofluid Mechanics – Viscous Flow

Aorta

$d_{aorta} = 1cm$

$l_{aorta} = 40cm$

Large Arteries

$N = 40$

$d_{arteries} = 0.3cm$

$l_{arteries} = 20cm$

$v_{arteries} = 23\,{}^{cm}\!/_{s}$

Blood

$\mu = 3.0cP = 0.03\,{}^{g}\!/_{cm\cdot s}$

$\rho = 1.057\,{}^{g}\!/_{cm^3}$

Reynolds number in large arteries

$$Re = \frac{\rho v d}{\mu} = \frac{\rho v_{arteries} d_{arteries}}{\mu} = \frac{\left(1.057\,{}^{g}\!/_{cm^3}\right)\left(23\,{}^{cm}\!/_{s}\right)(0.3cm)}{0.03\,{}^{g}\!/_{cm\cdot s}} = 243$$, so flow is laminar in large arteries.

a) Total volumetric flow rate

$$Q_{total} = n\cdot v\cdot A = N\cdot v_{arteries}\cdot \pi \cdot \left(\frac{d_{arteries}}{2}\right)^2 = 40\cdot 23\,{}^{cm}\!/_{s}\cdot \pi \cdot \left(\frac{0.3cm}{2}\right)^2 = \mathbf{65\ cm^3/s}$$

b) Pressure drop in large artery

The volumetric flow rate in one artery is:

$$Q = \frac{Q_{total}}{n} = \frac{65\,{}^{cm^3}\!/_{s}}{40} = 1.625\,{}^{cm^3}\!/_{s}$$

$$\Delta P = \frac{8\mu Q l}{\pi R^4} = \frac{8\left(0.03\,{}^{g}\!/_{cm\cdot s}\right)\left(1.625\,{}^{cm^3}\!/_{s}\right)(20cm)}{\pi\left(\frac{0.3cm}{2}\right)^4} = 4907\,{}^{g}\!/_{cm\cdot s^2}$$

$$\Delta P = 4907\,{}^{g}\!/_{cm\cdot s^2}\cdot\frac{1kg}{1000g}\cdot\frac{100cm}{m}\cdot\frac{1Pa}{{}^{kg}\!/_{m\cdot s^2}}\cdot\frac{760mmHg}{1.01\times 10^5\,Pa} = \mathbf{3.68\ mmHg}$$

c) Shear rate at wall of large artery

Since flow in large arteries is laminar, we say that:

$$\tau = \frac{1}{2}\cdot\frac{\Delta P}{l}R = \frac{1}{2}\cdot\frac{\Delta P}{l_{arteries}}\cdot\frac{d_{arteries}}{2} = \frac{1}{2}\cdot\frac{4907\,{}^{g}\!/_{cm\cdot s^2}}{20cm}\cdot\frac{0.3cm}{2} = 18.4\,{}^{g}\!/_{cm\cdot s^2}$$

Shear rate $= \dfrac{\tau}{\mu} = \dfrac{18.4\,{}^{g}\!/_{cm\cdot s^2}}{0.03\,{}^{g}\!/_{cm\cdot s}} =$ **613 s^{-1}**

d) Average velocity of blood in aorta
Q from part a is also the volumetric flow rate in the aorta.

$$\pi\cdot r^2\cdot v = Q = \pi\cdot\left(\frac{d_{aorta}}{2}\right)^2\cdot v_{aorta}$$

$$v_{aorta} = \frac{Q}{\pi\cdot\left(\frac{d_{aorta}}{2}\right)^2} = \mathbf{82\ cm/s}$$

(e) Typical value for Reynolds number

$\mathrm{Re} = \dfrac{(1.057\,g/cm^3)(82cm/s)(1cm)}{0.03\,{}^{g}\!/_{cm\cdot s}} =$ 2889, therefore flow is **turbulent**

2. Biomaterials – MMD 12.2
The equation for root mean square (rms) roughness is much like the equation for standard deviation:

$$\sigma_s = \sqrt{\frac{1}{n}\sum_{x=0}^{n}\left|s(x)-\overline{s(x)}\right|^2}\quad (1).$$

Given the following information and equation (1), we calculate rms roughness for the four materials using an Excel spreadsheet:

Distance	Height Material 1	Height Material 2	Height Material 3	Height Material 4
0	1	0	2	0
1	0	1	0	1
2	1	0	2	1
3	0	1	0	1
4	1	0	2	3
5	0	1	0	0
rms rough.	0.547723	0.547723	1.095445	1.095445

The rms roughness for materials 1 and 2 is the same, while the roughness for 3 and 4 is the same. The roughness for 3 and 4 is twice the roughness for 1 and 2. Profiles 3 and 4 show a limitation to this method of finding the surface roughness- despite the fact that 4 has larger peak than 3, the rms roughness is the same for these two materials. Thus, a better parameter to define roughness should be designed to characterize this data.

3. Biomaterials – MMD 12.15

We are given the following conditions/equation:

$L = 0.10$ m

$Q = 10^{-5}$ m^3/s

$\mu = 1$ cp $= 10^{-2}$ g cm^{-1} sec^{-1} = 1 g m^{-1} sec^{-1} = 10^{-3} kg m^{-1} sec^{-1}

$r = r_0(m) + 0.005\,\Delta P\,(Pa)$ (1)

We calculate pressure drop in the non-compliant tube using the following equation:

$$\Delta P = \frac{8\mu LQ}{\pi R^4} \quad (2)$$

$$\Delta P = \frac{8(10^{-3})(10^{-3})(10^{-5})}{\pi(10^{-2})^4} = 0.254648 \text{ kg / (m s}^2) = 0.254648 \text{ Pa}$$

Now we calculate the pressure drop in the compliant tube using equations (1) and (2):

$$\Delta P = \frac{8(10^{-3})(10^{-3})(10^{-5})}{\pi(r_0 + 0.005\Delta P)^4}$$

Using the solve function in Mathematica, we find the following:

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ro = 10^-2;  L = 0.1; μ = 10^-3; Q = 10^-5;
Solve[dP == (8*μ*L*Q)/(π*(ro + 0.005*dP)^4), dP] // N
{{dP→-2.81601-0.719796 i},{dP→-2.81601+0.719796 i},{dP→-1.27414-1.02542 i},{dP→-
1.27414+1.02542 i},{dP→0.1803}}
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The only real root (and solution for the compliant tube) is $\Delta P = 0.1803$ Pa.

Thus, the pressure drop needed to sustain flow through a totally noncompliant tube is greater than the drop needed to sustain flow through a compliant tube.