Spring 2005 (Przybycien) 42-101 Intro to BME NAME: ____________________

FINAL EXAM: SOLUTIONS and GRADING SCHEME

Format: open text, notes, homework and mind; closed neighbor.

Part I: Short Answer Questions (72 pts)

Short written-answer and short calculation questions - *No more than two or three sentences in an answer to any question in this section please.*

1. (6 pts) A circular coil of wire carrying a current is used to generate a magnetic field in a magnetic resonance imaging instrument. The magnitude of the magnetic flux density, B in Teslas [=] kg•s^{-2}•amperes^{-1}, along the axis of the coil is given by the relationship

$$B = \frac{\mu_0 N I a^2}{2(a^2 + y^2)^{3/2}},$$

where μ_0 is the permeability of free space, N is the number of turns of wire in the coil, I is the current in the wire in amperes, a is the radius of the coil in meters and y is the distance from the plane of the coil. What are the units of y and μ_0?

From the equation, y [=] a; therefore y must have units of meters (3 pts).
Also, μ_0 [=] $B\bullet a\bullet N^{-1}\bullet I^{-1}$ [=] (kg•s^{-2}•amperes^{-1})(m)(#$^{-1}$)(amperes^{-1}); therefore μ_0 must have units of kg•m•s^{-2}•amperes^{-2} (3 pts)

2. (6 pts) Give an example of a hydrophobic amino acid and a hydrophilic amino acid. State the reasoning behind your answers.

Hydrophobic amino acids: all have nonpolar (aliphatic or aromatic) side chains (1.5 pts); these include alanine, valine, leucine, isoleucine, proline, tryptophan, or phenylalanine (1.5 pts). Hydrophilic amino acids: all have polar (charged, hydroxyl-containing or guanidino-containing) side chains (1.5 pts); these include lysine, arginine, glutamic acid, aspartic acid, histidine, glutamine, asparagine, threonine, tyrosine or serine (1.5 pts)

3. (6 pts) A researcher is developing a reasonably detailed mathematical model to describe the composition and function of yeast cells. The systems (organelles and compartments) and their associated components that will be considered in the model are listed in the table. What is the number of independent mass balances that can be written for this model?

organelle/compartment	*components*
cell wall	glycoproteins, polysaccharides, proteins
plasma membrane	lipids, proteins
cytosol	lipids, glycoproteins, proteins, polyphosphates, polysaccharides, RNA, water
nucleus	DNA, lipids, proteins, RNA, water
endoplasmic reticulum	lipids, RNA, proteins, water
vacuole	lipids, polyphosphates, proteins, water
Golgi apparatus	lipids, proteins, water
mitochondrion	lipids, DNA, RNA, proteins, water
peroxisome	lipids, proteins, water

The number of independent mass balance equations in each system equals the number of components: (3 pts). 3(cell wall) + 2(plasma membrane) + 7(cytosol) + 5(nucleus) + 4 (ER) + 4(vacuole) + 3(Golgi) + 5(mitochondrion) + 3(peroxisome) = 36 independent mass balance equations (3 pts)

4. (6 pts) How does feedforward control differ from feedback control?

   In feedforward control, control decisions are made based on the value of the input signal; in feedback control, control decisions are made based on the value of the output signal. (6 pts)

5. (6 pts) What is the theoretical maximum concentration difference that could be generated across a cellular membrane for an uncharged species by an active transport process that involves the hydrolysis of one mole of ATP? You may assume that the transport occurs at body temperature or 310 K, that the internal concentration of solute is 100 μM and that the external concentration of solute is greater than the internal concentration; the universal gas constant is $1.987\ \text{cal}\bullet\text{mol}^{-1}\bullet\text{K}^{-1}$.

   The energy required to set up the concentration gradient, $\Delta G_{\text{gradient}} = RT \ln(C_1/C_2)$, will be provided by that resulting from the hydrolysis of ATP, $\Delta G_{ATP} = -7.3$ kcal/mol. (2 pts)

   $-\Delta G_{ATP} = \Delta G_{\text{gradient}} = RT\ln(C_{out}/C_{in}) = RT\ln((C_{in} + \Delta C)/C_{in})$
   Rearranging,
   $\Delta C = C_{in}[\exp\{-\Delta G_{ATP}/RT\}-1]$ (2 pts)

   $= (100\ \mu\text{M})[\exp\{-(-7.3\ \text{kcal/mol})/((1.987\ \text{cal}\bullet\text{mol}^{-1}\bullet\text{K}^{-1})(310\ \text{K})<1\ \text{kcal}/1000\ \text{cal}>)\} - 1]$
   $= 14{,}025{,}527\ \mu\text{M}$
   ≈ 14 M (2 pts)

6. (6 pts) Name the four tissue types and their functions.

   (1.5 pts each)
   1. Epithelial (cells that form layers over the surfaces of the body and internal organs) functions in protection, absorption and secretion
   2. Connective (widely dispersed, found under epithelial layers, in body spaces, in organs) function in connection, protection, insulation, support, internal transportation
   3. Muscle – (skeletal, cardiac (heart), and smooth (lining blood vessels and digestive tract)) function to provide structural support and permit motion
   4. Nervous - (neurons) function to conduct electrical signals for internal communication, and communication networking

7. (6 pts) A candidate anti-influenza virus drug, 4-guanidino-2-deoxy-2,3-didehydro-D-N-acetylneuraminic acid (4GDN), binds to a portion of one of the viral coat proteins, a neuraminidase (also known as sialidase), with $K_d \sim 10^{-10}$M; 4GDN also has a $K_d \sim 10^{-5}$M for mammalian sialidases. What is the significance of the relative values of these K_d's for viral and mammalian sialidases with respect to 4GDN's potential use as an anti-influenza drug?

   Since K_d(influenza sialidase) <<< K_d(mammalian sialidase), 4GDN will bind much more strongly to the viral enzyme than it will to the host enzyme. (3 pts) This provides a high degree of specificity (3 pts): low concentrations of 4GDN, say $\sim 10^{-8}$ M, will nearly completely inhibit the viral enzyme while having a negligible impact on the host enzyme.

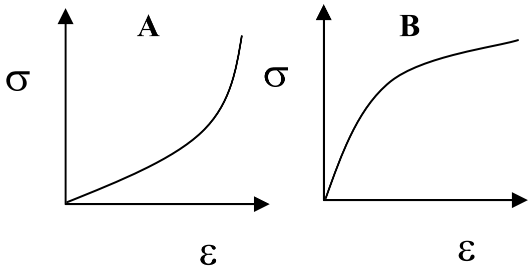
8. (6 pts) Capillaries are exceedingly small blood vessels that should have an extraordinarily large resistance to blood flow, yet Nature has managed to limit the pressure drop across capillary beds to about 90 mmHg. Drawing inspiration from the Hagen-Poiseuille equation, cite two physiological means by which Nature has limited the pressure drop across capillary beds.

   Inspiration: $\Delta P_{flow} = -8\mu \text{Vdot} L/(\pi r^4)$

   Capillaries are small and the viscosity of the blood is what it is – what's left is the volumetric flow through the capillaries and the length of the capillaries; Nature has addressed both. There are billions of capillaries – the volumetric flow is distributed among billions of vessels so that the volumetric flow through any given capillary is miniscule (3 pts) – this limits the ΔP_{flow}. Further, capillaries tend to be rather short (3 pts), on the order of a couple mm's – this also limits the ΔP_{flow}.

9. (6 pts) Would you expect the stress-strain curve for skin to look like plot A or plot B? Why?

   Plot A is representative of a material with rubbery elasticity behavior (3 pts); plot B is representative of crystalline materials. Given that skin is rather rubbery in its stress-strain behavior, plot A should represent the stress-strain behavior of skin (3 pts).



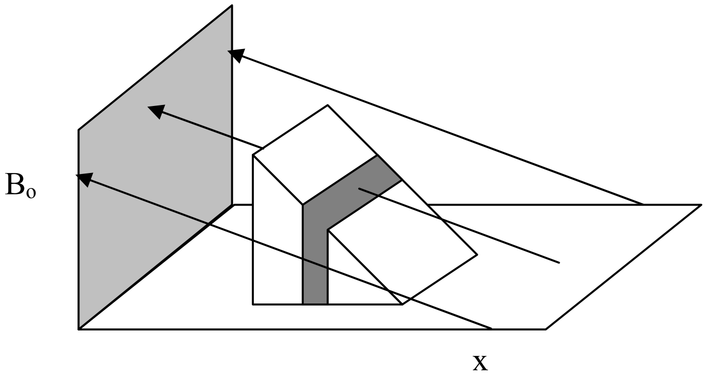
10. (6 pts) In the course of designing a part for a device that will come into contact with protein-containing solutions, an engineer is choosing between poly(tetrafluorethylene) - “PTFE” and poly(ether sulfone) - “PES” for the material of construction. While the surface roughness of the formed parts and the elastic properties of the bulk materials are similar, the contact angles are significantly different: 120° for PTFE and 55° for PES. Which material should the engineer choose for the part? Why is this choice appropriate?

    Since $\theta_{PTFE} >> \theta_{PES}$, PTFE is much less wettable than is PES; we expect PTFE to be more hydrophobic than PES. (3 pts) The extent of protein adsorption and subsequent denaturation tends to increase as surfaces become more hydrophobic. PES is probably the better choice as we’d expect less protein adsorption and less denaturation to occur. (3 pts)

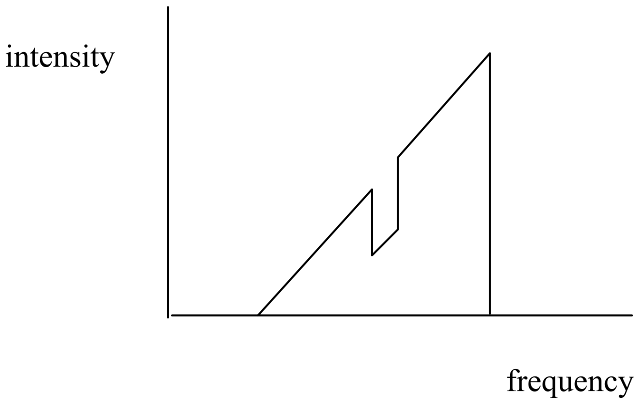
11. (6 pts) Describe three properties that may be used to distinguish white matter and grey matter in the brain using ^{1}H magnetic resonance imaging.

    The properties that enable MRI to distinguish different tissues (provide contrast between different tissues) are (1) differences in the concentrations of nuclei with net nuclear spins (^{1}H in this case – 2 pts), (2) differences in spin-lattice relaxation times (T_1 – 2 pts) and (3) differences in spin-spin relaxation times (T_2 – 2 pts).

12. (6 pts) The wedge-shaped solid object at the right is subjected to one-dimensional ^{31}P magnetic resonance imaging. The magnetic field gradient used is shown overlaid on the object. The ^{31}P concentration in the dark region of the object is half of that in the light regions. Plot what the one-dimensional “image” would look like on an intensity (y-axis) versus frequency (x-axis) plot.



    Recall that the signal intensity will be proportional to the concentration of, in this case, ^{31}P. Further, the frequency will be proportional to the field strength, B_o. (3 pts slope; 3 pts notch)



FINAL EXAM: SOLUTIONS and GRADING SCHEME

Part II: Detailed Questions (78 points)

1. (26 pts) A yeast ($CH_{1.66}N_{0.13}O_{0.40}$) is growing aerobically on arabinose ($C_5H_{10}O_5$) and ammonium hydroxide (NH_4OH) with a respiratory quotient of 1.4.
   a. (12 pts) Determine the yield coefficient, $Y_{X/S}$, for this system in grams biomass/gram substrate.
   b. (7 pts) What is the maximum biomass concentration that can be achieved in batch growth for this system if the initial substrate concentration is 10 g/L and the initial biomass concentration is 0.5 g/L?
   c. (7 pts) If the maximum biomass concentration from part b above is achieved in 4.0 hours of exponential growth, estimate the specific growth rate of the yeast.

a. Must write and balance the stoichiometric equation.

$aC_5H_{10}O_5 + bO_2 + cNH_4OH \rightarrow CH_{1.66}N_{0.13}O_{0.40} + dCO_2 + eH_2O$ (4 pts)

Equations for coefficients:

C atom balance: $5a = 1 + d$ (1)

H atom balance: $10a + 5c = 1.66 + 2e$ (2)

O atom balance: $5a + 2b + c = 0.40 + 2d + e$ (3)

N atom balance: $c = 0.13$ (4)

Respiratory quotient: $RQ = 1.4 = d/b$ (2 pts) (5)

Solving for the coefficients:

$2\times(3) - (2)$ $\Rightarrow 4b - 3c = -0.86 + 4d = 4b - 3(0.13) = -0.86 + 4(1.4b)$

$b = 0.2938$

(5) $\Rightarrow d = 1.4(0.2938)$

$d = 0.4113$

(1) $\Rightarrow 5a = 1 + 0.4113$

$a = 0.2823$

(3) $\Rightarrow 5(0.2823) + 2(0.2938) + (0.13) = 0.40 + 2(4.113) + e$

$e = 0.9065$

$0.2823C_5H_{10}O_5 + 0.2938O_2 + 0.13NH_4OH \rightarrow CH_{1.66}N_{0.13}O_{0.40} + 0.4113CO_2 + 0.9065H_2O$

$$Y_{x/s} = \left(\frac{1\text{mol x}}{0.2823\text{mol s}}\right)\left(\frac{12\times1+1\times1.66+14\times0.13+16\times0.40\text{g/mol x}}{12\times5+1\times10+16\times5\text{g/mol s}}\right) \text{ (4 pts)}$$

$= 0.517$

≈ 0.52 g biomass/g substrate (1 pt #, 1 pt sig figs)

b. A mass balance on biomass gives

$x(t) - x(0) = -Y_{x/s}(s(t) - s(0))$ (5 pts)

rearranging and noting that $s(t) = 0$,

$x(t) = x(0) + Y_{x/s}(s(0))$

$= (0.5 \text{ g x/L}) + (0.517 \text{ g x/g s})(10 \text{ g s/L})$

$= 5.67$

≈ 5.7 g biomass/L (1 pt #, 1 pt sig figs)

c. For exponential growth,

$x(t) = x(0)\exp\{\mu t\}$

rearranging,

$\mu = \ln(x(t)/x(0))/t$ (5 pts)

$= \ln((5.67 \text{ g x/L})/(0.5 \text{ g x/L}))/(4.0 \text{ h})$

$= 0.60708 \text{ h}^{-1}$

$\approx 0.6 \text{ h}^{-1}$ (1 pt #, 1 pt sig figs)

2. (26 pts) Consider the flow of blood in a vena cava with an internal diameter of 3.0 cm, length of 45 cm and blood flow rate of 5185 mL/min. For the purposes of this problem, assume that blood behaves as a Newtonian fluid with a density of 1.053 g/mL and a viscosity of 4.175 mPa•s. Recall 760 mmHg = 101.325 kPa.

a. (6 pts) What is the Reynolds number within the vena cava?
b. (10 pts) If the pressure at the right atrium of the heart (the outlet of the vena cava) is 2.0 mmHg, what is the pressure at the inlet of the vena cava, in mmHg? You may assume that the person to whom this vena cava belongs is lying down.
c. (10 pts) If the blood has a yield stress of 0.0080 Pa, at what radial distance, in cm, from the centerline of the vena cava would the blood behave as if it were a solid? Given that the diameter of a red blood cell is ~7.5 μm, is this region of solid behavior significant?

a. $\mathrm{Re} = \rho v d/\mu$ (2 pts)
need to calculate velocity, v, from volumetric flow rate, Vdot:
v = Vdot/A (2 pts)
$= \text{Vdot}/\pi r^2$
$= [(5185\ \text{mL/min})/(\pi(3.0\ \text{cm}/2)^2]<1\ \text{cm}^3/1\ \text{mL}>$
$= 733.527\ \text{cm/min}$
$\mathrm{Re} = [(1.053\ \text{g/mL})(733.527\ \text{cm/min})(3.0\ \text{cm})/(4.175\ \text{mPa}\bullet\text{s})]$
$\times<1\ \text{mL}/1\ \text{cm}^3><1\ \text{min}/60\ \text{s}><100\ \text{cm}/1\ \text{m}><1\ \text{mPa}/(\text{g}\bullet\text{m}^{-1}\bullet\text{s}^{-2})>$ (1 pt)
$= 925.03$
≈ 930 (1 pt)

b. $P_{vc\_inlet} = P_{vc\_outlet} - \Delta P_{flow}$ (4 pts)
need to calculate ΔP_{flow} from the Hagen-Poiseuille equation
$\Delta P_{flow} = -8\mu \text{Vdot} L/(\pi r^4)$ (4 pts)
$= [8(4.175\ \text{mPa}\bullet\text{s})(5185\ \text{mL/min})(45\ \text{cm})/(\pi(3.0\ \text{cm}/2)^4]$
$\times<1\ \text{cm}^3/1\ \text{mL}><1\ \text{min}/60\ \text{s}><\text{kPa}/10^6\ \text{mPa}><760\ \text{mmHg}/101.325\ \text{kPa}>$
$= -0.061255\ \text{mmHg}$ (1 pt)
$\approx -0.061\ \text{mmHg}$
$P_{vc\_inlet} = 2.0\ \text{mmHg} - (-0.061255\ \text{mmHg})$
$= 2.061255\ \text{mmHg}$
$\approx 2.1\ \text{mmHg}$ (1 pt)

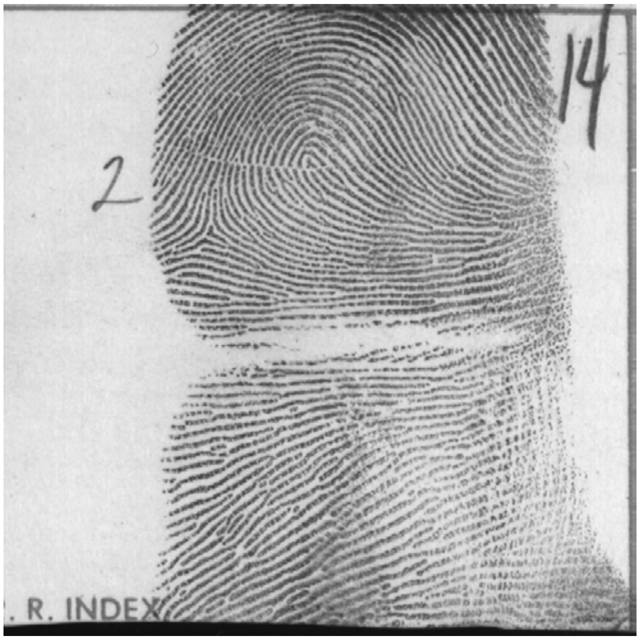
c. Find radial position at which $\tau = \tau_{yield}$
$\tau = \Delta P_{flow} r/(2L) = \tau_{yield}$ (4 pts)
Rearranging,
$r = -2L\tau_{yield}/\Delta P_{flow}$
$= [-2(45\ \text{cm})(0.0080\ \text{Pa})/(-0.061255\ \text{mmHg})]$
$\times<760\ \text{mmHg}/101{,}325\ \text{Pa}>$ (1 pt)
$= 0.088163\ \text{cm}$
≈ 0.088 cm from the centerline (1 pt)

0.088 cm from the centerline is equivalent to 88 μm, or roughly 12 red blood cell diameters. Thus, there will be a small region of slug-like flow that spans about 24 red blood cell diameters in the center of the vena cava. (4 pts)

3. (26 pts) The picture at the right represents a 1.5 inch x 1.5 inch fingerprint card from the FBI's fingerprint database. The ridge structures in the picture are spaced at a fairly regular interval of ~0.024 inch.

a. (13 pts) If this picture is to be digitized, at what *k*-space frequency, in inches^{-1}, should the picture be sampled in order to preserve the ridge spacing interval in the digitized image?

b. (13 pts) If this picture is sampled at a *k*-space frequency of 30 inches^{-1}, at what spacing, in inches, will the alias signal appear?



http://www.c3.lanl.gov/~brislawn/FBI/FBI.html

a. The minimum sampling frequency is given by the Nyquist frequency:

$f_{Nyquist}$ $= 2f_{signal,max}$ (5 pts)
$= 2(1/\text{feature spacing})$ [the *k*-space frequency] (5 pts)
$= 2\times(1/0.024 \text{ inches})$
$= 83.3333$
$\approx 83 \text{ inches}^{-1}$ (2 pts #. 1 pt sig figs)

b. Aliasing is described by the alternative form of the Nyquist criterion:

f_{alias} $= (f_{signal} - kf_{sample})$ (5 pts)
$= ((1/0.024 \text{ inches}) - k(30 \text{ inches}^{-1})$
$= 41.6667 - k30 \text{ inches}^{-1}$ where k is an integer

So for k = 1, 2, 3 etc.:
$= 11.6667 \text{ inches}^{-1}$, $-18.3333 \text{ inches}^{-1}$, $-48.3333 \text{ inches}^{-1}$ etc.

At a sampling frequency of 30 inches^{-1}, only signals with frequencies $\leq$15 inches^{-1} will be resolved. So, the 11.6667 inches^{-1} alias is the only frequency that will be observed. (5 pts)

Alias spacing $= 1/f_{alias}$
$= 1/(11.6667 \text{ inches}^{-1})$
$= 0.085714$ inches
≈ 0.086 inches (2 pts #. 1 pt sig figs)