Topics – Gait Analysis

- A simple model for power expenditure on walking
- Stride optimization
- Ergonomic analysis

Biomechanics refers primarily to the rules governing how biological systems move. Traditionally this has focused on force generation and power requirements for motion of the skeleto-muscular system, as well as the fluid mechanics of blood flow. More recently, there has been growing interest in cellular biomechanics – the study of force generation by elements of the cytoskeleton and the cellular response to externally applied physical stress.

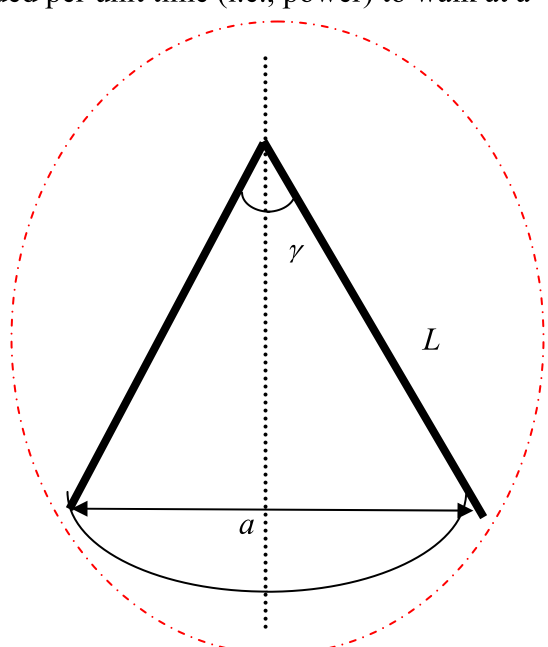
A sampling of topic in biomechanics:

Is there an optimal gait for walking? (How might this be used to design footwear?)
How is force transmitted throughout the body during an impact (automobile safety?)
How does an altered gait affect stress and movements of the upper body (orthopedics?)
How do blood flow patterns through a branching artery affect plaque deposition (atherosclerosis treatment?)
How do blood cells respond to shear stresses in blood vessels? (design of artificial heart valves?)
How do cells migrate through the extracellular matrix? (cancer treatment?)

Gait Analysis

We will develop a simple model that can predict how much power is required for a person to walk. At its core, this is an application of the energy balance concept. When a person walks, one foot is always in contact with the ground. At the right point in the stride, one foot is lifted and moved forward – it must gain kinetic energy. Another feature of walking is that the body tends to move up and down – the body must gain gravitational potential energy when it rises. This energy must be provided by the body – ultimately it is derived from metabolic energy sources, but in this analysis we will not concern ourselves with the metabolic energy source. Our goal is to figure out how much energy must be provided per unit time (i.e., power) to walk at a certain velocity.

If you observe different people walking in a group, you will notice that although they all move at the same velocity, not everyone in the group takes the same number of steps. If they take more steps to keep up with the group, they must be taking shorter steps. The velocity is equal to the step length a times the step frequency. What determines the optimal step length and frequency? This depends on how long a person's legs are and on the mass of the person.



Our model for walking is very simple: Two straight legs of length L that pivot around a common axis:

The full length of the stride is *a*, and the angle that the leg makes with the vertical line is γ.

How can we analyze the power requirements? The Energy Balance – let our system be the two legs and their pivot point, enclosed by the red border in the figure. Assume that the pivot point has a mass equal to that of the body minus that of the two legs.

$$\frac{dE_{sys}}{dt} = \dot{Q} + \dot{W} + \sum_{inlets} \dot{m}_i \left(u_i + gh_i + \tfrac{1}{2} v_i^2\right) - \sum_{outlets} \dot{m}_i \left(u_i + gh_i + \tfrac{1}{2} v_i^2\right)$$

There are no inlet or outlet streams, so the last two terms are zero. Likewise, we assume that there is no heat transfer during walking, so $\dot{Q} = 0$. Then we have

$$\frac{dE_{sys}}{dt} = \dot{W}$$

Let's break down the energy of the system into its parts:

$$\frac{dE_{sys}}{dt} = \frac{d\left(mu + mgh + \frac{1}{2} mv^2\right)}{dt} = \dot{W}$$

Since there is not a large temperature change, the internal energy u does not change. We are left with the statement that the power is being used to change the potential energy and the kinetic energy of the system. We will approximate the derivative by considering the change in potential energy and the kinetic energy over one stride of duration Δt (equal to the reciprocal of the step frequency).

$$\frac{dE_{sys}}{dt} = \frac{d\left(mgh + \frac{1}{2} mv^2\right)}{dt} = \dot{W}$$

$$\frac{mg\Delta h}{\Delta t} + \frac{\frac{1}{2} m\Delta v^2}{\Delta t} = \dot{W}$$

The kinetic energy change corresponds to swinging the leg forward. The mass m is the mass of the leg. The potential energy change corresponds to lifting the body.

The time interval depends on the walking velocity and the stride length:

$$\Delta t = \frac{a}{v}$$

The velocity of the leg goes from zero initially (in contact with ground) to v as it moves. Thus, the power requirement for the kinetic energy term is:

$$\frac{\frac{1}{2}m_{leg}v^2}{a/v} = \dot{W}_{ke}$$

$$\dot{W}_{ke} = \frac{m_{leg}v^3}{2a}$$

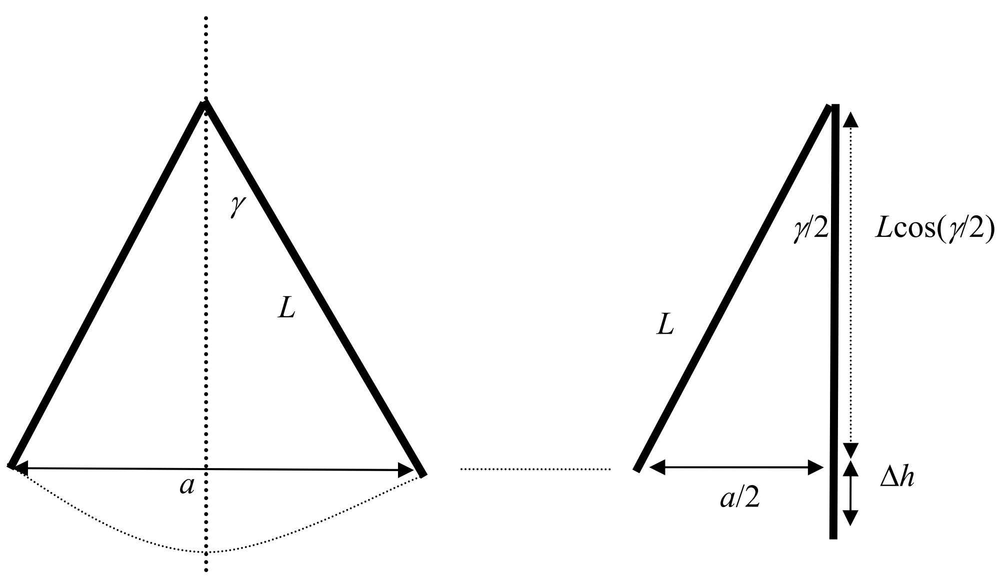
Note that we are assuming that it is the leg only that accelerates; the rest of the body is moving along at constant velocity. We will include a proportionality constant, α, which accounts for the ratio of leg mass to total body mass, so we can write our result in terms of the body mass.

$$\dot{W}_{ke} = \frac{\alpha m v^3}{2a}$$

Now, the potential energy change. The body elevation rises by some amount Δh during the step. Thus, the change in potential energy gives rise to a power term:

$$\dot{W}_{pe} = \frac{mg\Delta h}{\Delta t} = \frac{mg\Delta h}{a/v}$$

Determining Δh in terms of leg length, L, and stride angle, γ, is possible by trigonometry:



$$\Delta h = L - L\cos(\gamma/2) = L(1-\cos(\gamma/2))$$

So,

$$\dot{W}_{pe} = \frac{mg\Delta h}{\Delta t} = \frac{mgL(1-\cos(\gamma/2))}{a/v}$$

Combining the two power terms, we get

$$\dot{W} = \dot{W}_{ke} + \dot{W}_{pe}$$
$$= \frac{\alpha m v^3}{2a} + \frac{mgL(1-\cos(\gamma/2))}{a/v}$$

This can be analyzed to find the optimal stride length *a*, for a person whose legs have length *L* and whose body mass is *m*. For this analysis, the cosine term can be simplified by using a *Taylor series expansion*. This will replace a trigonometric function with a power series function that is easier to work with.

The Taylor series is commonly used for this purpose. The general definition of the Taylor series expansion of some function *f*(*x*) is

$$f(x) \approx f(x_o) + f'(x_o)(x-x_o) + \frac{1}{2}f''(x_o)(x-x_o)^2 + \frac{1}{6}f'''(x_o)(x-x_o)^3 + \ldots$$

The primes denote derivatives (first derivative, second derivative, etc.). We say that we are taking the expansion around some reference value, x_o. This expansion is valid as long as we apply it to values of x that are not too far from x_o. In each term, we evaluate the derivative at x_o.

So, if we assume that the angle γ is not too far from 0^o during the stride, we can expand

$$\cos(\gamma/2) \approx \cos(0) - \sin(0)(\gamma/2) - \frac{1}{2}\cos(0)(\gamma/2)^2 + \ldots$$
$$\cos(\gamma/2) \approx 1 - \frac{(\gamma/2)^2}{2}$$

So, the power requirement

$$\dot{W} = \frac{\alpha m v^3}{2a} + \frac{mgL(1-\cos(\gamma/2))}{a/v} =$$

$$W = \frac{\alpha m v^3}{2a} + \frac{mgL(1-\cos(\gamma/2))}{a/v} = \frac{\alpha m v^3}{2a} + \frac{mgL((\gamma/2)^2/2)}{a/v}$$

The angle γ can be related to L and a using trigonometry. Recall that the circumference of a circle of radius L is $2\pi L$. Recall also that a circle of 360° corresponds to an angle of 2π radians.

Then (using radians instead of degrees for angle measurements), recall that an arc length equals the product of the radius of the arc and the angle that subtends the arc. If we approximate the arc length subtended by $\gamma/2$ as $a/2$, then

$$\frac{a}{2} \approx \frac{\gamma}{2} L \quad \text{or} \quad \gamma \approx \frac{a}{L}$$

Now substitute that back into the power equation:

$$\dot{W} \approx \frac{\alpha m v^3}{2a} + \frac{mgL(a/L)^2}{8a/v}$$

$$\dot{W} \approx \frac{\alpha m v^3}{2a} + \frac{mgav}{8L}$$

To accommodate the numerous approximations made here, including the arc length, introduce a second proportionality constant β, such that

$$\frac{\alpha m v^3}{2a} + \frac{mgav}{8L} \approx \dot{W} = \frac{\alpha m v^3}{2a} + \frac{\beta mgav}{8L}$$

[Note that in our text, Domach prefers to lump the factor of ½ of the first term in to α and the factor of 1/8 of the second term in to β such that $\dot{W} = \frac{\alpha m v^3}{a} + \frac{\beta mgav}{L}$]

In application (and in our formulation rather than Domach's), $\alpha \sim 0.1$ and $\beta \sim 1.0$.

Stride Optimization

Notice that the kinetic energy and potential energy contributions each depend on the stride length in a different manner. If we increase the stride length *a*, the kinetic energy requirement decreases, but the potential energy requirement increases. (Picture a person walking – does this make sense?) Whenever we have opposing tendencies like this, there is an opportunity for *optimization*.

Optimization refers to the engineering design that maximizes some desirable outcome (profit, a large force, etc.) or minimizes some cost (e.g. power consumption). Let's find the stride that minimizes power consumption.

In calculus, this is sometimes called a *min-max* problem. We want to find the value of *a* that gives the minimum power. To find this, we apply the rule that at the minimum of any function, the first derivative must equal zero. The same rule applies to a maximum. Picture a function passing through a max or min. Does this make sense? To determine whether the point at which the first derivative is zero is a minimum or maximum (or a saddle point), can check the second derivative. For a minimum, the curvature at this point must be concave up and the second derivative will be positive. For a maximum, the curvature at this point must be concave down and the second derivative will be negative. (If the second derivative at this point is zero, the point will be a saddle point.)

The optimal power (minimal) is the power at that value of *a* that gives $dW/da = 0$ and $d^2W/da^2 > 0$:

$$\dot{W} = \frac{\alpha m v^3}{2a} + \frac{\beta m g a v}{8L}$$

$$\frac{d\dot{W}}{da} = \frac{-\alpha m v^3}{2a^2} + \frac{\beta m g v}{8L}$$

Set this equal to zero and solve for the optimal stride length a^*:

$$\frac{d\dot{W}}{da} = \frac{-\alpha m v^3}{2a^2} + \frac{\beta m g v}{8L} = 0$$

$$a^* = 2v\left(\frac{L\alpha}{g\beta}\right)^{1/2}$$

Check to see if this extremum is a minimum using the second derivative.

$$\left.\frac{d^2\dot{W}}{da^2}\right|_{a=a^*} = \frac{\alpha m v^3}{a^3} = \frac{\alpha m v^3}{a^{*3}} > 0$$ since $\alpha > 0$, $m > 0$, $v > 0$ and $a^* > 0$. $\therefore$ a^* is a minimum.

Substitute *a** into the power equation and find the minimal power requirement:

$$\dot{W}_{\min} = \frac{\alpha m v^3}{2\left(2v\left(\frac{L\alpha}{g\beta}\right)^{1/2}\right)} + \frac{\beta m g\left(2v\left(\frac{L\alpha}{g\beta}\right)^{1/2}\right)v}{8L}$$

$$\dot{W}_{\min} = \frac{\beta^{1/2} m g^{1/2} \alpha^{1/2} v^2}{2L^{1/2}}$$

Example stride analysis

Ergonomic Analysis - Scaling

We will now show how the gait model can be scaled to allow easy comparisons for different people. For example, how does the power requirement differ for different people walking at the same speed, etc.

For two people (1 and 2) walking at the same speed, the difference in their respective power requirements is

$$\Delta\dot{W} = \dot{W}_1 - \dot{W}_2 = \frac{\alpha m_1 v^3}{2a_1} + \frac{\beta m_1 g a_1 v}{8L_1} - \frac{\alpha m_2 v^3}{2a_2} - \frac{\beta m_2 g a_2 v}{8L_2}$$

$$= \frac{\alpha v^3}{2}\left(\frac{m_1}{a_1} - \frac{m_2}{a_2}\right) + \frac{\beta g v}{8}\left(\frac{m_1 a_1}{L_1} - \frac{m_2 a_2}{L_2}\right)$$

$$= \alpha v^3 \frac{m_1}{2a_1}\left(1 - \frac{m_2}{m_1}\frac{a_1}{a_2}\right) + \beta g v \frac{m_1 a_1}{8L_1}\left(1 - \frac{m_2 a_2 L_1}{m_1 a_1 L_2}\right)$$

$$= \alpha v^3 \frac{m_1}{2a_1}\left(1 - \frac{m_r}{a_r}\right) + \beta g v \frac{m_1 a_1}{8L_1}\left(1 - \frac{m_r a_r}{L_r}\right)$$

where we have introduced the *dimensionless variables* $m_r = m_2/m_1$, $a_r = a_2/a_1$ and $L_r = L_2/L_1$.

This type of analysis helps to rapidly answer questions of a relative nature. For example, how could two different people walking at the same velocity have equal power requirements? One way is for $m_r/a_r = m_r a_r/L_r = 1$.

Example ergonomic analysis