Ph.D. Econometrics I
Heinz School, Carnegie Mellon University
90-906, Spring 2000

Midterm

Instructions You may use any books, notes, calculators, and other aids you like. You may not converse, nor may you cooperate.

Please complete all questions.

Please show all relevant work.

Please interpret your results in plain English.

1. Consider the following regression model in which we assume that all the classical assumptions are satisfied:

$$Y = \beta_1 + \beta_2 X_2 + \epsilon \tag{1}$$

Let's think about two different estimators for β_2. The first is the OLS estimator, $\hat{\beta}_{2,OLS}$ and the second is the estimator we would have gotten by estimating the following equation by OLS, $\hat{\beta}_{3,OLS}$:

$$Y = \beta_3 X_2 + \epsilon \tag{2}$$

(a) Is $\hat{\beta}_{3OLS}$ biased? What is its bias, if so.

(b) What is the variance of $\hat{\beta}_{3,OLS}$?

(c) Is the variance of $\hat{\beta}_{3,OLS}$ always larger, always smaller, always the same, or sometimes larger or smaller than $\hat{\beta}_{2,OLS}$?

(d) How does your result relate to the Gauss-Markov Theorem?

(e) If β_1 is truly equal to zero, which of the two estimators for β_2 is better and why?

2. Consider the following model (for which we assume the classical assumptions hold):

$$price = \beta_1 + \beta_2 weight + \beta_3 domestic + \epsilon \qquad (3)$$

The data are on the subject of 1978 model year automobiles sold in the U.S. There are 69 observations. Price is the sticker price in dollars. Weight is the weight of the automobile in pounds. Domestic is a dummy variable equal to 1 for cars assembled in the U.S.

Instead of estimating Equation 3, we will instead estimate Equation 4 below:

$$price = \beta_4 + \beta_5(weight - \overline{weight}) + \beta_6(domestic - \overline{domestic}) + \epsilon \qquad (4)$$

It is easy to prove (and you should do so after the test is over) that estimating these two equations gives you:

$$\begin{aligned} \hat{\beta}_5 &= \hat{\beta}_5 \\ \hat{\beta}_6 &= \hat{\beta}_6 \\ \hat{\beta}_4 &= \hat{\beta}_1 + \hat{\beta}_2 \overline{weight} + \hat{\beta}_3 \overline{domestic} \end{aligned}$$

A useful fact to know below (which uses our formula for partitioned matrixes) is that:

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}^{-1} = \begin{bmatrix} A_1^{-1} & 0 \\ 0 & A_2^{-1} \end{bmatrix} \qquad (5)$$

We know the following facts about these data:

$\overline{price} = 6146$ $\qquad$ $\overline{weight} = 3032$

$\overline{domestic} = 0.70$

$V(\widehat{price}) = 8482308$ $\qquad$ $V(\widehat{weight}) = 628613$

$V(\widehat{domestic}) = 0.215$

$\widehat{Cov}(price, weight) = 1265034$ $\qquad$ $\widehat{Cov}(price, domestic) = 23.44$

$\widehat{Cov}(weight, domestic) = 237.4$

(a) Are prices higher or lower, on average, for domestically produced cars?

(b) Are prices higher of lower, on average, for domestically produced cars when we compare cars of the same size (ie weight)?

(c) Test the theory that, in terms of affecting price, changing a car from domestic to foreign assembly is equivalent to increasing its weight by 800 pounds.