Problem Set on Difference Equations

1. (A supply and demand model). Solve the following problem using the method of undetermined coefficients.

$$y_t = \alpha_0 - \alpha_1 p_t + e_t \,,$$

$$y_t = \beta_0 + \beta_1 E\left[p_t \mid I_{t-1}\right],$$

$$e_t = \rho e_{t-1} + \varepsilon_t \,, \quad E\left[\varepsilon_t = 0\right].$$

Hint: **S**olve for price first, assuming a solution of the form $p_t = \pi_0 + \pi_1 \varepsilon_t + \pi_2 e_{t-1}$.

2. (Market equilibrium with price-setting firms). Let demand be given by

$$y_t = -p_t + aE\left[p_{t+1} \mid I_t\right] + u_t \,,$$

$$u_t = \rho u_{t-1} + \varepsilon_t \,, \quad 0<a<1,\ 0<\rho<0.$$

and let aggregate supply be given by

$$p_t = cE\left[y_t \mid I_{t-1}\right].$$

The demand equation says that demand will be high if (i) it is subject to a positive shock, and (ii) prices are expected to rise next period. The supply equation says that firms will raise prices if they expect output to be high. The coefficient c can be interpreted as a measure of price flexibility. Let $\overline{y}_t = E\left[y_t \mid I_{t-1}\right]$ be the one-period conditional expectation of output, and let $\tilde{y}_t = y_t - \overline{y}_t$ be the unanticipated component of output. Show that an increase in flexibility reduces the variance of $\overline{y}_t$ but increases the variance of $\tilde{y}_t$.

3. (Market equilibrium with partial adjustment). Consider the following supply and demand system:

$$D_t = \alpha_0 - \alpha_1 p_t + \varepsilon_t \,,$$

$$S_t = S_{t-1} + \gamma\left(S_t^* - S_{t-1}\right),$$

$$S_t^* = \beta_0 + \beta_1 E\left[p_t \mid I_{t-1}\right],$$

where S_t is quantity supplied and S_t^* is the quantity that firms would like to supply.

(a) Derive the basic difference equation for price.

(b) Assume that expectations are adaptive, with the form

$$E\left[p_t \mid I_{t-1}\right] = E\left[p_{t-1} \mid I_{t-2}\right] + \phi\left(p_{t-1} - E\left[p_{t-1} \mid I_{t-2}\right]\right).$$

Solve for price as a function of lagged supply, lagged prices, and the disturbance term.

(c) Assume now that expectations are rational. Solve again for price as a function of lagged supply, lagged prices, and the disturbance term.

(d) Compare your answers in (b) and (c).

4. Consider the following model

$D_t = -\beta p_t$ (demand)

$S_t = \gamma E\left[p_t \mid I_{t-1}\right] + \varepsilon_t$ (supply)

$I_t = \alpha\left(E\left[p_{t+1} \mid I_t\right] - p_t\right)$ (inventory demand)

$S_t = D_t + \left(I_t - I_{t-1}\right)$ (market clearing)

Suppose there is perfect foresight. This implies that $E\left[p_{t+j} \mid I_t\right] = p_{t+j}$ for all j. In other words, agents in the model can predicts the time path of the exogenous shocks, ε_t with perfect accuracy.

(a) Write the basic difference equation for price.

(b) Use the **method of factorization** to solve for p_t as a function of all past and future values of εt. Hint: Depending on how you decide to present the solution, you may be able to make use of the following relationship, which holds for $\lambda \neq 1$,

$$\frac{1}{(1-\lambda L)(1-\lambda^{-1}L)} = \left(\frac{1}{\lambda - \lambda^{-1}}\right)\left(\frac{\lambda}{1-\lambda L} - \frac{\lambda^{-1}}{1-\lambda^{-1}L}\right).$$

5. (Partial adjustment, again). Recall the partial adjustment equations under rational expectations from section 1:

$$y_t - y_{t-1} = \left(\frac{a}{a+b}\right)\left(\hat{y}_t - y_{t-1}\right) + \beta\left(\frac{b}{a+b}\right)\left(y_{t+1} - y_t\right).$$

Solve it.

6. (Hunting Deer). Suppose that, in the absence of hunting, a deer population evolves according to the logistic equation $x_t = 1.8x_{t-1} = 0.8x_{t-1}^2$, where x is measured in thousands.

a) Analyze the steady state properties of this system.

b) Assume that permits are issued to hunt 72 deer per time period year. What happens to the steady state population? How does your answer depend on the initial population?

c) Assume now that permits are issued to hunt 240 deer per time period year. What happens to the steady state population? How does your answer depend on the initial population?
Answer these questions using Theorems 3.1 and 3.2 as well as graphically.