Homework 9

1. Show that a functor category $\mathbf{D}^{\mathbf{C}}$ has binary products if $\mathbf{D}$ does (construct the product of two functors F and G "objectwise": $(F \times G)(C) = F(C) \times G(C)$).

2. Let $\mathbf{C}$ be a locally small category. Show that there is a functor

$$\mathrm{hom} : \mathbf{C}^{\mathrm{op}} \times \mathbf{C} \to \mathbf{Sets}$$

such that for each object C of $\mathbf{C}$,

$$\mathrm{hom}(C, -) : \mathbf{C} \to \mathbf{Sets}$$

is the covariant representable functor and

$$\mathrm{hom}(-, C) : \mathbf{C}^{\mathrm{op}} \to \mathbf{Sets}$$

is the contravariant one. (*Hint: use the Bifunctor Lemma*)

3. (a) Show that equivalence of categories is an equivalence relation.

   (b) A category is *skeletal* if isomorphic objects are always identical. Show that every category is equivalent to a skeletal subcategory. (Every category has a "skeleton".)

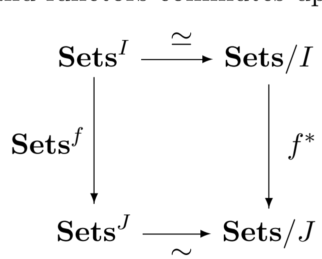
4. (a) Complete the proof that, for any set I, the category of I-indexed families of sets, regarded as the functor category $\mathbf{Sets}^I$, is equivalent to the slice category $\mathbf{Sets}/I$ of sets over I.

$$\mathbf{Sets}^I \;\simeq\; \mathbf{Sets}/I$$

   (b) * Show that reindexing of families along a function $f : J \to I$, given by precomposition,

$$\mathbf{Sets}^f((A_i)_{i \in I}) = (A_{f(j)})_{j \in J}$$

   is represented by pullback, in the sense that the following diagram of categories and functors commutes up to natural isomorphism.



Here $f^* : \mathbf{Sets}/I \to \mathbf{Sets}/J$ is the pullback functor along the function $f : J \to I$.