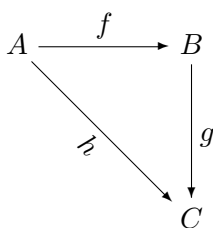


Homework 2

1. Show that a function between sets is an epimorphism if it is surjective. Conclude that the isos in **Sets** are exactly the epi-monos.
2. (Inverses are unique) If an arrow $f : A \rightarrow B$ has inverses $g, g' : B \rightarrow A$ (i. e. $g \circ f = 1_A$ and $f \circ g = 1_B$, and similarly for g'), then $g = g'$.
3. With regard to a commutative triangle,



in any category \mathbf{C} , show

- a. if f and g are isos (resp. monos, resp. epis), so is h ;
 - b. if h is monic, so is f ;
 - c. if h is epic, so is g ;
 - d. (by example) if h is monic, g need not be.
4. In any category with binary products, show directly from the UMP that:

$$A \times (B \times C) \cong (A \times B) \times C.$$

5. In any category \mathbf{C} with products, define the *graph* of an arrow $f : A \rightarrow B$ to be the monomorphism

$$\Gamma(f) = \langle 1_A, f \rangle : A \rightarrow A \times B$$

(why is this monic?). Note for $\mathbf{C} = \mathbf{Sets}$ this determines a functor $\Gamma : \mathbf{Sets} \rightarrow \mathbf{Rel}$ to the category \mathbf{Rel} of relations, as defined in the exercises to Chapter 1. (To get an actual subset of $A \times B$, take the image of A under $\Gamma(f)$.) Can you generalize the construction of \mathbf{Rel} to $\mathbf{Rel}(\mathbf{C})$ so that the graph construction above is a functor $\mathbf{C} \rightarrow \mathbf{Rel}(\mathbf{C})$?

6. * Show that the epis among posets are the surjections (on elements), and that the one-element poset $\mathbf{1}$ is projective. Show that sets, regarded as discrete posets, are projective in the category of posets (use the foregoing exercise). Give an example of a poset that is not projective. Show that every projective poset is discrete, i.e. a set. Conclude that **Sets** is (isomorphic to) the “full subcategory” of projectives in **Pos**, consisting of all projective posets and all monotone maps between them.