

HOMEWORK 9

Due Thursday, November 30

1. Determine whether each of the following arguments is valid. Justify your answer either way.
 - (a) All whales are fish.
Some mammals are not fish.
No mammals that are whales are fish.
Therefore, no mammals are whales.
 - (b) If you serve in the National Guard, you get paid.
W got paid for serving in the National Guard.
Therefore, W served in the National Guard.
2. In the language $\mathcal{L} = \{0, S, +, \cdot\}$ of Arithmetic (see van Dalen, 2.7, example 6) consider the set \mathbf{T} of sentences satisfied by the Natural Numbers \mathbb{N} ,

$$\mathbf{T} = \{\sigma \mid \mathbb{N} \models \sigma\}.$$

Is \mathbf{T} a Henkin theory? Justify your answer!
3. Does the theory \mathbf{T} above have any finite models? Does it have any models other than the natural numbers? Justify your answers!
4. Consider the language $\mathcal{L} = \{0, S, \leq\}$ with a constant symbol, a unary function symbol and a binary relation symbol.
 - (a) Can you give a finite axiom set for a theory having only finite models of a given size? If so, do so; if not, say why.
 - (b) Can you give a finite axiom set for a theory having only infinite models? If so, do so; if not, say why.
 - ★ (c) Can you write down an axiom set for a theory having finite models of every finite size, but no infinite models? If so, do so; if not, say why.