

## HOMEWORK 9

Due Thursday, November 5

1. In van Dalen, do problem 15 on p. 81.
2. Read section 2.7 of van Dalen. With example 6 in mind, do problem 10 on page 91.
3. Suppose the formula  $\psi$  has no free  $x$ . Show that  $\forall x\varphi(x) \rightarrow \psi \vdash \forall x(\varphi(x) \rightarrow \psi)$  is *not* provable.
4. Determine whether the following arguments are valid (and justify your answers).
  - (a) All whales are fish.  
Some mammals are not fish.  
No mammals that are whales are fish.  
Therefore, no mammals are whales.
  - (b) If you serve in the National Guard, you get paid.  
W got paid for serving in the National Guard.  
Therefore, W served in the National Guard.
  - (c) All Greeks are mortal.  
Some Greeks are not women.  
Therefore, some mortals are not women.
  - (d) Romeo loves Juliet.  
Romeo is a Montague.  
Juliet is a Capulet.  
Anyone who loves a Capulet comes to a bad end.  
Therefore, some Montague comes to a bad end.
5. Using just the language of equality  $=$ , give formulas expressing the following conditions on structures (i.e. sets)  $A$ :
  - (a)  $A$  is not empty.
  - (b)  $A$  has at least  $n$  elements (for an arbitrary natural number  $n$ ).
  - (c)  $A$  has at most  $n$  elements (for an arbitrary natural number  $n$ ).

Can you find a sentence that expresses that  $A$  is infinite? Can you find a theory  $\mathbb{T}$  that expresses this?

- ★ 6. Given any language  $\mathcal{L}$  and any  $\mathcal{L}$ -structure  $\mathcal{A}$ , consider the “total theory of  $\mathcal{A}$ ”,

$$\mathbb{T}(\mathcal{A}) = \{\sigma \mid \mathcal{A} \models \sigma\}.$$

Show that  $\mathbb{T}(\mathcal{A})$  is maximally consistent.