Homework 8

Due Thursday, November 2

1. Give natural deduction proofs of the following formulas (using the 4 quantifier rules, and not defining $\exists\varphi$ as $\neg\forall\neg\varphi$!).

   (a) $\neg\exists x\varphi \to \forall x\neg\varphi$

   (b) $\exists x\neg\varphi \to \neg\forall x\varphi$

   (c) $(\exists x\varphi \to \psi) \to \forall x(\varphi \to \psi)$, where x is not free in ψ.

   (d) $(\exists x\varphi \wedge \psi) \to \exists x(\varphi \wedge \psi)$, where x is not free in ψ.

   (e) $(\forall x\varphi \vee \psi) \to \forall x(\varphi \vee \psi)$, where x is not free in ψ.

   Note that in each case, the converse also holds, but you need not show this.

2. Verify that the following syllogism is correct by giving a deduction.

   No Greeks are slaves.
   Some slaves are women.
   Therefore, some women are not Greek.

3. Formalize the following argument and prove that it is correct.

   Romeo loves Juliet.
   Romeo is a Montague.
   Juliet is a Capulet.
   Any Montague who loves a Capulet will come to a bad end.
   Therefore, Romeo will come to a bad end.

⋆ 4. Taking only the introduction and elimination rules for the existential quantifier as given, derive the corresponding rules for the universal quantifier, defined by $\forall = \neg\exists\neg$ (in the way done in Lemma 2.9.1 of van Dalen). Be sure to take account of the side conditions on the rules.