

HOMEWORK 8
Due Thursday, October 29

1. Let L be a language with two unary predicates, A and B . Consider the equivalence

$$\forall x(A(x) \vee B(x)) \leftrightarrow \forall xA(x) \vee \forall xB(x).$$

- (a) Show that one direction is valid. In particular, your answer should make it clear that you know what “valid” means!
- (b) Show that the other direction is not valid.
2. Show that the following equivalences hold.

- (a) If there is no x free in ψ ,

$$\forall x(\varphi \vee \psi) \equiv (\forall x\varphi) \vee \psi.$$

- (b) If there is no x free in ψ ,

$$\exists x(\varphi \wedge \psi) \equiv (\exists x\varphi) \wedge \psi.$$

3. Find a prenex sentence (i.e. one where all the quantifiers occur up front) equivalent to

$$\neg(\exists xA(x) \rightarrow \forall y(B(y) \vee \exists xR(y, x))).$$

4. Fix a language L with one binary relation symbol R . Which of the following statements are true and which are false? Justify your answers.

- (a) If φ is any sentence, either $\models \varphi$ or $\models \neg\varphi$.
- (b) If φ is any sentence and \mathcal{A} is any L -structure, either $\mathcal{A} \models \varphi$ or $\mathcal{A} \models \neg\varphi$.
- (c) If φ is any sentence and Γ is any set of sentences, then either $\Gamma \models \varphi$ or $\Gamma \models \neg\varphi$.
- (d) If φ and ψ are any sentences, $\models \varphi \wedge \psi$ implies $\models \varphi$ and $\models \psi$.
- (e) If φ and ψ are any sentences, $\models \varphi$ and $\models \varphi \rightarrow \psi$ implies $\models \psi$.
- (f) If φ and ψ are any sentences, $\models \varphi \vee \psi$ implies $\models \varphi$ or $\models \psi$.

5. Determine whether the following syllogism is valid (justify your answer).

Some Greeks are not slaves.

No slaves are women.

Therefore, some women are not Greek.

- ★ 6. Do problem 5 on page 72 of van Dalen.