Homework 8

Due Thursday, November 8

1. Fix a language, L, which has two predicate symbols A and B. For each of the following formulas, find an interpretation that satisfies it and one that does not. Justify your answers.

   (a) $\forall x(A(x) \vee B(x))$

   (b) $\forall x A(x) \vee \forall x B(x)$

   (c) $\exists x(A(x) \vee B(x))$

   (d) $\exists x A(x) \vee \exists x B(x)$ and the same for $\wedge$ in place of $\vee$.

2. Let L be a language with two unary predicates, A and B. Consider the equivalence

$$\forall x(A(x) \vee B(x)) \leftrightarrow \forall x A(x) \vee \forall x B(x).$$

   (a) Show that one direction is valid. In particular, your answer should make it clear that you know what "valid" means!

   (b) Show that the other direction is not valid.

3. Show that the following equivalences hold.

   (a) If there is no x free in ψ,

$$\forall x(\varphi \vee \psi) \equiv (\forall x \varphi) \vee \psi.$$

   (b) If there is no x free in ψ,

$$\exists x(\varphi \wedge \psi) \equiv (\exists x \varphi) \wedge \psi.$$

4. Find a prenex sentence (i.e. one where all the quantifiers occur up front) equivalent to

$$\neg\big(\exists x A(x) \to \forall y(B(y) \vee \exists x R(y, x))\big).$$

⋆ 5. Determine whether the following syllogism is valid (justify your answer).

> Some Greeks are not slaves.
> No slaves are women.
> Therefore, some women are not Greek.