Homework 7

Due Thursday, October 22

1. Prove $\neg\exists x\neg\varphi(x) \leftrightarrow \forall x\varphi(x)$.

2. Give natural deduction proofs of the following formulas (using the 4 quantifier rules, and not defining $\exists\varphi$ as $\neg\forall\neg\varphi$!).

   (a) $\neg\exists x\varphi(x) \rightarrow \forall x\neg\varphi(x)$

   (b) $\exists x\neg\varphi(x) \rightarrow \neg\forall x\varphi(x)$

   (c) $(\exists x\varphi \rightarrow \psi) \leftrightarrow \forall x(\varphi \rightarrow \psi)$, where x is not free in ψ.

   (d) $(\exists x\varphi \wedge \psi) \leftrightarrow \exists x(\varphi \wedge \psi)$, where x is not free in ψ.

   (e) $(\forall x\varphi \vee \psi) \leftrightarrow \forall x(\varphi \vee \psi)$, where x is not free in ψ.

3. Verify that the following syllogisms are valid by giving deductions.

   (a) All Greeks are mortal.
   Some Greeks are women.
   Therefore, some women are mortal.

   (b) No Greeks are slaves.
   Some slaves are women.
   Therefore, some women are not Greek.

4. Prove that following argument is correct.

   Romeo loves Juliet.
   Romeo is a Montague.
   Juliet is a Capulet.
   Any Montague who loves a Capulet will come to a bad end.
   Therefore, Romeo will come to a bad end.

5. The language of *monoids* has a constant symbol 1 and a binary function symbol, written $x \cdot y$. The axioms for monoids are associativity, $\forall x, y, z\ (x \cdot (y \cdot z) = (x \cdot y) \cdot z)$, and 1 is a (two-sided) unit, $\forall x\ (1 \cdot x = x)$ and $\forall x\ (x \cdot 1 = x)$.

(a) Use deduction to show that every monoid can be partially ordered by defining:

$$x \leq y =_{\mathsf{df}} \exists z(x \cdot z = y),$$

i.e. show that this relation is reflexive and transitive.

(b) Prove $\forall x, y, z(x \leq y \to z \cdot x \leq z \cdot y)$

⋆ 6. Taking only the introduction and elimination rules for the existential quantifier as given, derive the corresponding rules for the universal quantifier (in the way done in Lemma 2.9.1 of van Dalen). Be sure to take account of the side conditions on the rules.