

HOMEWORK 7

Due Thursday, October 26

1. Consider a first-order language, with relation symbols $<$ and $=$, and constant symbol 0 . Consider the interpretation: the natural numbers, with “less-than” and “equality”, and the distinguished element 0 . Formalize the following statements:
 - (a) “ x is less than or equal to y ”
 - (b) “ 0 is the smallest number”
 - (c) “there is a smallest number”
 - (d) “there is no largest number”
 - (e) “every number has an immediate successor” (in other words, for every number, there is another one that is the “next largest”)
 - (f) “every number is greater than some (other) number”
 - (g) “there is some number that every (other) number is greater than”
 - (h) Add the operations $+$ and \cdot , and the constant 1 , and formalize the following:
 - i. “all square numbers are positive”
 - ii. “there’s just one even prime number”
 - iii. “between every two squares there is always a prime”
2. Consider a first-order language with predicate symbols L, P, M and F . The intended interpretation is all people, living or dead, with $L(x, y)$ meaning “ x loves y ”, $P(x, y)$ meaning “ x is a parent of y ”, $M(x)$ meaning “ x is male”, $F(x)$ meaning “ x is female”. Formalize the following statements:
 - (a) “everyone loves their grandmother”
 - (b) “some fathers love women who are not the mothers of their (the fathers’) children”
 - (c) “not all aunts are loved”
 - (d) “to love and to be loved, that is to be a parent!”