1. Finish the proof of the Model Existence Lemma from the lecture by showing that if $M$ is a maximally consistent set of formulas, then $\varphi \rightarrow \psi \in M$ if and only if $\varphi \in M$ implies $\psi \in M$.

2. Show that if $\Gamma$ is any consistent set, and $\varphi$ is any formula, then either $\Gamma \cup \{\varphi\}$ or $\Gamma \cup \{\neg \varphi\}$ is consistent. (Hint: suppose they are both inconsistent...)

3. In van Dalen, do exercise 9 on p. 45, namely: Consider an infinite set $\{\varphi_1, \varphi_2, \varphi_3, \ldots\}$. If for each valuation $v$ there is an $n$ such that $[\varphi_n]_v = 1$, then there is an $m$ such that $\vdash \varphi_1 \lor \ldots \lor \varphi_m$. (Hint: consider the negations $\neg \varphi_1, \neg \varphi_2, \ldots$, and use compactness.)

4. A formula $\varphi$ is said to be independent of a set of formulas $\Gamma$ if $\Gamma \not\vdash \varphi$ and $\Gamma \not\vdash \neg \varphi$. Suppose $\Gamma$ is a consistent set of formulas, $\varphi$ is independent of $\Gamma$, and $\psi$ is independent of $\Gamma \cup \{\varphi\}$. Show that there are at least three different maximally consistent sets containing $\Gamma$. 

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