

## HOMEWORK 6

Due Thursday, October 12

1. Do problem 1 on page 44-5 of van Dalen.  
Hint: If you claim the set is inconsistent, show it by deriving a contradiction from those assumptions. If you claim the set is consistent, show this by providing a valuation under which all the formulas are true. (Reason from soundness that a set of formulas is consistent if there is such a valuation.)
2. Do problem 3 on page 45.
3. Do problem 6 on page 45.
4. Let  $\Gamma$  be a maximally consistent set of formulas. Show that, for any formulas  $\varphi, \psi$ , we have  $\varphi \vee \psi \in \Gamma$  iff  $\varphi \in \Gamma$  or  $\psi \in \Gamma$ .
- ★ 5. Define an *ultrafilter* of formulas to be a set  $\Gamma$  such that:
  - if  $\varphi \in \Gamma$  and  $\varphi \models \psi$ , then  $\psi \in \Gamma$ ,
  - if  $\varphi \in \Gamma$  and  $\psi \in \Gamma$ , then  $\varphi \wedge \psi \in \Gamma$ ,
  - $\neg\varphi \in \Gamma$  iff  $\varphi \notin \Gamma$

Show that a maximally consistent set of sentences is the same thing as an ultrafilter.