

HOMEWORK 5  
Due Thursday, October 4

1. Show  $\vdash (\varphi \vee \psi) \leftrightarrow (\neg\varphi \rightarrow \psi)$ .
2. Do problem 11 on page 38 of van Dalen.
3. Show that the system of natural deduction is *free from contradiction*, in the sense that there is no propositional formula  $\varphi$  for which one has both  $\vdash \varphi$  and  $\vdash \neg\varphi$ . (Hint: use soundness.)
4. In section 2.5 of van Dalen, do problem 1 (pp. 44-5).  
Hint: If you claim the set is inconsistent, show it by deriving a contradiction from those assumptions. If you claim the set is consistent, show this by providing a valuation under which all the formulas are true (and reason from soundness that a set of formulas is consistent if there is such a valuation).
- ★ 5. Extend the proof of soundness, Lemma 2.5.1 in van Dalen, to cover the full language of propositional logic, by proving that the  $\forall I$  and  $\forall E$  rules are sound.