Homework 4

Due Thursday, September 28

1. Recall that a (positive) rational number is an equivalence class of pairs of natural numbers $\frac{n}{m} = [(n, m)]$, under the equivalence relation $(n, m) \sim (n', m') \iff n \cdot m' = n' \cdot m$.

   (a) Show that the function $inv : \mathbb{Q} \to \mathbb{Q}$ giving the multiplicative inverse of a rational number, defined as $inv(\frac{n}{m}) = (\frac{m}{n})$, is well-defined.

   (b) Show that the attempted definition of a function $add : \mathbb{Q} \to \mathbb{N}$ as $add(\frac{n}{m}) = n + m$ is not well-defined.

2. The *Lindenbaum-Tarski algebra* of propositional logic $LT(Prop)$ is by definition the quotient set

   $$LT(Prop) = (PROP/\equiv)$$

   of the set $PROP$ of propositional formulas, modulo logical equivalence $\varphi \equiv \psi$ (together with a Boolean algebra structure). The elements of $LT(Prop)$ are thus equivalence classes $[\varphi]$, with $[\varphi] = [\psi]$ iff $\varphi \equiv \psi$. The Boolean operations on $LT(Prop)$ are defined by:

   $$\begin{aligned} 1 &= [\top] \\ 0 &= [\bot] \\ \overline{[\varphi]} &= [\neg\varphi] \\ [\varphi] \cdot [\psi] &= [\varphi \wedge \psi] \\ [\varphi] + [\psi] &= [\varphi \vee \psi] \end{aligned}$$

   Prove that this *is* a Boolean algebra. (Hint: you may use anything proved in class, but cite the results used. The main point is to show that the operations are all well-defined.)

3. Do problems 1d,e,f and 3a,b,c on page 37 of van Dalen.

⋆ 4. On page 37, do problem 5.