Homework 4

Due Thursday, October 1

1. Do problems 1d,e,f and 3a,b,c on page 39 of van Dalen.

2. In section 1.6 of van Dalen, do problems 1 and 5 (using the intro and elim rules for disjunction).

3. Extend the proof of soundness, Lemma 1.5.1 in van Dalen, to cover the full language of propositional logic, by proving that the $\vee I$ and $\vee E$ rules (see p. 50) are sound.

4. Do problem 1 on page 47 of van Dalen.
   Hint: If you claim the set is inconsistent, show it by deriving a contradiction from those assumptions. If you claim the set is consistent, show this by providing a valuation under which all the formulas are true. (Reason from soundness that a set of formulas is consistent if there is such a valuation.)

5. Show that the system of natural deduction is *free from contradiction*, in the sense that both $\vdash \varphi$ and $\vdash \neg\varphi$ cannot be proved, for any propositional formula φ. (Hint: use soundness.)

6. Do problem 6 on page 48 of van Dalen.
   (Note: what is to be shown is that if Γ is a consistent set of formulas such that for every formula φ, either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$, then Γ is maximally consistent.)

⋆ 7. On page 39, do problem 5.