1. Recall that a (positive) rational number is an equivalence class of pairs of natural numbers $\frac{n}{m} = [(n, m)]$, under the equivalence relation $(n, m) \sim (n', m') \iff n \cdot m' = n' \cdot m$.

(a) Show that the function $\text{inv} : \mathbb{Q} \to \mathbb{Q}$ giving the multiplicative inverse of a rational number, defined as $\text{inv}(\frac{n}{m}) = (\frac{m}{n})$, is well-defined.

(b) Show that the attempted definition of a function $\text{add} : \mathbb{Q} \to \mathbb{Q}$ as $\text{add}(\frac{n}{m}) = n + m$ is not well-defined.

2. The Lindenbaum-Tarski algebra of propositional logic $LT(\text{Prop})$ is by definition the quotient set $LT(\text{Prop}) = (\text{PROP}/\equiv)$ of the set $\text{PROP}$ of propositional formulas, modulo logical equivalence $\varphi \equiv \psi$ (together with a Boolean algebra structure). The elements of $LT(\text{Prop})$ are thus equivalence classes $[\varphi]$, with $[\varphi] = [\psi]$ iff $\varphi \equiv \psi$.

The Boolean operations on $LT(\text{Prop})$ are defined by:

\[
\begin{align*}
1 &= [\top] \\
0 &= [\bot] \\
[\neg \varphi] &= \overline{[\varphi]} \\
[\varphi] \cdot [\psi] &= [\varphi \land \psi] \\
[\varphi] + [\psi] &= [\varphi \lor \psi]
\end{align*}
\]

Prove that this is a Boolean algebra. (Hint: you may use anything proved in class, but cite the results used. The main point is to show that operations are all well-defined.)

3. Do problems 1d,e,f and 3a,b,c on page 37 of van Dalen.

* 4. On page 37, do problem 5.