1. Do problem 1 on page 27 of van Dalen.

2. Determine conjunctive and disjunctive normal forms for the following formulas:

   \neg(p \leftrightarrow q), \quad ((p \rightarrow q) \rightarrow p) \rightarrow p

   Use these normal forms to determine whether each formula is a tautology.

3. (a) Show that all of the truth functions (on \{0, 1\}) can be defined in terms of \{\rightarrow, \bot\}, i.e. that this is a functionally complete set of connectives.

   (b) Show that \{\rightarrow, \lor, \land\} is not a functionally complete set of connectives.

   (c) Conclude that \{\rightarrow, \lor, \land, \leftrightarrow, \top\} is not a functionally complete set of connectives. (Hint: define the last two in terms of the others.)

4. Prove the following proposition:

   Let \( X \) be a set with an equivalence relation, written \( x \sim y \). Suppose given a set \( A \), and a function \( f : X \rightarrow A \) that respects \( \sim \), in the sense that \( f(x) = f(y) \) whenever \( x \sim y \). Then there is a function

   \[ \overline{f} : X/\sim \rightarrow A \]

   such that for all \( x \in X \),

   \[ \overline{f}([x]) = f(x). \]

   Where the quotient set \( X/\sim = \{[x] \mid x \in X\} \) is the set of all equivalence classes \([x] = \{y \mid x \sim y\}\).

4. Use the foregoing to show that one can define the multiplication of a (positive) rational number \( \frac{n}{m} \) by a natural number \( k \) by the usual formula,

   \[ k \cdot \frac{n}{m} = \frac{k \cdot n}{m}. \]

(Recall that a (positive) rational number is an equivalence class of pairs of natural numbers \( \frac{n}{m} = [(n, m)] \), under the equivalence relation \((n, m) \sim (n', m') \iff n \cdot m' = n' \cdot m\).)