Homework 3
Due Thursday, September 21

1. Do problem 1 on page 27 of van Dalen.

2. Using algebraic means, determine conjunctive and disjunctive normal forms for the following formulas:
   \[-(p \leftrightarrow q), \quad ((p \rightarrow q) \rightarrow p) \rightarrow p\]
   Use these normal forms to determine whether each formula is a tautology.

3. (a) Show that all of the truth functions (on \{0, 1\}) can be defined in terms of \{\rightarrow, \bot\}, i.e. that this is a functionally complete set of connectives.
(b) Show that \{\rightarrow, \lor, \land\} is not a functionally complete set of connectives.
(c) Conclude that \{\rightarrow, \lor, \land, \leftrightarrow, \top\} is not a functionally complete set of connectives. (Hint: define the last two in terms of the others.)

4. Let \(X\) be a set with an equivalence relation, written \(x \sim y\). Suppose given a set \(A\), and a function \(f : X \rightarrow A\) that respects \(\sim\), in the sense that \(f(x) = f(y)\) whenever \(x \sim y\). Show that there is a function
   \[\overline{f} : X/\sim \rightarrow A\]
such that for all \(x \in X\),
   \[\overline{f}([x]) = f(x),\]
where the quotient set \(X/\sim = \{[x] \mid x \in X\}\) is the set of all equivalence classes \([x] = \{y \mid x \sim y\}\).

5. Use the foregoing to show that one can define the multiplication of a (positive) rational number \(\frac{n}{m}\) by a natural number \(k\) by the usual formula,
   \[k \cdot \frac{n}{m} = \frac{k \cdot n}{m}.\]
(Recall that a (positive) rational number is an equivalence class of pairs of natural numbers \(\frac{n}{m} = [(n, m)]\), under the equivalence relation \((n, m) \sim (n', m') \iff n \cdot m' = n' \cdot m\).)