Undergraduates should do only the unstarred problems. Graduate students should also do the starred problem.

1. Do problems 1 and 2 on page 14 of van Dalen.

2. A binary truth function is any function of the form:

   \[ f : \{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\} \]

   Note that a binary truth function is defined uniquely by its truth table.

   (a) How many different binary truth functions are there?

   (b) Define \( \oplus \) (exclusive or), \( \mid \) (nand, the sheffer stroke), and \( \downarrow \) (nor), where \( p \mid q \) is “not both \( p \) and \( q \)”, and \( p \downarrow q \) is “neither \( p \) nor \( q \)”. Give the definitions by truth tables.

   (c) Can the truth functions from part (b) be defined just in terms of \( \lor \) and \( \neg \)? (proof!)

   (d) Can they be defined just in terms of \( \lor \) and \( \land \)? (proof!)

3. In van Dalen, do problem 1a and problems 2, 3, and 6 on page 20.

4. Use semantic arguments (rather than truth tables) to prove each of the following:

   (a) \( \phi \lor \psi \models \neg \psi \rightarrow \phi \)

   (b) \( \{\phi \rightarrow \psi, \neg \psi\} \models \neg \phi \)

   (c) it’s not the case that \( \{\phi \rightarrow \psi, \neg \phi\} \models \neg \psi \)

   (d) it’s not the case that \( \{p \land \neg q, \neg r, p \lor \neg s\} \models \neg q \rightarrow r \lor s. \)