

HOMEWORK 2

Due Thursday, September 11

1. Do problems 1 and 2 on page 14 of van Dalen.
2. A *binary truth function* is any function of the form:

$$f : \{0, 1\} \times \{0, 1\} \longrightarrow \{0, 1\}$$

Note that a binary truth function is defined uniquely by its truth table.

- (a) How many different binary truth functions are there?
 - (b) Define \oplus (exclusive or), $|$ (nand, the sheffer stroke), and \downarrow (nor), where $p | q$ is “not both p and q ”, and $p \downarrow q$ is “neither p nor q ”. Give the definitions by truth tables.
 - (c) Can the truth functions from part (b) be defined just in terms of \vee and \neg ? (proof!)
 - (d) Can they be defined just in terms of \vee and \wedge ? (proof!)
3. In van Dalen, do problem 1a on page 20, and problems 2, 3, and 6 on page 21.
 4. (a) Let $x \equiv y$ be an equivalence relation on a set X . For any element $x \in X$, let $[x]$ denote its equivalence class, defined by

$$[x] = \{a \in X \mid a \equiv x\}.$$

Show that for any $a, b \in X$, one has $a \equiv b$ iff $[a] = [b]$.

- (b) Conclude from problems 2 and 6 on page 21 of van Dalen that the logical equivalence classes of propositional formulas are partially ordered by entailment.
5. Use semantic arguments (rather than truth tables) to prove each of the following:
 - (a) $\varphi \vee \psi \models \neg\psi \rightarrow \varphi$
 - (b) $\{\varphi \rightarrow \psi, \neg\psi\} \models \neg\varphi$
 - (c) it's not the case that $\{\varphi \rightarrow \psi, \neg\varphi\} \models \neg\psi$
 - (d) it's not the case that $\{p \wedge \neg q, \neg r, p \vee \neg s\} \models \neg q \rightarrow r \vee s$.
 - ★ 6. In van Dalen, do problem 5 on page 21.