

HOMEWORK 3

Due Thursday, September 21

1. Do problems 1 and 2 on page 14 of van Dalen.
2. A *binary truth function* is any function of the form:

$$f : \{0, 1\} \times \{0, 1\} \longrightarrow \{0, 1\}$$

Note that a binary truth function is defined uniquely by its truth table.

- (a) How many different binary truth functions are there? How many don't depend on any of their arguments? How many depend only on one of their two arguments?
 - (b) We've seen several binary truth functions that depend on both arguments, namely those corresponding to the connectives \wedge , \vee , \rightarrow . Define \oplus (exclusive or), \mid (nand, the sheffer stroke), and \downarrow (nor), where $p \mid q$ is "not both p and q ", and $p \downarrow q$ is "neither p nor q ". Give the definitions by truth tables.
3. In van Dalen, do problem 1a on page 20; problems 2, 3, and 6 on page 21; and problem 1 on page 27.
 4. (a) Let $x \equiv y$ be an equivalence relation on a set X . For any element $x \in X$, let $[x]$ denote its equivalence class, defined by

$$[x] = \{a \in X \mid a \equiv x\}.$$

Show that for any $a, b \in X$, one has $a \equiv b$ iff $[a] = [b]$.

- (b) Conclude from problems 2a and 2b on page 21 of van Dalen that the equivalence classes of propositional formulas are partially ordered.
- ★ 5. Consider equivalence modulo 5 on the natural numbers: $m \equiv n$ iff $m - n = 5z$ for some integer z .

- Define addition \oplus on equivalence classes by:

$$[m] \oplus [n] = [m + n].$$

Show that this operation is well defined: if $m \equiv m'$ and $n \equiv n'$ then $m + n \equiv m' + n'$.

- Try to define exponentiation \uparrow on equivalence classes by:

$$[m] \uparrow [n] = [m^n].$$

Show that exponentiation is *not* well-defined.