

HOMEWORK 10
Due Thursday, November 16

1. Find a prenex sentence (i.e. one where all the quantifiers occur up front) equivalent to

$$(\exists x A(x) \wedge y = t) \rightarrow \forall y (B(y) \vee \exists x R(y, x)).$$

2. Suppose the formula ψ has no free x . Show that

$$\forall x \varphi(x) \rightarrow \psi \vdash \forall x (\varphi(x) \rightarrow \psi)$$

is *not* in general provable (exhibit specific formulas for which it fails).

3. Show that the following formulas are not valid by exhibiting counter-models:

(a) $\forall x (P(x) \rightarrow \exists y R(x, y))$

(b) $\exists x \forall y (R(f(x), y))$

(c) $P(a) \vee \forall x (R(x, a) \rightarrow \exists y (P(x) \wedge R(x, y)))$

Show that each of these formulas cannot be proved.

- ★ 4. Using just the language of equality $=$, give formulas expressing the following conditions on structures (i.e. sets) A :

(a) A is not empty.

(b) A has at least n elements (for an arbitrary natural number n).

(c) A has at most n elements (for an arbitrary natural number n).