1. Determine whether the following syllogism is valid (and justify your answer).

Some Greeks are not slaves.
No slaves are women.
Therefore, some women are not Greek.

2. Using just the language of equality $=,$ give formulas expressing the following conditions on structures (i.e. sets) $A$:

(a) $A$ is not empty.
(b) $A$ has at least $n$ elements (for an arbitrary natural number $n$).
(c) $A$ has at most $n$ elements (for an arbitrary natural number $n$).
(d) Can you find a sentence that expresses that $A$ is infinite?
(e) Can you find a theory $T$ (a set of sentences) that expresses this?

3. Consider the natural numbers as a monoid $N = (\mathbb{N}, +^N, 0^N)$ in the obvious way. Show that there is a monoid $P = (P, +^P, 0^P)$ satisfying all the same first-order sentences as $N$, but having a larger cardinality $\#P > \#\mathbb{N}$. Is there a model having a smaller cardinality?

4. The language of linear orders with endpoints has two constant symbols 0, 1 and a binary relation symbol, written $x \leq y$. The axioms for linear orders with endpoints are:

reflexivity: $\forall x \ (x \leq x),$
transitivity: $\forall x, y, z \ ((x \leq y \land y \leq z) \rightarrow x \leq z),$
antisymmetry: $\forall x, y \ ((x \leq y \land y \leq x) \rightarrow x = y),$
linearity: $\forall x, y \ (x \leq y) \lor (y \leq x),$
endpoints: $\forall x \ (0 \leq x) \land (x \leq 1).$
Consider the following models of the theory of linear orders with endpoints:

\[ \mathcal{I} = ([0, 1] \subseteq \mathbb{R}, 0, 1, \leq) \]

(the usual ordering of the unit interval in the reals)

\[ \mathcal{N} = (\mathbb{N} \cup \{\infty\}, 0, \infty, \leq) \]

(the usual ordering of the natural numbers,
  but with a new element \( \infty \) added at infinity)

\[ \mathcal{Z} = (\mathbb{Z} \cup \{-\infty, +\infty\}, -\infty, +\infty, \leq) \]

(the usual ordering of the integers,
  but with new elements \(-\infty, +\infty\) added at both ends)

(a) Show that these models are distinguishable in first-order logic by producing sentences that are satisfied by each, but not the others.

(b) Can there be a model that satisfies all the same first-order sentences as \( \mathcal{N} \) and is uncountable? Justify your answer!

(c) Can there be a model that satisfies all the same first-order sentences as \( \mathcal{I} \) and is countable? Justify your answer!

\( \star \) (d) (for Grad Students) Give an explicit model that has cardinality less than that of \( \mathcal{I} \) but still satisfies the sentence that you found for \( \mathcal{I} \) in part (a).