

HOMEWORK 1  
Due Thursday 3 September

Undergraduates are to do only the unstarred problems. Graduate students should also do the starred problem.

1. Use the least number principle to show that every natural number greater than 1 can be factored into primes.
2. Show that for inductively defined sets,  $C_* \subseteq C^*$  where the latter is the intersection of all inductive sets and the former is the set of elements having a construction sequence.
3. Define the set of “babble-strings” inductively, as follows:
  - “ba” is a babble-string
  - if  $s$  is a babble-string, so is “ab” $\hat{\ }s$
  - if  $s$  and  $t$  are babble-strings, so is  $s\hat{\ }t$

Prove by induction that every babble-string has the same number of  $a$ 's and  $b$ 's, and that every babble-string ends with an “a”.

4. Referring to the previous problem, show that the set  $B$  of babble-strings is not freely generated. Give a different specification of  $B$  such that  $B$  is freely generated. Use that specification to define a length function  $f : B \rightarrow \mathbb{N}$  giving the number of letters in the string.
- ★ 5. The set of (unlabelled, binary) trees is defined inductively as follows:
  - $*$  is a tree
  - if  $s, t$  are trees, so is  $[s, t]$

Define functions on trees that count the height of a tree and the width of a tree, as well the number of subtrees. Show that the number of subtrees is always at least as great as the width of the tree.