Homework #9

Due Wednesday, October 31

1. Read through section 2.6 in van Dalen.

⋆ 2. (Repeated from last assignment.) Do problem 4 on page 68. In each case, just indicate whether the term is "free" or "not free" for the specified variable in the specified term, and carry out the substitution either way.

⋆ 3. Do problem 1 on page 72. Use the symbols P, T, S, and c to denote addition, multiplication, successor ("+1"), and 0 respectively.

4. Do problems 2 and 3 on page 72.

○ 5. Prove unique readability for terms and formulas in a given first-order language (and/or write a parser).

⋆ 6. Consider the equivalence

$$\exists x\,(\varphi(x) \wedge \psi(x)) \leftrightarrow (\exists x\,\varphi(x) \wedge \exists x\,\psi(x)).$$

   a. Show that one direction of this equivalence is valid (i.e. true in every structure). Prove this carefully; you can use Lemma 2.4.5.

   b. Find examples of φ and ψ where the other direction is not valid (and justify this claim).

⋆ 7. Fix a language, L, which has one binary relation symbol, R. Which of the following statements are true and which are false? Justify your answers.

   a. If φ is any sentence, either $\models \varphi$ or $\models \neg\varphi$.

   b. If φ is any sentence and $\mathfrak{A}$ is any structure, either $\mathfrak{A} \models \varphi$ or $\mathfrak{A} \models \neg\varphi$.

   c. If φ is any sentence and Γ is any set of sentences, then either $\Gamma \models \varphi$ or $\Gamma \models \neg\varphi$.

8. Do problems 6 and 7 on page 72.