Homework #3

Due Wednesday, September 12

1. Read through section 1.2 of van Dalen.

$\star$ 2. Write down explicit definitions of the functions f and g, where

   a. f is defined recursively by $f(0) = 0$, $f(n+1) = 3 + f(n)$, and

   b. g is defined recursively by $g(0) = 1$, $g(n+1) = (n+1)^2 g(n)$. (Hint: use "factorial" notation: $m! = 1 \times 2 \times \ldots \times m$.)

3. Write down an explicit definition of the function h, where $h(0) = 0$ and $h(n+1) = 3 \cdot h(n) + 1$. (Hint: compare to the sequence $1, 3, 9, 27, 81, \ldots$)

$\star$ 4. Suppose g is a function from $\mathbb{N}$ to $\mathbb{N}$. Write down a recursive definition of the function $f(n)$, defined by $f(n) = \sum_{i=0}^{n} g(i)$.

5. Do problem 1 on page 30 of the Enderton handout.

$\circ$ 6. Do problem 3 on page 30 of the Enderton handout.

$\star$ 7. Suppose, as Section 2.2 of the notes, we are given a set U, a subset $B \subseteq U$, and some functions $f_1, \ldots, f_k$. Say a set is *inductive* if it contains B and is closed under the f's, and let C^* be the intersection of all the inductive subsets of U. Show C^* is inductive.

$\star$ 8. Define the set of "babble-strings" inductively, as follows:

   - "ba" is a babble-string
   - if s is a babble-string, so is "ab"$\hat{\ }s$
   - if s and t are babble-strings, so is $s\hat{\ }t$

   Prove by induction that every babble-string has the same number of a's and b's, and that every babble-string ends with an "a". Is the set of babble-strings freely generated?