Combinations and permutations

Formula: Given a sequence of k choices to make, with n_1 choices for the first, n_2 choices for the second, and so on, the number of combined choices is $n_1 n_2 \dots n_k$.

Example: A Denny's breakfast special with 3 choices of entree, 3 choices of bread, and 2 choices of side yields 18 possible breakfasts.

Formula: The number of ways of choosing k objects from n, with repetitions, where the order counts, is n^k.

Example: Suppose we can choose three scoops of Bakin Robbins' ice cream, with a total of 31 flavors. Then $31 \times 31 \times 31$ cones are possible.

Formula: The number of ways of choosing k objects from n, without repetitions, where the order counts is

$$\frac{n!}{(n-k)!} = n(n-1)\cdots n-(k-1).$$

This is called the number of *permutations* of k objects from n.

Example: If repetitions are not allowed in the previous example, there are $31 \times 30 \times 29$ possibilities.

Formula: The number of ways of choosing k objects from n, without repetitions, where the order doesn't count, is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots n-(k-1)}{k(k-1)\cdots 1}$$

This is called the number of *combinations* of k objects from n, and happens to be the kth entry of the nth row of Pascal's triangle (assuming you start counting with 0).

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
\end{array}$$

Example: If order does not count in the preceeding examples, the number of possible 3-scoop choices is $\frac{31\times30\times29}{3\times2\times1}$

The binomial theorem:

$$\begin{aligned}
(x+y)^n &= \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i} \\
&= x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \dots \binom{n}{n-1} x y^{n-1} + y^n
\end{aligned}$$