Econ 73-250A-F
Spring 2001
Prof. Daniele Coen-Pirani

Class #25 - Review Exercise

(a) The supply firm of each firm is given by the portion of the MC curve that lies above the AC curve. The MC curve is $MC(y) = 8y$,and a firm that produces chooses a quantity y such that

$$8y = p.$$

In order to make non-negative profits the price that a typical firm receives must be such that

$$\Pi(p) = p\left(\frac{p}{8}\right) - 4\left(\frac{p}{8}\right)^2 - 16 \geq 0.$$

This implies

$$\frac{p^2}{16} - 16 \geq 0 \Rightarrow p \geq 16.$$

Thus a firm's supply curve is

$$y = \begin{cases} \frac{p}{8} \text{ if } p > 16 \\ 0 \text{ if } p \leq 16 \end{cases}.$$

Since there are N firms in the industry, aggregate supply is

$$Q_S(p) = \begin{cases} N\left(\frac{p}{8}\right) \text{ if } p > 16 \\ 0 \text{ if } p \leq 16 \end{cases}.$$

(b) In equilibrium, each firm will produce at the minimum of its AC curve:

$$AC(y) = \frac{4y^2 + 16}{y}.$$

The minimum is the point at which $AC(y) = MC(y)$:

$$\frac{4y^2 + 16}{y} = 8y \Rightarrow 16 = 4y^2 \Rightarrow y^* = 2.$$

At $y^* = 2$, $AC(y^*) = MC(y^*) = 16$. Since each firm sets price equal to marginal cost, it must be the case that $p^* = 16$ in a long-run equilibrium. The equilibrium number of firms is found by setting demand equal to supply at $p^* = 16$:

$$200 - 2(16) = 2N \Rightarrow N^* = 84.$$

Aggregate output is therefore $2(84) = 168$. A typical firm makes zero profits. Consumers' surplus is given by the area of the triangle above $p^* = 16$ and below the inverse demand curve

$$p = 100 - \frac{Q_D}{2}.$$

Computing it yields: $(100 - 16)\,168/2 = 7056$.

(c) When $N = 84$, industry supply is equal to $84(p/8) = 10.5p$. Under the subsidy, consumers pay $0.5p$ for a unit of the good, where p is the price received by firms. Therfore under the subsidy the industry demand curve is

$$200 - 2\,(0.5p) = 200 - p.$$

Set industry supply equal to industry demand and solve for the equilibrium price p^{**}:

$$10.5p = 200 - p,$$

which implies that $p^{**} = 200/11.5 \approx 17.39$. Consumers pay the price

$$p^{**}\,(0.5) \approx 8.70.$$

Each firm produces $p^{**}/8 \approx 2.17$ units of output. Each firm earns positive profits:

$$p^{**}y - c(y) \approx 2.90.$$

(d) Once again, in a long-run equilibrium, all firms produce at the minimum of their AC curves, i.e., where $y^* = 2$. The price is therefore going to be equal to

$$p^* = MC(2) = 16.$$

When $p = 16$, consumers pay 8 for each unit of the good. Total industry demand is therefore $200 - 2\,(8) = 184$. Since each firm produces 2 units of output, the long-run equilibrium number of firms is $184/2 = 92$. The subsidy leads to the entry of $92 - 84 = 8$ new firms into the industry.

(e) A monopolist would optimally produce at a point where $MC(y) = MR(y)$. The $MR(y)$ curve is

$$\frac{d\left[\left(100 - \frac{y}{2}\right)y\right]}{d} = 100 - y.$$

Therefore $100 - y = 8y$ leads to $y_M = 100/9 \approx 11.11$, i.e., the quantity produced by the monopolist. The price charged by the monopolist is

$$\begin{aligned} p_M &= 100 - \frac{y_M}{2} \\ &= 100\left(\frac{17}{18}\right) \approx 94. \end{aligned}$$

The monopolist's profits are

$$\begin{aligned} \Pi_M &= 100\left(\frac{17}{18}\right)\frac{100}{9} - 4\left(\frac{100}{9}\right)^2 - 16 \\ &= \left(\frac{100}{9}\right)^2\frac{9}{2} - 16 \approx 539. \end{aligned}$$

Consumer's surplus is the area of the triangle below the demand curve and above p_M

$$\frac{1}{2}\frac{100}{9}\left(100 - 100\left(\frac{17}{18}\right)\right) = \frac{50}{9}\left(\frac{100}{18}\right) \approx 30.86.$$

(f) The monopolist now faces the (inverse) demand curve

$$p = 200 - y.$$

The $MR = MC$ condition is

$$200 - 2y = 8y,$$

which yields $y_M = 20$. The price is $200 - 20 = 180$. The price paid by consumers is $180/2 = 90$. Therefore quantity produced increases, and price received by monopolist increases.