

## 73-250 - Intermediate Microeconomics

### Recitation #7 - March 23, 2001

**Exercise #1.** Consider a small price-taking firm which produces a single output using two inputs according to the following production function:

$$y = 0.5K^{\frac{1}{2}}L^{\frac{1}{4}},$$

where  $y$  is the firm's yearly output,  $K$  is machinery (measured in machine-hours), and  $L$  is labor supply (measured in person-hours). Let  $w_k$  be the rental rate of one machine-hour and let  $w_l$  be the wage rate (i.e. the cost of one person-hour). Suppose that  $w_k = 32$  and  $w_l = 1$ .

(a) Suppose that the firm owns its plant and that the rental market is competitive, so that the user cost of capital is equal to the rental rate of capital  $w_k$ . The firm's machinery can run for 10,000 hours in a year. The firm has bought the machine several years ago and plans to complete amortizing its cost within a year. In that year the machine represents a fixed cost for the firm. Suppose that the firm can freely adjust its labor input (i.e., labor is not a fixed factor). Determine the firm's short run cost function  $c_s(y)$ .

(b) Using your answer from part (a), derive an expression for the firm's short run average cost function  $AC_s(y)$  and the firm's short run marginal cost function  $MC_s(y)$ . On a diagram plot the short run average and marginal cost functions.

(c) Consider the problem of the firm after the year has passed and the machine has been fully amortized. Assuming that the firm can now freely choose the values of both of its inputs, derive an algebraic expression for the firm's long-run cost function  $c_l(y)$ .

(d) Using your answer from part (c), derive an expression for the firm's long-run average cost function  $AC_l(y)$  and the firm's long run marginal cost function  $MC_l(y)$ . On a diagram plot the long run average and marginal cost functions.