73-250 - Intermediate Microeconomics
Recitation #4 - February 9, 2001

Exercise #1. Tom's preferences over consumption bundles of the form (x,y) are represented by the utility function

$$u(x,y) = xy,$$

where x is the quantity of good x that Tom consumes and y is the quantity of good y that he consumes.

(a) Suppose that Tom's income is $m = 120$ and that the prices of the two goods are both equal to 1. What is Tom's optimal consumption bundle?

(b) Suppose now that the government imposes a tax of \$1 on each unit of good y purchased, thereby raising the price of good y from 1 to 2. What is Tom's optimal consumption bundle in this case? How much tax revenue does the government collect?

(c) Suppose that rather than tax the consumption of good y, the government instead imposes a lump-sum income tax of \$30, thereby reducing Tom's income to 90 but leaving prices unchanged from part (a). What is Tom's optimal consumption bundle in this case? Is Tom better off than he was in part (b)?

(d) Illustrate your answers to parts (b) and (c) in a diagram showing the budget lines and the optimal consumption bundles under the two different taxation schemes.

Exercise #2. Bill's preferences over consumption bundles of the form (x,y) are represented by the utility function

$$u(x,y) = \log(x) + y,$$

where x is the quantity of good x that Bill consumes and y is the quantity of good y that he consumes. Let Bill's income be m and let the prices of the two goods be p_x and p_y respectively. Suppose that

$$\frac{m}{p_y} > 1.$$

(a) Derive Bill's demand functions for x and y. What does the assumption above guarantee?

(b) Is good x an ordinary good or a Giffen good? Explain.

(c) Is good x a normal good or an inferior good? Explain.

(d) Graph the Engel curve for good y for $m > p_y$. What is the slope of the Engel curve?

(e) Is good x a substitute for good y?

(f) Is good y a substitute for good x?

(g) Is good y a luxury good or a necessary good?

Suppose now that

$$m \leq p_y.$$

(h) On a clearly-labelled graph with good x on the x-axis and good y on the y-axis, draw some of Bill's indifference curves. On the same graph also draw a budget line. On this graph mark Bill's demanded bundle.

Consider now a comparison between a quantity tax on good x and an income tax that raises the same amount of revenues as the quantity tax. Let t denote the tax per-unit of good x.

(i) Compute Bill's demanded bundle when good x is taxed, under the assumption $m \leq p_y$. On the same graph of part (h) draw the budget line with the quantity tax and Bill's demanded bundle. Compute the revenue collected by the government, denoting it by R^*.

(j) Suppose now that the government taxes Bill's income instead of his consumption of good x. To compare the quantity tax with the income tax, suppose that the amount of the income tax is R^*. Bill's after tax income is therefore $m - R^*$. Compute Bill's demanded bundle in this case. On the same graph of part (k) draw the budget line with the income tax and Bill's demanded bundle.

(k) Is Bill better-off under the quantity tax or the income tax? Why is this conclusion different from the one we have seen in class? [Reminder: in class we have seen a case where the consumer is strictly better-off under the income tax than under the quantity tax.]