73-250 - Intermediate Microeconomics

Recitation #3 - February 2, 2001

Exercise #1. Let Julia's preferences over consumption bundles (x_1,x_2) be represented by the utility function

$$u(x_1,x_2) = -\frac{4}{x_1} - \frac{1}{x_2}.$$

In addition, suppose that $m = 240$, $p_1 = 8$, and $p_2 = 2$, where m is Julia's income, p_1 is the price of good 1, and p_2 is the price of good 2.

(a) Calculate Julia's marginal rate of substitution (MRS) of good 2 for good 1.

(b) Use the MRS computed from part (a) to show that Julia's preferences are strictly convex. [Hint: show that the absolute value of the MRS diminishes with x_1.]

(c) Calculate Julia's optimal choices for x_1 and x_2. Locate the optimal consumption bundle in a carefully labelled diagram showing Julia's budget line and indifference curves.

(d) Suppose now that the price of good 2 increases to 8. Calculate Julia's optimal choices for x_1 and x_2. Again, locate the optimal consumption bundle in a carefully labelled diagram showing Julia's budget line and indifference curves. Is Julia better or worse off than she was in part (c)?

Exercise #2. This is a continuation of exercise #1 of the 01/19/01 recitation. Consider an individual, call him Tom, who derives utility from two goods. Good 1 is the number of hours of leisure that Tom "consumes" in a given week. Good 2 is a basket of commodities whose value is measured in dollars. Tom earns income by working at the wage of 10$ per hour. (Tom has no other source of income.) He spends his income on good 2. Since the number of hours in a week is fixed at $24 \times 7 = 168$, the number of hours that he spends working plus the number of hours that he spends in leisure must equal 168 in any given week. Let x_1 and x_2 denote, respectively, the amount of good 1 and the amount of good 2 that the Tom consumes in a given week. Note that since good 2 is measured in dollars, the dollar price p_2 of a unit of good 2 is 1. Recall that the equation of the budget line for this problem reads

$$10x_1 + x_2 = 1680.$$

Tom's preferences over leisure and consumption of good 2 are represented by the Cobb-Douglas utility function

$$u(x_1,x_2) = x_1^a x_2,$$

where a is a positive constant.

Tom's goal is to choose the number of weekly hours of leisure and consumption of good 2 that maximizes his utility subject to his budget constraint.

(a) Calculate Tom's marginal rate of substitution between leisure and consumption.

(b) Use your answer from part (a) to determine Tom's optimal choices for consumption and leisure as a function of the parameter a.

(c) Let $a = 3$. How many hours does Tom choose to work? How much does he consume (measured in dollars)? How does Tom's optimal consumption-leisure bundle change if $a = 11$?

(d) Now suppose that the government implements the following incentive scheme: the government will give Tom \$100 in cash provided that he works at least 20 hours per week. What is Tom's optimal consumption-leisure bundle if a = 3? Does the incentive scheme induce Tom to work more or less than he did in its absence? Illustrate your answer with a clearly labelled diagram.

(e) Now suppose that a = 11. Under the government incentive scheme, what is Tom's optimal consumption-leisure bundle? Does the incentive scheme induce Tom to work more or less than he did in its absence? Illustrate your answer with a clearly labelled diagram.

(f) In light of your findings in parts (e) and (f), what conclusions can you draw about the effectiveness of the government incentive scheme in enticing people to work more hours?