Econ 73-250A-F
Spring 2001
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Suggested Answers to Practice Exam #3

Part III

Exercise #1. The monopolist maximizes

$$\Pi = p(y)\,y - c(y)$$

with respect to y. The optimal choice of output is such that

$$MR(y) = MC(y),$$

where

$$\begin{aligned} MR(y) &= \frac{d\left[p(y)\,y\right]}{dy} = 12 - 2y \\ MC(y) &= 2y. \end{aligned}$$

Therefore, optimal output is such that

$$12 - 2y = 2y,$$

or $y^* = 3$. The price charged by a monopolist is $p(3) = 12 - 3 = 9$.

Exercise #2. (a) If admission to the park were free and you had to choose the profit maximizing price of a ride p^*, which value would you set?

The aggregate demand curve for rides is

$$x = 1{,}000\,(50 - 50p).$$

The inverse aggregate demand curve is

$$p = 1 - \frac{x}{50{,}000}.$$

As a monopolist you would sell x^* rides, where x^* solves

$$MC(x^*) = MR(x^*).$$

The marginal cost of a ride is 0. The marginal revenue is

$$\frac{d\left[x - \frac{x^2}{50{,}000}\right]}{d} = 1 - \frac{x}{25{,}000}.$$

Thus, at an optimum

$$1 - \frac{x}{25{,}000} = 0,$$

or $x^* = 25{,}000$. The price of each ride is

$$\begin{aligned} p &= 1 - \frac{x^*}{50{,}000} \\ &= 0.5. \end{aligned}$$

Each person takes $50 - 50\,(0.5) = 25$ rides. Total profits for the monopolist are $0.5\,(25)\,(1{,}000) = 12{,}500$.

(b) As seen in class, when the monopolist is allowed to charge a two-part tariff, is going to sell each ride at its marginal cost, i.e., zero. It is also going to charge an entry fee to a consumer equal to its consumer surplus. The consumer surplus is represented by the area ABC in Figure 1. To compute it, notice that the inverse supply curve for each consumer is

$$p = 1 - \frac{x}{50}.$$

A consumer's surplus is equal to $(1)\,50/2 = 25$. The monopolist will charge an entry fee of 25. Total profits for the monopolist are $25\,(1000) = 25{,}000$.

Exercise #3. (a) First, you need to find the aggregate demand curve:

$$q = \begin{cases} 50{,}000 - 2{,}000p \text{ if } p \geq 20, \\ 60{,}000 - 2{,}500p \text{ if } p \leq 20. \end{cases}$$

Then, you find the inverse aggregate demand curve:

$$p = \begin{cases} 25 - \frac{q}{2{,}000} \text{ if } q \leq 10{,}000 \\ 24 - \frac{q}{2{,}500} \text{ if } q > 10{,}000 \end{cases}.$$

The marginal revenue curve is

$$MR(q) = \begin{cases} 25 - \frac{q}{1{,}000} \text{ if } q \leq 10{,}000 \\ 24 - \frac{q}{1{,}250} \text{ if } q > 10{,}000 \end{cases}.$$

In principle, marginal revenue equal to marginal cost at two levels of q:

$$\begin{aligned} 25 - \frac{q}{1{,}000} &= 10 \text{ if } q \leq 10{,}000 \\ 24 - \frac{q}{1{,}250} &= 10 \text{ if } q > 10{,}000. \end{aligned}$$

It is easy to check that the first equation yields $q_1^* = 15{,}000 > 10{,}000$. Therefore, there is no intersection between MR and MC in the first segment of the marginal revenue curve. The second equation yields $q_2^* = 17{,}500 > 10{,}000$. Since this is the only intersection between MR and MC, the firm will choose this level of production. The price associated with this quantity is $24 - q_2^*/2{,}500 = 17$. Profits associated with this pricing policy are $\Pi_2 = (17 - 10)\,35{,}000 = 245{,}000$.

(b) First, find the inverse demand curves

$$\begin{aligned} p_{US} &= 25 - \frac{q_{US}}{2{,}000} \\ p_{UK} &= 20 - \frac{q_{UK}}{500}. \end{aligned}$$

In the US you would sell q_{US}^* units so that

$$MC(q_{US}^*) = MR_{US}(q_{US}^*),$$

where $MC(q_{US}^*) = 10$, and

$$\begin{aligned} MR_{US}(q) &= \frac{d\left[25q - \frac{q^2}{2{,}000}\right]}{dq} \\ &= 25 - \frac{q}{1{,}000}. \end{aligned}$$

Thus,

$$10 = 25 - \frac{q}{1{,}000}$$

yields

$$q_{US}^* = 15{,}000.$$

In the UK you would sell q_{UK}^* units so that

$$MC(q_{UK}^*) = MR_{UK}(q_{UK}^*),$$

where $MC(q_{UK}^*) = 10$, and

$$\begin{aligned} MR_{UK}(q) &= \frac{d\left[20q - \frac{q^2}{500}\right]}{dq} \\ &= 20 - \frac{q}{250}. \end{aligned}$$

Thus,

$$10 = 20 - \frac{q}{250}$$

yields

$$q_{UK}^* = 2{,}500.$$

Exercise #4. (a) First find the firm's supply curve assuming that the firm will produce some positive amount of output:

$$p = MC(y) = 2y \;\Rightarrow\; y = \frac{p}{2}.$$

The lowest price at which this company will supply a positive amount of product A in the long-run is found by computing this company's profits as function of price p

$$\Pi(p) = p\left(\frac{p}{2}\right) - \left(\frac{p}{2}\right)^2 - 10.$$

The lowest price is p^* such that

$$\begin{aligned}\Pi(p^*) &= 0 \Rightarrow \left(\frac{p^*}{2}\right)^2 = 10 \\ &\Rightarrow p^* = 2\sqrt{10}.\end{aligned}$$

(b) Demand for the product in equilibrium must equal supply. Aggregate supply is given by

$$100y^* = 100\left(\frac{2\sqrt{10}}{2}\right) = 100\sqrt{10}.$$

Part II

Exercise #5. To find the cost function you need to solve the problem

$$\begin{aligned}&\min_{K,L} 10K + 6L \\ \text{s.t. } y &= 10K + L.\end{aligned}$$

Replace the constraint into the objective function, so that the problem becomes

$$\min_{K} 10K + 6(y - 10K).$$

To minimize this expression, it is easy to see that it must be $K^*(y) = y/10$ (notice that $y - 10K \geq 0$). Therefore $L^*(y) = y - 10K^* = 0$. The firm will only use capital in production, and the cost function will be

$$c(y) = 10K^*(y) = y.$$

Part I

Exercise #6. John's problem is

$$\begin{aligned}&\max_{x_b, x_m} (x_b)^{0.2}(x_m)^{0.4} \\ &s.t. \\ 1{,}000 &= p_b x_b + p_m x_m,\end{aligned}$$

where p_b and p_m are respectively the nominal prices of books and restaurant meals. We know that $p_m = 40$, and that $p_b/p_m = 0.4$. Therefore, $p_b = 0.4\,(40) = 16$.

We know that for a Cobb-Douglas utility function, the optimal choices are given by

$$\begin{aligned} x_b^* &= \frac{0.2}{0.2+0.4}\frac{m}{p_b} = \frac{1}{3}\frac{1{,}000}{16} \sim 20.833, \\ x_m^* &= \frac{0.4}{0.2+0.4}\frac{m}{p_m} = \frac{2}{3}\frac{1{,}000}{40} \sim 16.666. \end{aligned}$$

Notice here that, since utility is ordinal, and not cardinal, you can use a positive monotonic transformation to re-write John's utility function as

$$u = (x_b)^{\frac{0.2}{0.6}}\,(x_m)^{\frac{0.4}{0.6}}\,.$$

We know that for this Cobb-Douglas utility function, the expenditure share of a good is given by its exponent in the utility function.

FIGURE 1