Econ 73-250A-F
Spring 2001
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Suggested Answers to Problem Set #6

Please refer to the end of this document for the Figures.

Exercise #1. (a) The optimal level of output is found at the point where marginal cost equal marginal revenue: $MC(y) = MR(y)$. The marginal cost is just 2. Marginal revenue is obtained by taking the derivative of total revenue with respect to y. Total revenue is given by

$$2y\,(11 - y)\,.$$

Thus,

$$MR(y) = \frac{d\,[2y\,(11 - y)]}{dy} = 2(11 - y) - 2y = 2\,(11 - 2y)\,.$$

The optimality condition is

$$2 = 2\,(11 - 2y)\,,$$

which yields $y^* = 5$. The price charged by the monopolist is obtained by replacing this quantity into the inverse demand curve: $p^* = 2\,(11 - y^*) = 12$.

(b) The consumer's surplus is obtained by calculating the area of the triangle ABC in Figure 1: $5\,(22 - 12)\,/2 = 25$. The producer's surplus is represented by the area of the rectangle ACDE in Figure 1: $5\,(12 - 2) = 50$. The total amount of surplus is therefore: $50 + 25 = 75$.

(c) The monopolist is required to set a price equal to marginal cost: $p^{**} = 2$. At this price quantity demanded is: $Q_D(p^{**}) = 11 - \frac{p^{**}}{2} = 10$. The consumer's surplus is represented by the area BDF in Figure 1: $10\,(22 - 2)\,/2 = 100$. The producer's surplus is zero because the monopolist sells the good at a price equal to its production cost. Total surplus is therefore 100, which is larger than 75. The deadweight loss of monopoly is given by the difference between total surplus under competitive pricing and total surplus under monopoly pricing: $100 - 75 = 25$.

Exercise #2. (a) To find the US price, set the marginal cost of producing drug P equal to the marginal revenue in the US:

$$MC(y) = MR_{US}(y),$$

where

$$MR_{US}(y) = \frac{d\,[(1{,}000 - y)\,y]}{dy} = 1{,}000 - 2y.$$

Therefore, the optimal quantity to be sold in the US satisfies

$$1{,}000 - 2y = 100,$$

or $y^*_{US} = 450$. The price charged to US consumers is $p^*_{US} = 1{,}000 - 450 = 550$.

To find the optimal quantity to be sold in Canada find y that solves

$$MC(y) = MR_{CA}(y),$$

where

$$MR_{CA}(y) = \frac{d\left[(500 - y)\, y\right]}{dy} = 500 - 2y.$$

Therefore, the optimal quantity to be sold in Canada satisfies

$$500 - 2y = 100,$$

or $y^*_{CA} = 200$. The price charged to Canadian consumers is $p^*_{CA} = 500 - 200 = 300$.

(b) The profits that Pharmacia makes in the US market are given by

$$\Pi_{US} = 450\,(550 - 100) = 202{,}500.$$

The profits that Pharmacia makes in the Canadian market are given by

$$\Pi_{CA} = 200\,(300 - 100) = 40{,}000.$$

The surplus of US consumers is given by the area ABC in Figure 2:

$$\frac{(1{,}000 - 550)^2}{2} = 101{,}250.$$

The surplus of Canadian consumers is given by the area ABC in Figure 3:

$$\frac{(500 - 300)^2}{2} = 20{,}000.$$

(c) We first need to compute the aggregate demand curve for the drug. First, let's find the demand curves in the two countries:

$$\begin{aligned} y_{US} &= 1{,}000 - p \\ y_{CA} &= 500 - p. \end{aligned}$$

The aggregate demand curve is

$$y = \begin{cases} 1{,}500 - 2p \text{ for } 0 \le p \le 500, \\ 1{,}000 - p \text{ for } 500 \le p \le 1{,}000, \end{cases}.$$

The aggregate inverse demand curve is (see Figure 4).

$$p = \begin{cases} 1{,}000 - y \text{ for } 0 \le y \le 500 \\ 750 - \frac{y}{2} \text{ for } 500 \le y \le 1{,}500 \end{cases}.$$

The marginal revenue curve is given by

$$MR(y) = \begin{cases} 1{,}000 - 2y \text{ for } 0 \le y \le 500 \\ 750 - y \text{ for } 500 \le y \le 1{,}500 \end{cases},$$

and is represented in Figure 4. There are two points at which $MC(y) = MR(y)$. The first corresponds to the case where Pharmacia sets a price that is too high for Canadians, whose demand wold be zero. It is found by setting the marginal cost equal to the first segment of marginal revenue ($0 \leq y \leq 500$):

$$100 = 1{,}000 - 2y$$

and yields the quantity $y_1^* = 450$. The price corresponding to this quantity is $p_1^* = 1{,}000 - y_1^* = 550$. This is just the solution found above for the US. In the other case, Pharmacia can set a price below 500 in order to sell some units of the drug also to Canadian customers. The optimal quantity is found by setting marginal cost equal to the second segment of marginal revenue ($500 \leq y \leq 1{,}500$):

$$750 - y = 100$$

and yields $y_2^* = 650$, with a corresponding price of $p_2^* = 750 - \frac{y_2^*}{2} = 425$. Pharmacia is going to choose option 1 (setting a price $p_1^* = 550$ and not selling anything to Canadians) if this yields higher profits than selling to both US and Canadian customers at a price $p_2^* = 425$. The profits associated with the first option are $450\,(550 - 100) = 202{,}500$, while profits associated with the second option are $650\,(425 - 100) = 211{,}250$. Therefore, Pharmacia will choose the second pricing option $p_2^* = 425$. Figure 4 represents Pharmacia's options.

(d) Pharmacia optimally decides to set a price that allows both US and Canadian consumers to buy the good. This is more convenient than just charging the price 550, which would have excluded the Canadian consumers. The profits of Pharmacia are 211,250, i.e., less than the sum of US and Canadian profits that it used to make when price discriminating between these two markets (242,500). Consumer's surplus in the US is given by

$$\frac{1}{2}\,(1{,}000 - 425)^2 = 165{,}312.5.$$

For Canadian consumers the surplus is:

$$\frac{1}{2}\,(500 - 425)^2 = 2{,}812.5.$$

US consumers are better-off (they enjoy a lower price and will buy more units of the good), while Canadian consumers are worse-off (for the opposite reason).

Exercise #3. (a) A solves:

$$\max_{s,x} \Pi_A\,(s,x) = 10s - 5s^2 - (1-x)^2\,.$$

The first order conditions for profit maximization are

$$\begin{aligned} \frac{\partial \Pi_A\,(s,x)}{\partial s} &= 10 - 10s = 0, \\ \frac{\partial \Pi_A\,(s,x)}{\partial x} &= 2\,(1-x) = 0. \end{aligned}$$

Therefore, $s^* = 1$, and $x^* = 1$. A's profits are

$$\Pi_A(1,1) = 10(1) - 5(1) - 0 = 5.$$

(b) B solves:

$$\max_f \Pi_B(f,1) = f - f^2 - 2(1),$$

where the quantity of pollutants $x^* = 1$ is taken as given by B because determined by A. The first order conditions for profit maximization is

$$\frac{\partial \Pi_B(f,1)}{\partial f} = 1 - 2f = 0.$$

Therefore, $f^* = 0.5$. B's profits are

$$\Pi_B(0.5,1) = 0.5 - 0.5^2 - 2(1) = -1.75.$$

(c) A&B solves:

$$\begin{aligned}\max_{s,x,f} \Pi_{A\&B}(s,f,x) &= \Pi_A(s,x) + \Pi_B(f,x) \\ &= 10s + f - 5s^2 - f^2 - (1-x)^2 - 2x.\end{aligned}$$

The first order conditions for profit maximization are

$$\begin{aligned}\frac{\partial \Pi_{A\&B}(s,f,x)}{\partial s} &= 10 - 10s = 0, \\ \frac{\partial \Pi_{A\&B}(s,f,x)}{\partial f} &= 1 - 2f = 0 \\ \frac{\partial \Pi_A(s,f,x)}{\partial x} &= 2(1-x) - 2 = 0.\end{aligned}$$

Therefore, $s^{**} = 1$, $f^{**} = 0.5$, and $x^{**} = 0$. A&B's profits are

$$\begin{aligned}\Pi_{A\&B}(1,0.5,0) &= 10 + 0.5 - 5 - 0.25 - (1)^2 \\ &= 4.25 \\ &> \Pi_A(1,1) + \Pi_B(0.5,1) \\ &= 5 - 1.75 = 3.25.\end{aligned}$$

FIGURE 1

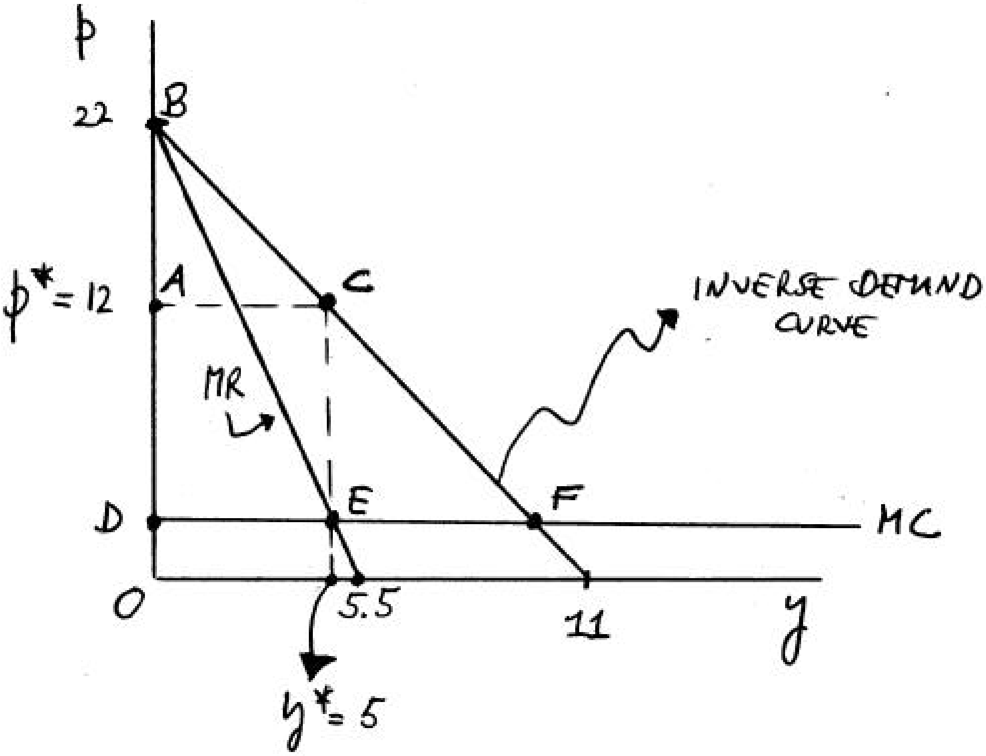


FIGURE 2

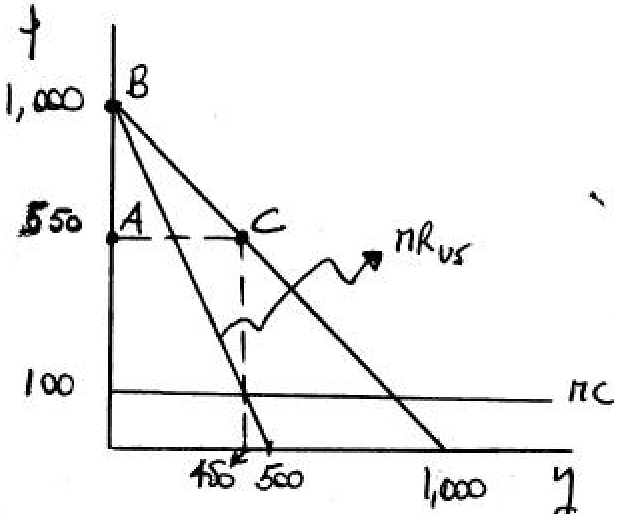


FIGURE 3

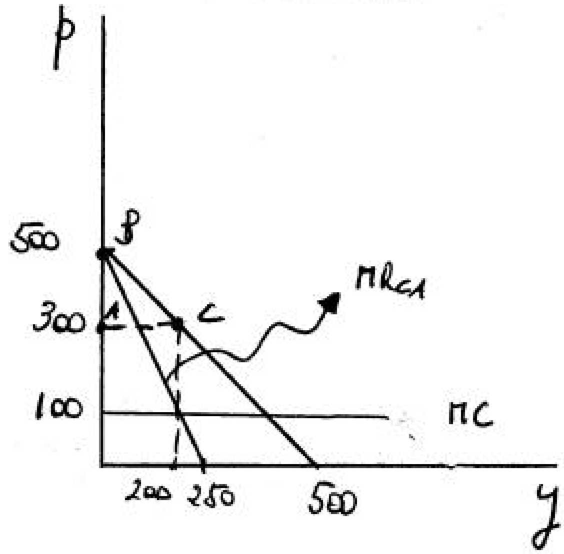
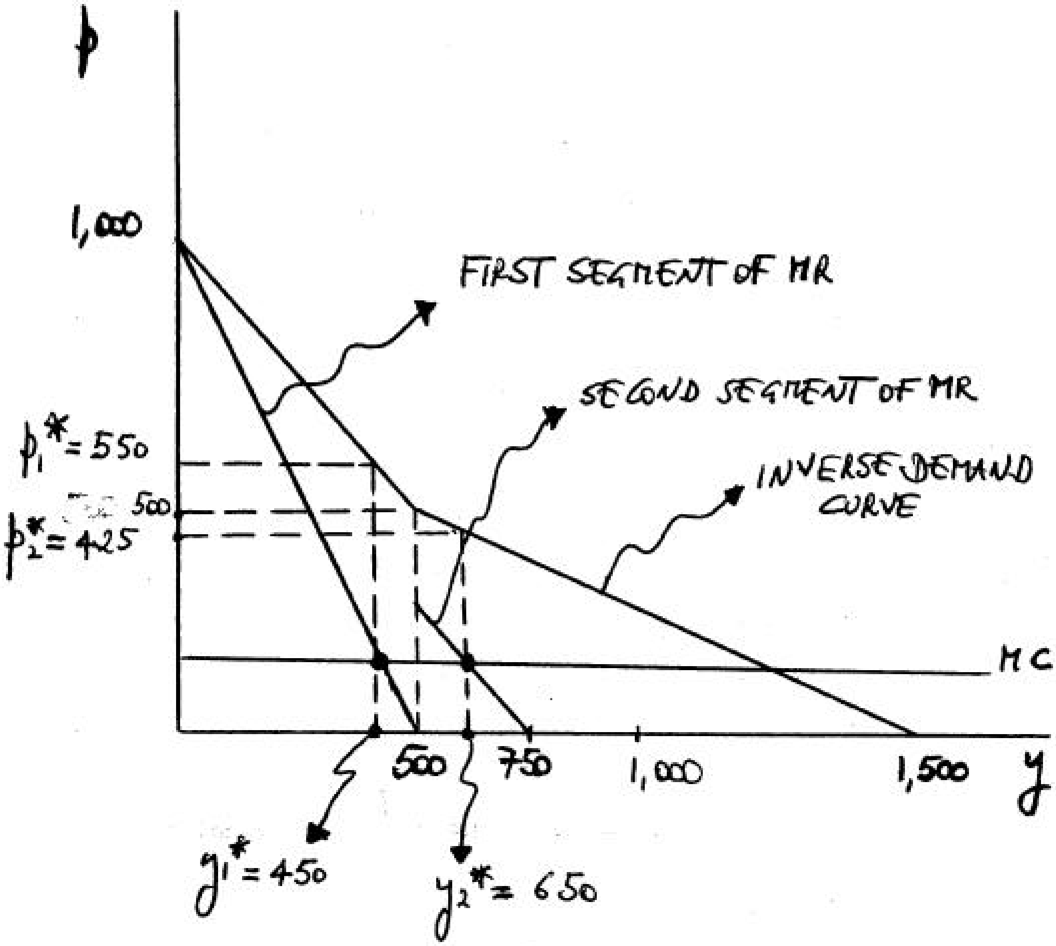


FIGURE 4



p
1,000
FIRST SEGMENT OF MR
SECOND SEGMENT OF MR
$p_1^* = 550$
500
$p_2^* = 425$
INVERSE DEMAND CURVE
MC
500
750
1,000
1,500
y
$y_1^* = 450$
$y_2^* = 650$