Econ 73-250A-F
Spring 2001
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Suggested Answers to Problem Set #5

Please refer to the end of the document for all diagrams.

Exercise #1. (a) The equation of the marginal cost curve is $MC(y) = y/2$. The equation of the average cost curve is $AC(y) = c(y)/y = y/4 + F/y$.

(b) The value of y at which $MC(y) = AC(y)$ is found by setting

$$\frac{y}{2} = \frac{y}{4} + \frac{F}{y}.$$

Solving this equation for y yields $y^* = 2\sqrt{F}$. The value of y which minimizes the firm's average cost satisfies the first-order condition:

$$\frac{d\left[AC(y)\right]}{dy} = \frac{1}{4} - \frac{F}{y^2} = 0.$$

The solution to this equation is the same as above: $y^* = 2\sqrt{F}$.

(c) Since the firm takes prices as given, the firm sets $p = MC(y) = y/2$. Given the price p, the firm will therefore supply $y = 2p$ units of output, provided that is making at least zero profits. If the firm supplies $2p$ units of output, its profits are $py - c(y) = p(2p) - c(2p) = p^2 - F$. The firm's profits are non-negative only if $p^2 - F \geq 0$, i.e., $p \geq \sqrt{F}$. Thus the firm will supply a positive amount of output as long as $p > \sqrt{F}$. If $p = \sqrt{F}$ the firm is indifferent between supplying $2\sqrt{F}$ units of output and shutting down (in which case it supplies zero units of output).

(d) If $F = 25$, the firm will supply a positive amount of output only if $p > \sqrt{25} = 5$. In this case, the firm will supply $2p$ units of output. If $p = 5$, the firm will be indifferent between, on the one hand, supplying $2(5) = 10$ units of output and, on the other hand, shutting down and supplying zero units of output. If $p < 5$, then the firm will supply zero units of output. The firm's supply decision as a function of p is therefore:

$$y = \left\{ \begin{array}{c} 2p \text{ if } p > 5 \\ 0 \text{ or } 10 \text{ if } p = 5 \\ 0 \text{ if } p < 5 \end{array} \right. .$$

See Figure 1.

(e) If $p = 10$, then the firm supplies $2p = 20$ units of output. Its revenues are 10(20)=200, its costs are $20^2/4 + 25 = 125$, and its profits are $200 - 125 = 75$. See Figure 1.

Exercise #2. (a) Each firm sets price equal to marginal cost: $p = MC(y) = 2y$. Solve for y to obtain the supply function of an individual firm: $y = p/2$.

(b) Since all firms in this industry have identical cost functions, industry supply is equal to N times the output of a typical firm, i.e. $N_y = (Np)/2$.

(c) In a long-run equilibrium, all firms produce at the minimum of their average cost curves. The equation of the average cost curve is $AC(y) = c(y)/y = y + 4/y$. Take the derivative of this equation with respect to y and set it equal to zero to find the minimum of the AC curve: $1 - 4y^{-2} = 0$, which implies that $y = 2$. At this level of output, $AC(y) = MC(y) = 4$. Since each firm sets price equal to marginal cost, it must be the case that $p = 4$ in a long-run equilibrium.

When $p = 4$, total demand in the industry is 400-4(4)=384. Since each firm produces 2 units of output, the long-run equilibrium number of firms is 384/2=192. By construction, $p = AC(2)$, so each firm makes zero profits in the long-run equilibrium.

(d) When $N = 192$, the industry supply curve is $(Np)/2 = (192/2)p = 96p$. To find the new (short-run) equilibrium price, set $Q_D(p) = 96p$, or $600 - 4p = 96p$. The solution is $p = 6$. At this price, industry supply is $96(6) = 576$. The supply of a typical firm is therefore $576/192 = 3$. The profits of a typical firm are: $py - c(y) = 6(3) - 3^2 - 4 = 5$. Each firm therefore makes positive profits in the short-run.

(e) Once again, in a long-run equilibrium all firms produce at the minimum of their average cost curves, i.e. where $y = 2$. At this level of output, $AC(y) = MC(y) = 4$, so the long-run equilibrium price is again equal to 4 (since each firm sets price equal to marginal cost). When $p = 4$, industry supply is now $600 - 4(4) = 584$. The total number of firms in the industry is 584/2, or 292. The increase in demand therefore leads $292 - 192 = 100$ new firms to enter the industry. Again, each firm earns zero profits in a long-run equilibrium. As in part (c), revenues of a typical firm are $4(2) = 8$ and costs of a typical firm are $c(2) = 8$.

Exercise #3. (a) If B=100, then $f(B) = 1{,}200{,}000/10 = 120{,}000$. The annual profits of a typical fishing boat are therefore \$120,000 – \$60,000 = \$60,000.

(b) If B remains at 100 indefinitely, then the net present value of the profit stream generated by a typical fishing boat is \$60,000/0.1 = \$600,000.

(c) The annual profits of a typical fishing boat are given by the expression

$$\Pi(B) = f(B) - 60{,}000.$$

If a typical fishing boat is to make zero profits, then the number of boats B must satisfy the zero-profit condition $\Pi(B) = 0$, or

$$\frac{1{,}200{,}000}{B^{1/2}} - 60{,}000 = 0.$$

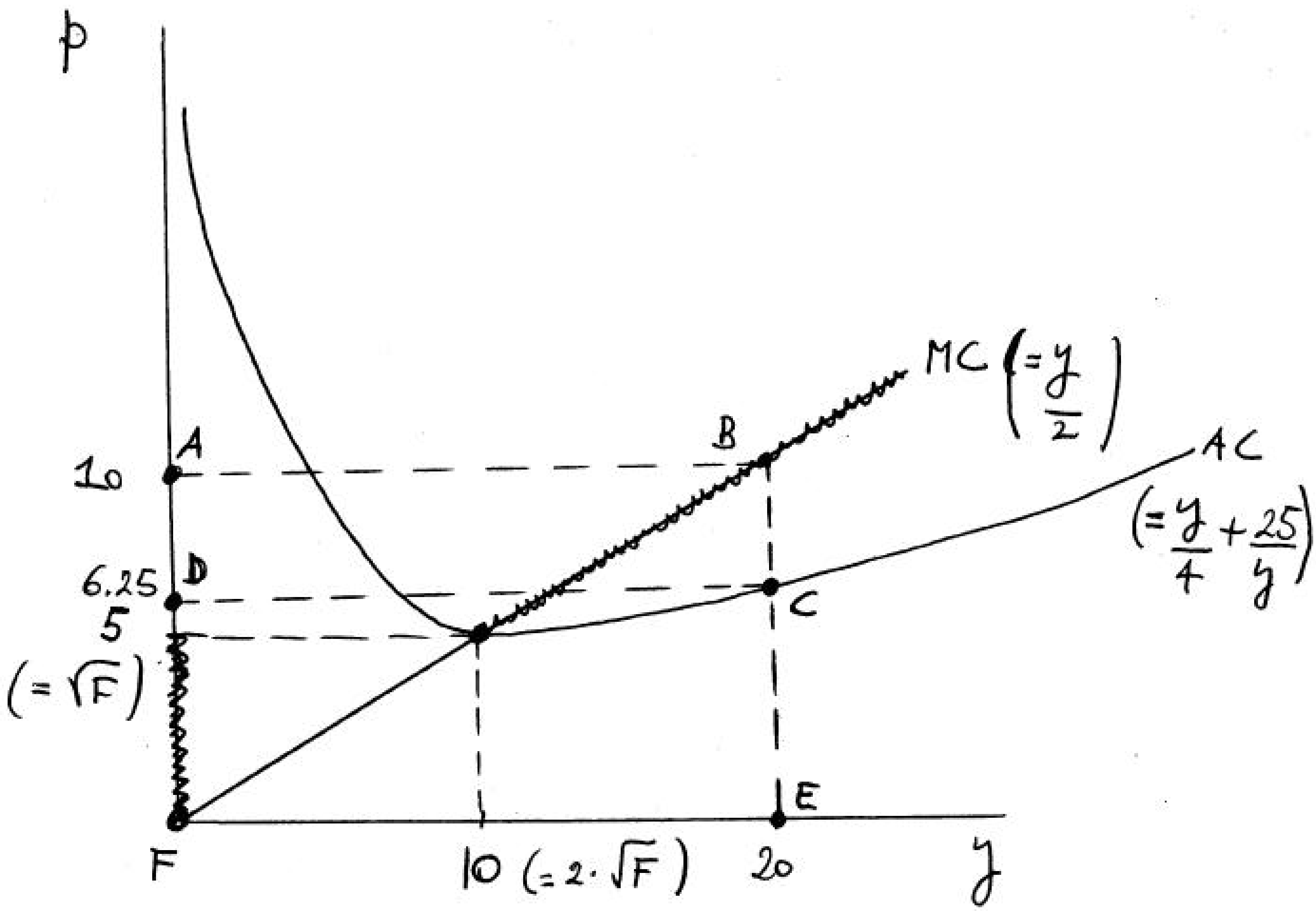
The solution to this equation is $B = 400$. In a long-run equilibrium with free entry and exit, 400 fishing boats will be in operation, each earning zero profits.

(d) If the government issues 144 licenses, and if all 144 licenses are purchased by the owners of fishing boats, then each boat earns annual profits equal to $\Pi(144) = \$40{,}000$. The net present value of the profit stream generated by a typical boat is therefore $40{,}000/0.1 = \$400{,}000$. If a typical boat is to earn zero profits (net of the cost of acquiring a fishing license), then the market price of a fishing license must be exactly equal to the net present value of the profit stream that a fishing boat can generate, or \$400,000. In other words, in order to enter the fishing industry, the owner of a fishing boat must give up (by purchasing a fishing license) exactly the amount of the profits that the fishing boat will earn over the course of its lifetime.

(e) With 225 boats in operation, the annual profits of a typical boat are $\Pi(225) = \$20{,}000$. The net present value of the boat's profit stream is $20{,}000/0.1 = \$200{,}000$. The market price of a fishing license is \$200,000.

(f) Suppose that all 625 fishing licenses are purchased by the owners of fishing boats, so that 625 boats are in operation. The the annual profits of a typical fishing boat are negative: $\Pi(625) = 48{,}000 - 60{,}000 = -\$12{,}000$. This outcome cannot be a long-run equilibrium, since fishing boats earning negative profits will exit the industry. Recall from the answer to part (c) above that in a long-run equilibrium with free entry and exit (and with no fishing licenses), exactly 400 boats will enter the industry, with each boat earning zero profits. Thus 400 is the maximum number of boats that the industry can sustain in long-run equilibrium. When the government issues 625 fishing licenses, the number of licenses therefore does not limit the number of fishing boats in the industry. In other words, the price of a fishing license in this case is zero, which is precisely the net present value of the profit stream generated by a typical fishing boat.

FIGURE 1



∿∿∿∿ = SUPPLY CURVE

ABCD = PROFITS

ABEF = REVENUES

DCEF = COSTS